ABSTRACT
Biometric authentication eliminates the need for users to remember secrets and serves as a convenient mechanism for user authentication. Traditional implementations of biometric-based authentication store sensitive user biometry on the server and the server becomes an attractive target of attack and a source of large-scale unintended disclosure of biometric data. To mitigate the problem, we can resort to privacy-preserving computation and store only protected biometrics on the server. While a variety of secure computation techniques is available, our analysis of privacy-preserving biometric authentication constructions revealed that available solutions fall short of addressing the challenges of privacy-preserving biometric authentication. Thus, in this work we put forward new constructions to address the challenges.

Our solutions employ a helper server and use strong threat models, where a client is always assumed to be malicious, while the helper server can be semi-honest or malicious. We also determined that standard secure multi-party computation definitions are insufficient to properly demonstrate security in the two-phase (enrollment and authentication) entity authentication application. We thus extend the model and formally show security in the multi-phase setting, where information can flow from one phase to another and the set of participants can change between the phases. We implement our constructions and show that they exhibit practical performance for authentication in real time.

CCS CONCEPTS
• Security and privacy → Privacy-preserving protocols; Biometrics.

KEYWORDS
secure computation, biometric authentication, multi-phase secure execution, garbled circuit evaluation, oblivious transfer

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1 INTRODUCTION
Biometric-based authentication provides a convenient user authentication mechanism which does not require users to remember passwords or maintain other secrets. Biometric-based authentication is also now more easily accessible than before to the average user for a variety of application due to proliferation of smartphones equipped with sufficient sensors. Biometric data, however, requires strong protection because, unlike password-based authentication, biometry cannot be replaced if the data becomes compromised. Enhancing protection of biometric data used in biometric-based authentication is the focus of this work.

We consider the problem of privacy-preserving biometric authentication in a system where users authenticate to a server using their biometric data, but the authentication server does not have access to the users’ biometric data in the clear. If the information stored on the server does not allow one to recover user’s biometric samples, user biometric data cannot be easily abused by insiders or through computer break-ins. Large-scale leakage of sensitive biometric data is of growing concern due to increasing availability of large-scale biometric data sets. Thus, this work targets designing a robust and practical solution to privacy-preserving biometric-based authentication which can be employed in place of traditional biometric-based authentication mechanisms and which makes abuse of sensitive biometric data more difficult.

In the context of privacy-preserving biometric-based authentication, we can consider two types of solutions: (i) those based on secure sketches and fuzzy extractors and (ii) solutions based on secure multi-party computation (SMPC). The former has a disadvantage that it discloses partial information about each biometric sample, the implications of which are hard to quantify, and we focus on the latter that can guarantee that no biometric-related information of a user is disclosed to any party.

Now if we consider secure two-party computation between a user and an authentication server, we can distinguish between two types based on the amount of interaction: (i) interactive two-party computation where the user carries the full burden of participating in secure evaluation of biometric matching and (ii) non-interactive computation on encrypted data. Note that in the context of user authentication, we must assume that a user can act maliciously in the attempt to circumvent the authentication mechanism and obtain access to the system at any cost. This means that when modeling SMPC, we must provide security in the presence of malicious users, which increases the protocols’ cost. This is undesirable for clients operating from computationally-limited battery-powered
devices and thus may present usability concerns. On the other hand, non-interactive computation that employs fully homomorphic encryption (HE) permits comparing an encrypted biometric sample provided by the user at authentication time and the encrypted sample captured at the enrollment time. A concern with this solution is that, in order for the user biometric data to stay private from the server, the decryption key must be available only to the client. This means that it is not possible for a client to enroll once and later be able to authenticate from any computer or device because each device has to have the user’s private decryption key. This nullifies the advantages of biometric-based authentication which permits authentication without the need to remember passwords or maintain secret keys and brings us back to requiring the user to use secrets together with their biometry.

To mitigate these issues, our approach is to introduce a helper server. This is not a new idea by itself, but it makes a significant difference for this application. The helper server does not contribute any inputs and does not learn any information about user biometric data or the result of user authentication, but rather contributes its computational power and can store protected biometric data. Multiple authentication servers can use the same helper server. This setup improves both usability and efficiency, as we demonstrate in this work. In particular, that expands the set of techniques we can use for privacy-preserving biometric matching and authentication and consequently aids efficiency. It also permits minimal involvement of users and removes the need for storing any keys or other secrets on user devices. This improves usability and enables a user to authenticate from different devices and a variety of platforms including weak battery-powered devices.

