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REPLICATION SECTION

CALCULUS ATTAINMENT AND GRADES RECEIVED IN INTERMEDIATE ECONOMIC THEORY

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SUMMARY

We revisit the work of Butler *et al*. (1998) who examine the effect of mathematical preparation on grades received in intermediate economic theory courses. Using a Bayesian approach under reasonably 'diffuse' priors, we are able to replicate their two-step point estimates almost exactly. We also introduce a new model specification that accounts for the censoring and discrete nature of the outcome variable (grade received). The results from this specification echo the conclusions of the original paper—the level of calculus attained plays an important role in explaining grades received in intermediate micro theory. Copyright © 2006 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In a thoughtful paper of general interest to all economists, Butler *et al*. (1998) take data from a sample of Vanderbilt University students and estimate the impact of mathematical preparation (as quantified by number of calculus courses taken) on grades received in intermediate micro and macro theory. In this study, the authors carefully handled two primary econometric concerns: (1) that calculus attainment should be treated as potentially *endogenous* when trying to extract its effect on intermediate economic theory grades, and (2) that the level of calculus attained should be modelled as an *ordered endogenous* variable.

Though not explicitly written in the paper by Butler *et al*., the authors determined the impact of calculus preparation on intermediate theory grades through a model of the form

$$y_{gi} = x_{gi}\beta_g + \varepsilon_{gi} \tag{1}$$

$$z_{ci} = x_{ci}\beta_c + \varepsilon_{ci} \tag{2}$$

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where y_{gi} denotes the grade received by student i in intermediate micro or macro theory (quantified in terms of GPA for the course) and z_{ci} is an unobserved latent variable that generates the level of calculus attained. We denote the observed calculus level as y_{ci}, and note that indicator functions denoting the various values of y_{ci} are included in x_{gi}. In Butler *et al.*, there are seven possible ordered categories of mathematical preparation (i.e., $y_{ci} \in \{1, 2, \ldots, 7\}$), ranging from a one-semester calculus survey course ($y_{ci} = 1$) up to four semesters of calculus ($y_{ci} = 7$).[1] The set of dummy variable coefficients on these variables in (1) describe the 'structural' impact of calculus preparation on grade received—the primary parameters of interest.

The observed calculus attainment levels y_{ci} are related to the unobserved latent z_{ci} in equation (2) through the function $y_{ci} = j$ if $\gamma_{cj} < z_{ci} \leq \gamma_{cj+1}$, $j = 1, 2, \ldots, 7$.[2] To permit the possible *endogeneity of calculus choice*, it is also assumed that

$$\begin{bmatrix} \varepsilon_{gi} \\ \varepsilon_{ci} \end{bmatrix} \overset{\text{i.i.d.}}{\sim} N \begin{bmatrix} \sigma_g^2 & \sigma_{gc} \\ \sigma_{gc} & 1 \end{bmatrix} \tag{3}$$

Of course, when $\sigma_{gc} \neq 0$, OLS estimates from (1) are biased and inconsistent.

Butler *et al.* estimated the model in (1)–(3) using a two-step approach. In the first step an ordered probit was fit for the calculus attainment equation in (2). Consistent estimates of the cutpoints ($\hat{\gamma}_{cj}$) and regression parameters ($\hat{\beta}_c$) were obtained from this first step of the process. For the second step, the selection-corrected conditional mean functions of (1) can be written as follows:

$$\begin{aligned} E(y_{gi}|x_{gi}, y_{ci} = j) &= x_{gi}\beta_g + E(\varepsilon_{gi}|y_{ci} = j) && (4) \\ &= x_{gi}\beta_g + \sigma_{gc} \frac{\phi[\gamma_{cj} - x_{ci}\beta_c] - \phi[\gamma_{cj+1} - x_{ci}\beta_c]}{\Phi[\gamma_{cj+1} - x_{ci}\beta_c] - \Phi[\gamma_{cj} - x_{ci}\beta_c]} \\ &\equiv x_{gi}\beta_g + \sigma_{gc}\lambda_i && (5) \end{aligned}$$

