Review Problems for the Midterm 2

1. Solve the diffusion equation on half-line and express the answer in terms of the error function:

$$u_t = ku_{xx}, \ x \geq 0$$
$$u(0,t) = 1,$$
$$u(x,0) = \begin{cases} 0, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

2. Solve the diffusion equation on the line $(-\infty < x < \infty)$ and express the answer in terms of the error function:

$$u_t = ku_{xx},$$
$$u(x,0) = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$$

3. Solve the wave equation on half–line $(x \geq 0)$:

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = \sin x$$
$$u_t(x,0) = c \cos x$$
$$u(0,t) = t$$

4. Solve the wave equation with source on half–line $(x \geq 0)$:

$$u_{tt} - c^2 u_{xx} = \sin \sqrt{3} \, (x - ct)$$
$$u(x,0) = u_t(x,0) = 0$$
$$u(0,t) = 0$$

5. Expand a function $f(x)$ in:

1. Fourier sine series on $0 < x < l$,

2. Fourier cosine series on $0 < x < l$.

$$f(x) = 1,$$
$$f(x) = x,$$
$$f(x) = x^2$$

6. Solve the heat equation on an interval $0 \le x \le 2$

$$u_t = u_{xx},$$
$$u(x,0) = \sin \pi x + \sin 4\pi x$$
$$u(0,t) = u(2,t) = 0$$

7. Solve the heat equation on the interval $(0 \le x \le 1)$:

$$u_t = 4u_{xx}$$
$$u(x,0) = 1$$
$$u_x(0,t) = u_x(1,t) = 0$$

Hint: You may also guess what the solution will be from physical intuition

8. Solve the wave equation on the interval $(0 \le x \le \pi)$:

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = x$$
$$u_t(x,0) = 1$$
$$u(0,t) = u_x(\pi,t) = 0$$

9. Solve the wave equation on the interval $(0 \le x \le 1)$:

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = x(1-x)$$
$$u_t(x,0) = 0$$
$$u(0,t) = u(1,t) = 0$$