An interesting aspect of this work is that we found that employing traditional SMPC security definitions for (privacy-preserving) authentication is insufficient and there is a need for new definitions. In particular, SMPC is concerned with a single evaluation of a function, during which the set of participants does not change. In the context of authentication, on the other hand, we deal with two phases: enrollment and authentication. Furthermore, the participants themselves can change because a malicious user might attempt to impersonate another user during authentication (while enrollment was carried out by the authentic user). While we can use traditional security definitions to ensure that the participants do not learn unauthorized information during a given phase, there is still a need to link the two phases together and ensure that no biometric-based information is available to the participants as a result of information flow from one phase to another. This is because the servers will obtain certain output after the enrollment phrase, but the output of function evaluation is never protected under the standard definition and is not treated as leakage. Thus, in this application we need to consider the overall view of the two-step process, conceptually treating the output of the first stage as an intermediate result (which must reveal no information) and not as the target output (which is allowed to reveal information). This will also permit us to demonstrate that a malicious user is unable to learn sensitive biometric data of any enrolled user.

We determined that a few prior publications that treat the topic of biometric-based authentication [1, 3, 21] use a two-phase model; however, the definitions have custom interfaces and are not applicable to other functionalities. We provide a more detailed comparison in the related work section.

Our solution is based on garbled circuit evaluation (GCE) [36] and we use two strong threat models, in both of which the client is malicious and can behave arbitrary. In the weaker model, the server is semi-honest (follow the prescribed protocols) and do not collude with each other or the clients. In the stronger model, the helper server can act maliciously and can additionally collude with clients. When building our constructions, we introduce a variant of oblivious transfer (OT), termed oblivious transfer with bit operations (OTB), which may be of independent interest, and consider over-the-threshold cosine similarity and Euclidean distance as the basis for biometric matching. We formally prove security of our solutions under standard security definitions, expanded as discussed above to accommodate multi-phase computation where the participants can change between the phases. We also implement and empirically evaluate performance of our solutions and show that they are well suited for authentication in real time.

**Related Work.** The first line of work that employ cryptography to protect confidentiality of biometric data during matching uses secure sketches and fuzzy extractors, e.g., [10, 24, 28, 30, 35], some of which make use of an additional secret or password to improve the properties of the solution. The second line of research—closer to this work—uses SMPC or secure outsourcing. Constructions for different biometric modalities have been developed. For instance, they include face [15, 20, 31, 33], iris [11, 12], fingerprints [7, 12, 14, 19], voice [4], and others. The computation itself widely differs in the complexity, ranging from simple Hamming or Euclidean distance over integers to hidden Markov model evaluation on floating-point values. A variety of techniques have been used including GCE, secret sharing, encryption with special properties (e.g., HE and predicate encryption), and a combination thereof. Most results above above focused on privacy-preserving biometric matching or identification in the semi-honest security setting. Authentication, however, demands a stronger security model in which clients must be assumed to be malicious.

Publications that treat privacy-preserving biometric authentication in the presence of a malicious client include [1, 2, 17, 22, 34]. Several of them use HE to perform a simple distance computation and disclose it to one of the parties. For instance, in [16, 17, 22] the server computes $\text{dist} \cdot r_0 + r_1$, where $\text{dist}$ is (Hamming or Euclidean) distance between the enrollment and current biometric samples and $r_0, r_1$ are large random values. The use of the randomizing values prevents a malicious client from making meaningful changes to the distance prior to sending it to the server. This structure has two disadvantages: (i) each client has to maintain a secret key on each device he/she wants to use for authentication (which our solution is set to mitigate) and (ii) the computed distance is revealed to one of the parties, commonly the server who can compile distributions of this information for each user over time or, worse, to the (malicious) client who can use the distance as the guide for improving

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1 In addition, [29, 37] are also said to provide privacy-preserving authentication. However, in the solution of [37] an authority obtains cleartext access to user biometrics and thus does not achieve privacy, while in [29] the client is considered fully trusted and consequently the construction does not correspond to authentication.
its strategy for impersonating the authentic user. [34] employs HE in their semi-honest protocol and GCE when the client can be malicious. In both cases, the client has to maintain a secret key or another state. Further, none of these solutions connect enrollment and authentication to protect enrollment data from malicious users.