Using the $\hat{\gamma}_{cj}$ and $\hat{\beta}_c$ estimates obtained from the first-step ordered probit model, Butler *et al.* generate the estimated selection correction term $\hat{\lambda}_i = \hat{\lambda}_i[\hat{\gamma}_c, \hat{\beta}_c, x_{ci}]$ appearing on the right-hand side of (5), and then run a regression of y_{gi} on x_{gi} and $\hat{\lambda}_i$ to obtain consistent estimates of β_g.

Instead of adopting this two-step approach, we chose to replicate the results of Butler *et al.* (1998) using a simulation-based Bayesian algorithm based on the *augmented likelihood* function implied by (1) and (2). To make our results closely comparable to those originally obtained and reported by the authors (i.e., to minimize the influence of the prior on our posterior results), we specify priors that are quite 'flat' and have little information relative to information contained in the data. For the sake of brevity, we do not present our complete set of replication results,[3] but simply note that posterior means of parameters of interest were found to be *virtually identical* to the point estimates reported in tables II (p. 193) and III (p. 195) of Butler *et al.*

[1] The authors also differentiate between calculus courses intended for math or math-oriented science majors and standard calculus courses. See table I, p. 189 of their paper for a complete description of these categories.
[2] For identification purposes, some values of the *cutpoints* $\{\gamma_{cj}\}_{j=1}^{8}$ are restricted as follows: $\gamma_{c1} = -\infty$, $\gamma_{c2} = 0$ and $\gamma_{c8} = \infty$.
[3] These, however, are available upon request.

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2. SENSITIVITY ANALYSIS: AN ORDERED OUTCOME MODEL WITH AN ORDERED ENDOGENOUS VARIABLE

In this section, we consider an extension of the model described in (1) and (2) that was not considered in Butler *et al.* (1998), and seek to determine if key results change within this new model specification. Specifically, we recognize that in addition to the calculus attainment variable y_{ci}, the observed grade outcome y_{gi} could also be treated as a discrete ordered variable (rather than continuous), since grade received can take only one of 12 possible outcomes (A, A−, B+, B, B−, C+, C, C−, D+, D, D−, F). To incorporate this added feature into the model, we introduce a latent variable version of (1) of the form

$$z_{gi} = x_{gi}\beta_g + \tilde{\epsilon}_{gi} \tag{6}$$

and impose $y_{gi} = j$ if $\gamma_{gj} < z_{gi} \leq \gamma_{gj+1}$, $j = 1, 2, \ldots, 12$.[4] Our model for the observed grade and calculus level data is then defined by (2) and (6).

Extending this model to the case where the outcome is also ordered requires some additional thought and care, however, since the nonlinearity induced by the ordered outcome in (6) precludes the two-step estimation approach originally used by the authors in (4) and (5). Fortunately, by simply appending some additional steps to our algorithm that was used to fit the continuous outcome model in (1) and (2), it is possible to handle the ordered nature of the outcome y_g. This algorithm utilizes a *rescaling transformation* to improve the mixing of the posterior simulations and satisfies the ordering restriction on the cutpoints by sampling their *differences* from a Dirichlet proposal density. Finally, blocking steps are employed to jointly sample cutpoints and latent data from their respective equations. Details of this algorithm are not presented here, but they and additional results can be found in Li and Tobias (2005).

