

MTH 537 HW 1

Problem 1. Binary representation

Convert the repeating binary number

$$(101.\overline{01})_2$$

to decimal. Give your answer as an exact fraction and show your work.

Problem 2. Loss of significance in the quadratic formula

Consider the quadratic equation

$$x^2 + x + 10^{-16} = 0.$$

Compute both roots in double precision using the standard quadratic formula.

Investigate the accuracy of the computed roots. Explain any significant difference in their accuracy, and find a way to compute both roots accurately.

Support your conclusions with appropriate numerical results.

Problem 3. Bisection method

Consider the equation

$$x^3 + x - 1 = 0.$$

Starting from the interval

$$[0, 1],$$

carry out the first four iterations of the bisection method by hand.

Then implement the bisection method and use it to approximate the root with accuracy 10^{-10} .

Report the computed root and the number of iterations required. Explain how you determine when the desired accuracy has been achieved, and compare the observed number of iterations with the convergence estimate for the bisection method.

Problem 4. Fixed-point iteration

Consider the following two nonlinear equations:

$$x = \frac{1}{2} \sin x - \frac{1}{\pi} \sqrt{|x+3|}, \quad x = \frac{1}{2} \sin x - 2\sqrt{|x+3|}.$$

Use graphs to obtain reasonable initial guesses for the fixed point iterations.

Do the corresponding fixed-point iterations converge? Why?

For any iteration that does not converge, rearrange the terms in the equation to obtain a fixed-point iteration that is convergent. Justify your choice.