HE together with digital signatures, GCE, and zero-knowledge proofs are used in [2] for evaluating cosine similarity as a distance metric. While it does treat malicious adversaries, active tampering is limited to client devices only, and the remaining participants are semi-honest. Specifically, these participants are service providers and terminals, where a terminal is an external device outside client control which obtains the authentication phase biometric. This setting is equivalent to our weaker (non-colluding) security model. Both secret keys and encoded templates are stored on the devices, which can reveal non-revocable template information to adversaries with access to the device. Two of their constructions also leak the information that the server handles. This includes information that the server is interested in extracting biometric information about the users from different services. Security requirements are such that $S_1$ has no access to sensitive biometric information about any user $C$ and only determines the outcome of each authentication (i.e., whether the supplied biometric was a close match and is considered authentic using the over-the-threshold computation described above). $S_2$ learns no information about any biometrics and no information about authentication outcomes, i.e., its purpose is to improve efficiency and usability of the protocols for the client and the service.

Because we utilize a helper server for usability and efficiency reasons, we denote the main authentication server as $S_1$ and a helper server as $S_2$ (recall that the same helper server can be employed by different services). Security requirements are such that if $S_1$ is the authentication server, it must properly enforce access control and correctly perform the computation (as otherwise no meaningful guarantees can be maintained in the presence of a malicious client). However, someone with access to the server (e.g., a dishonest insider or in the case of a computer break-in) might be interested in extracting biometric information about the users from the information that the server handles. This includes information stored at the server and the server’s view during all registration and authentication protocols. For that reason, we begin with a model of the servers being semi-honest and non-colluding.

In addition, because the helper server $S_2$ is not controlled by the service and may not be as trustworthy, we consider the possibility of $S_2$ behaving maliciously. For that reason, we consider a stronger security model, in which $S_1$ remains to be semi-honest and non-colluding with other parties, while $S_2$ can be malicious and possibly colluding with clients $C$ (who are always assumed to be malicious). This stronger model has implications on the cost of the protocols in order to maintain security.

### 2 Preliminaries

#### 2.1 Problem Statement

We consider a setting where a client $C$ uses a service that employs biometric data for entity authentication. At the enrollment time, the client registers with the service, which involves $C$ capturing its biometric sample, extracting features to produce representation $B$, and storing the result in a privacy-preserving way with the service. At the time of authentication, the client captures a new biometric sample and produces representation $\hat{B}$, after which the client and the service engage in a protocol. As a result, the client is either authenticated and gains access to the service or is denied access. The computation involves performing biometric matching by first computing the distance between the enrollment and current biometric samples, $d = \text{dist}(B, \hat{B})$, and consequently comparing the distance to a predefined threshold $t$.

Because we utilize a helper server for usability and efficiency reasons, we denote the main authentication server as $S_1$ and a helper server as $S_2$ (recall that the same helper server can be employed by different services). Security requirements are such that $S_1$ has no access to sensitive biometric information about any user $C$ and only determines the outcome of each authentication (i.e., whether the supplied biometric was a close match and is considered authentic using the over-the-threshold computation described above). $S_2$ learns no information about any biometrics and no information about authentication outcomes, i.e., its purpose is to improve efficiency and usability of the protocols for the client and the service.

Because we work with authentication, we must assume that the client is malicious, i.e., it will try all means at its disposal in the attempt to successfully authenticate without sufficient credentials. The servers, on the other hand, can be more trustworthy and can be expected to follow the prescribed computation. In particular, because $S_1$ is the authentication server, it must properly enforce access control and correctly perform the computation (as otherwise no meaningful guarantees can be maintained in the presence of a malicious client). However, someone with access to the server (e.g., a dishonest insider or in the case of a computer break-in) might be interested in extracting biometric information about the users from the information that the server handles. This includes information stored at the server and the server’s view during all registration and authentication protocols. For that reason, we begin with a model of the servers being semi-honest and non-colluding.