In Table I we present posterior means, standard deviations and point estimates of marginal effects from the model treating both y_g and y_c as ordered variables. For the sake of brevity, we focus on parameter estimates associated with grades received in intermediate micro theory, as calculus attainment was found to have a significant effect on grade outcomes in micro (but not macro) theory in Butler *et al.*[5]

Generally speaking, the results reported in Table I are completely consistent with those obtained from the linear outcome model. Key coefficients retain their signs and 'significance' and, most importantly, the number of calculus courses taken remains clearly related to grade outcomes in intermediate micro theory. The only difference worth noting is that our estimated impacts of calculus attainment were found to be slightly smaller than those implied from the linear outcome model. For example, the coefficients on the Math 171A through Math 221B/222 dummies in Butler *et al.* were [0.39, −0.18, 1.02, 1.52, 1.33, 0.75], respectively, while ours are [0.27, −0.3, 0.68, 0.92, 0.84, 0.5]. This reduction seems reasonable given that our specification formally imposes a ceiling and a floor on the grade outcome, and thus compresses the possible impact of changes in covariates. It is most important to note, however, that substantive results were not changed when considering this generalized model specification.

[4] Again, we must impose restrictions on some cutpoints, namely $\gamma_{g1} = -\infty$, $\gamma_{g2} = 0$ and $\gamma_{g13} = \infty$.

[5] Like Butler *et al.*, we found no evidence that calculus attainment played a significant role in intermediate macro theory grades. Table I presents coefficient and marginal effect estimates from (6) only—parameter estimates from (2) were found to be similar to those reported in Butler *et al.* and are available upon request.

Table I. Ordered probit with endogenous covariates: grade in Micro-2 course

Variable	$E(\beta \mid D)$	$Std(\beta \mid D)$	$P(\beta > 0 \mid D)$	Marginal effect[a]
Constant	−3.69	0.711	0	
Level of calculus attained				
Math 171A	0.461	0.559	0.797	0.273
Math 172A	−0.257	1.28	0.422	−0.296
Math 171B	1.17	0.434	0.996	0.68
Math 172B	1.76	0.647	0.997	0.918
Math 221A	1.66	0.874	0.973	0.841
Math 221B or 222	0.853	0.668	0.901	0.497
Grade in last calculus course				
Math 170	0.444	0.123	1	0.284
Math 171A	0.329	0.176	0.969	0.213
Math 172A	0.616	0.41	0.933	0.379
Math 171B	0.208	0.0854	0.993	0.136
Math 172B	0.122	0.19	0.74	0.0778
Math 221A	−0.104	0.247	0.334	−0.0762
Math 221B or 222	0.21	0.155	0.912	0.135
Grade deflator: Micro-2	1.29	0.165	1	0.751
Taken in Sophomore year	0.116	0.114	0.846	0.0767
Taken in Senior year	−0.0635	0.183	0.36	−0.047
Timing of Micro-1 and Micro-2				
In same academic year	0.0228	0.125	0.573	0.014
At least one semester gap	0.202	0.109	0.967	0.131
Grade in Macro-1	0.294	0.0771	1	0.191
Grade in Micro-1	0.45	0.0744	1	0.288
Grade deflator: Macro-1	−0.446	0.227	0.0233	−0.322
Grade deflator: Micro-1	−0.217	0.316	0.244	−0.157
Class size: Micro-2	−0.00302	0.00819	0.358	−0.00205
Freshman GPA	0.439	0.146	0.999	0.28
Female	0.181	0.0929	0.972	0.119
SAT-math $\times 10^{-2}$	0.123	0.0981	0.894	0.0806
SAT-verbal $\times 10^{-2}$	0.05	0.0699	0.762	0.0328
Correlation[b]				
ρ_{gc}	0.0817	0.103	0.789	

[a] To calculate the marginal effect from a one-unit increase in any control variable x_g, we calculate the probability of each discrete grade outcome based on (6) with x_g evaluated at its original level, and this value plus one, respectively, holding other covariates constant. The GPAs associated with each discrete grade outcome are then multiplied by their corresponding probabilities and summed together. Marginal effects are constructed as differences between the two average GPA values.

[b] The parameter characterizes the correlation between the unobservables in equations (2) and (6) and thus quantifies the degree of selectivity in the model.

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