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#### 2.2 Security Definitions

We use the standard formulation of security that relies on the real/ideal paradigm in the presence of malicious adversaries and guarantees correctness and no unintended information disclosure. The definition requires that the view of any adversary in real protocol execution is computationally indistinguishable from its view.
in an ideal world execution, where an ideal functionality produces the output and parties not controlled by the adversary are not participating in the computation.

In a general setup, let parties $P_1, \ldots, P_n$ engage in a secure multiparty protocol $\Pi$ that computes function $f$. We specify $f$ as taking $n$ inputs $x_1, \ldots, x_n$ and producing $n$ outputs $y_1, \ldots, y_n$, i.e., $f(x_1, \ldots, x_n) = (y_1, \ldots, y_n)$. Each $x_i$ and $y_i$ is treated as a vector to permit entering and receiving multiple values, but some participants may not provide any inputs and/or receive no output (in which case the corresponding $x_i$ and/or $y_i$ is empty).

Adversary $A$ is permitted to corrupt one or more participants based on the threat model. The remaining parties are honest and denoted by $H$. We let $\text{VIEW}_{\Pi, A}$ denote the view of adversary $A$ after an execution of $\Pi$. The view is the union of the views of the parties controlled by $A$, which include their inputs, randomness used during the computation, and all messages received during the computation from other participants. We also let $\text{OUT}_{\Pi, H}$ denote the output of the honest parties after the execution, i.e., the produced $y_i$ that correspond to the honest parties. Let $\kappa$ denote a security parameter and define

$$\text{REAL}_{\Pi, A}(1^{\kappa}, (x_i)_{i=1}^n) \overset{\$}{=} \text{VIEW}_{\Pi, A} \cup \text{OUT}_{\Pi, H}$$

In the ideal world, there is no protocol execution and instead a probabilistic polynomial time (PPT) simulator $S$ interacts with $A$. The simulator is able to query an ideal functionality $F$ which computes function $f$ on behalf of the participants and the goal is to simulate $\Pi$’s execution without access to the data of non-corrupt participants. As before, the view of $A$ corresponds to the inputs, random choices, and the messages received by the parties controlled by $A$ during the simulation, which we denote by $\text{VIEW}_f, A$.

The ideal functionality evaluates function $f$ on behalf of the participants. It uses inputs of honest participants and obtains inputs of corrupt participants from $S$. When $A$ is semi-honest, $S$ obtains access to inputs of the corrupt parties controlled by $A$ and supplies them to $F$. When $A$ is malicious, it can instruct the parties it controls to deviate from the prescribed computation and enter their inputs into the computation in a different form. Thus, it is $S$’s task to extract the corrupt parties’ inputs the way they are entered into the computation and communicate the inputs to $F$, who will evaluate the function using the supplied inputs. Note that $S$ or $F$ can abort if either of them obtain empty or malformed inputs or messages. If the evaluation is successful, the parties obtain the output of $f$, and we denote the output of honest parties by $\text{OUT}_f, H$.

Similar to the real execution, we define

$$\text{IDEAL}_{f, S}(\mathcal{A})(1^{\kappa}, (x_i)_{i=1}^n) \overset{\$}{=} \text{VIEW}_f, A \cup \text{OUT}_f, H$$

Given above, we formulate the security definition as:

**Definition 1.** An $n$-party protocol $\Pi$ between $P_1, \ldots, P_n$ securely evaluates function $f$ if for all PPT adversaries $A$ controlling a subset of the participants, all input vectors $x_i$, and $\kappa \in \mathbb{Z}$, there exists a PPT simulator $S$ such that

$$\text{REAL}_{\Pi, A}(1^{\kappa}, (x_i)_{i=1}^n) \overset{c}{=}= \text{IDEAL}_{f, S}(\mathcal{A})(1^{\kappa}, (x_i)_{i=1}^n)$$

where $c$ denotes computational indistinguishability.

Because in our context information produced in one phase of the computation is used as input into another phase, we extend the standard definition to support multi-stage computation. For simplicity, we consider computation consisting of two phases, but the concept easily generalizes to any number of phases. To accomplish this, we define the outputs of the first phase to be additional, auxiliary inputs $u_i$ (which may be empty) into the second phase. Conceptually, this can be pictured as we pause after the first phase, save the output as the current state, and resume the computation once the inputs into the second phase are received. Note that each phase receives inputs from the parties and the second phase additionally receives the outputs from the first phase in the form of auxiliary inputs.

It is important to take into account that the participating parties might change between the phases of the computation. This is the case for authentication applications, where a malicious user (imposter) might attempt to authenticate impersonating another user who previously enrolled in the system (authentic user). For that reason, we define two different, overlapping sets of participants $p_1^{(1)}, p_2^{(1)}, \ldots, p_{n_1}^{(1)}$ and $p_1^{(2)}, p_2^{(2)}, \ldots, p_{n_2}^{(2)}$. Here superscript $(j)$ denote data associated with phase $j$ and $n_1$ (respectively, $n_2$) denote the number of participants in the first (resp., second) phase.

If $p_i^{(1)} = p_j^{(2)}$ for some $i$ and $j$, i.e., the party is involved in both phases, then it will have an auxiliary input for the second phase. The remaining participants, i.e., those who are involved only in one of the phases, contribute their input and receive the output as in the conventional formulation of an execution.

The more complex participant structure requires that we also carefully specify adversarial corruptions. If a party is controlled by an adversary, the adversary controls it in both stages of the computation. If an adversary controls multiple conspiring participants, it will control them in all phases in which the parties are active protocol participants.

Let $f$ denote the multi-phase functionality and $f^{(1)}$ and $f^{(2)}$ denoted the functions we evaluate in phases 1 and 2, respectively. The auxiliary input is set for each $p_i^{(1)}$ as $u_i^{(1)} = y_j^{(1)}$ if $p_i^{(1)} = p_j^{(1)}$ for some $j$ and $u_i^{(2)}$ is empty if $p_i^{(2)}$ was not a protocol participant in phase 1. Given this, we define real and ideal views in the second (or any subsequent) phase of the computation as

$$\text{REAL}_{\Pi^{(2)}, A}(1^{\kappa}, (x_i^{(2)}, u_i^{(2)})_{i=1}^{n_2}) \overset{\$}{=} \text{VIEW}_{\Pi^{(2)}, A} \cup \text{OUT}_{\Pi^{(2)}, H}$$

and

$$\text{IDEAL}_{f^{(2)}, S}(\mathcal{A})(1^{\kappa}, (x_i^{(2)}, u_i^{(2)})_{i=1}^{n_2}) \overset{\$}{=} \text{VIEW}_f^{(2)}, A \cup \text{OUT}_f^{(2)}, H$$

**Definition 2.** A sequence of two protocols $\Pi^{(1)}$ and $\Pi^{(2)}$, executed by parties $p_1^{(1)}, \ldots, p_{n_1}^{(1)}$ and $p_1^{(2)}, \ldots, p_{n_2}^{(2)}$, respectively, securely evaluates the sequence of functions $f^{(1)}$ and $f^{(2)}$ if for all PPT adversaries $A$ controlling a subset of the parties, all input vectors $x_i^{(1)}$, $1 \leq i \leq n_1$, and $x_i^{(2)}$, $1 \leq i \leq n_2$, all auxiliary input vectors $u_i^{(2)} = y_j^{(1)}$ subject to $p_i^{(2)} = p_j^{(1)}$, and $\kappa \in \mathbb{Z}$, there exists PPT simulator $S$ such that

$$\text{REAL}_{\Pi^{(1)}, A}(1^{\kappa}, (x_i^{(1)})_{i=1}^{n_1}) \overset{\$}{=} \text{IDEAL}_{f^{(1)}, S}(\mathcal{A})(1^{\kappa}, (x_i^{(1)})_{i=1}^{n_1})$$

and

$$\text{REAL}_{\Pi^{(2)}, A}(1^{\kappa}, (x_i^{(2)}, u_i^{(2)})_{i=1}^{n_2}) \overset{\$}{=} \text{IDEAL}_{f^{(2)}, S}(\mathcal{A})(1^{\kappa}, (x_i^{(2)}, u_i^{(2)})_{i=1}^{n_2})$$
For the purposes of this work, the computation participants are C, S1, and S2, i.e., we are dealing with three-party computation. As described earlier, we consider two threat models:

1. The minimal meaningful security model that treats C as malicious and S1 and S2 as semi-honest and non-colluding. For the purposes of showing security, this means that A can corrupt one party at a time with the specified semi-honest/malicious abilities and our solutions need to be secure for each instantiation of A.

2. A stronger security model in which, in addition to malicious client C, helper server S2 can behave maliciously and collude with some clients. Recall that it is not meaningful to assume that S1 is malicious in the context of this application, and S1 also does not collude with other parties. This means that A has two instantiations: semi-honest S1 and malicious and colluding C and S2.

The user who participates in the registration is called authentic Cauth. The same or a different user might attempt to authenticate with E. For the purposes of this work, the computation participants are (2) A stronger security model in which, in addition to malicious and 1, S2.

2.3 Building Blocks

In this work, we use the following cryptographic primitives:

- **Oblivious Transfer (OT)** is a protocol between two parties, sender S and receiver R. In 1-out-of-2 OT; OT2[1], the sender holds two strings, m0 and m1, while the receiver holds bit b and learns m_b. The security requirements are that the sender learn nothing about b, while the receiver learns nothing about the remaining string m_1-b.

- Additionally, we employ a new (to the best of our knowledge) generalization of OT, which we refer to as **Oblivious Transfer with Bit Operations (OTB)**. In this setting, the sender additionally holds an input bit c, and the receiver obtains m_0 XOR c, where ⊕ is a previously agreed upon boolean binary function. Details are provided in Section 2.3.1.

OT extensions are commonly used for efficiency when multiple calls to OT are needed. OT and OT extensions take a computational security parameter k, and constructions secure in the malicious model can also rely on a statistical security parameter ρ.

- **Garbled circuit (GC) evaluation** is a secure two-party protocol parameterized by computational security parameter k that evaluates some function f, represented as a boolean circuit, on private inputs. One party, garbler G, chooses two random labels l_i[1], l_i[2] to represent each (boolean) wire i in the circuit. For each binary gate of the circuit, G derives an encryption key from each of the four possible input wire label pairs and uses these to encrypt the label of the corresponding output wire. This collection of per-gate tables constitutes the garbled circuit G_f. The other party, evaluator E, receives from G both G_f and the set of input wire labels corresponding to G’s input values (which are required to not reveal anything about the input they represent). E then engages in an OT^2 protocol with G to obtain the wire labels corresponding to E’s input values. Finally, E evaluates the circuit gates beginning with the input labels and obtains the final output label(s). At the end of the protocol, E sends the corresponding output label(s) to G (which necessarily reveals the actual output to G). For the construction to comply with the security definition, it must be the case that
- G and E learn nothing about each others’ input and
- G learns the function output.

The literature contains a number of well known optimizations to the original Yao construction [36]. This includes the use of the “free XOR” gates introduced in [26] which imposes a certain relationship between the two labels corresponding to a wire, namely, that l_i[1] XOR l_i[2] = Δ for each wire i. The labels are also commonly generated as pseudorandom strings. In our implementation discussed in Section 4, we use garbling as in the JustGarble work [9]. The conventional variant of GC for semi-honest adversaries provides resilience against malicious evaluators, as long as the appropriate variant of OT is used. We do not require a strengthened variant secure against malicious adversaries, since within our protocols it is possible to arrange for the circuit garbler to be semi-honest.

- A commitment scheme is parameterized by a security parameter k and characterized by two algorithms, commit and open. The commit algorithm is randomized and denoted by c = com(x, r), where x is the value being committed and r is randomness specified explicitly. We call c to be a commitment to x. Commitment c can later be opened (typically by revealing x and r), which exposes the value of x. The security requirements are hiding and binding properties of the commitment scheme. Namely, hiding requires that the release of c does not disclose information about x and binding requires that it is infeasible to open a commitment c to any value other than the value x used to produce the commitment. The security guarantees can be information-theoretic or computational.

2.3.1 Oblivious Transfer with Bit Operations. This generalization of OT works with any already proven secure OT scheme. Here, the parties agree upon a binary boolean function, denoted as ⊕ : F_2 x F_2 → F_2. In addition to the sender holding messages m_0 and m_1 and the receiver holding bit b, the sender now also holds an input bit c. Then the receiver obtains m_0 XOR c without learning anything else, while the sender learns nothing about receiver’s input b.

This operation is realized using regular OT, where instead of entering (m_0, m_1), the sender enters (m_0, m_1) specified as follows for the three most common binary boolean operations:

- **AND**: the sender sets m_0 = m_0 and m_1 = m_1
- **OR**: the sender sets m_0 = m_0 and m_1 = m_1
- **XOR**: the sender sets m_0 = m_0 and m_1 = m_1

In terms of correctness, notice that in the case of AND, if the sender holds c = 0, then the receiver obtains m_0 regardless of their input (b XOR 0 = b), and m_1 otherwise (b XOR 1 = b). Similarly, in the case of OR, if the sender holds c = 1, then the receiver obtains m_1 regardless of their input and m_0 otherwise. And in the case of XOR, b XOR c = c if and only if b = 0, while b XOR c = ¬c = c XOR 1 if and only if b = 1.
In terms of security, nothing in this modification allows the sender to learn \( b \) if the OT being used already predicts this (although the sender may know which string the receiver gets; this is a function of the sender’s input). Similarly, the receiver receives exactly one string \( m_{b \in \sigma} \) without learning \( m_{c \notin \sigma} \) and does not learn anything about \( c \) (but the receiver may know they are getting \( m_b \) based on their input and the function \( \sigma \) being computed).

We will be using this OT variant while transferring GC labels that correspond to an XOR-share of the clients’ private biometric data and denote it as OTB.

### 2.4 DeepPrint Fingerprint Matching

We are interested in supporting authentication based on popular biometric modalities with good distinguishing properties such as fingerprints and iris codes. Iris codes are represented as binary strings and their matching is based on the Hamming distance. As a result, iris matching does not introduce significant complexity. On the other hand, conventional minutiae-based comparison of fingerprints is complex and not well suited for use in secure computation.

For this reason, DeepPrint [18] that uses deep neural network for fingerprint feature selection with excellent discriminating properties is of interest to us. The resulting fingerprint representations are fixed length and can be compared using simple conventional distance metrics, making it easier to use the representation with cryptographic tools.

DeepPrint encodes a fingerprint biometric as a vector of 192 single-precision floating-point values, which is normalized to be unit length. A unit-length vector is defined as having its \( L^2 \) norm, denoted by \( \|B\| \) for a biometric vector \( B \), equal to 1. Concretely, for vector \( B = (B[i])_{i=1}^{w} \) it is required that \( \|B\| = \sqrt{\sum_{i=1}^{w} B[i]^2} = 1 \).

The distance between two unit-length DeepPrint representations \( B \) and \( \tilde{B} \) can be determined using the cosine similarity between the two vectors, defined as the dot product of the vectors divided by the product of their \( L^2 \) norms \( \left(\frac{\sum_{i=1}^{w} B[i] \tilde{B}[i]}{\|B\| \|\tilde{B}\|}\right) \). Of course, when normalizing to unit length, this division is unnecessary.

The range of values the cosine similarity distance metric may take on normalized inputs is \([-1, 1]\), with 1 representing an exact match. Thus, to determine if two representations are within a close distance, treated as a “match,” it suffices to determine if their dot product is within the range \((1 - t, 1)\) for a desired threshold value \( t \).

The authors of [18] also used Euclidean distance as a distance function. For two unit-length vectors \( B \) and \( \tilde{B} \), Euclidean distance defined as \( \sqrt{\sum_{i=1}^{w} (B[i] - \tilde{B}[i])^2} \) yields values in the range \([0, 2]\), with 0 representing an exact match. Thus, a match is determined by checking if the distance is within \([0, t]\) for some threshold \( t \). For performance reasons, we work with squared Euclidean distance, in which case the threshold \( t \) needs to be adjusted accordingly. We use notation \( d \sim t \) to denote the result of comparing the distance \( d \) to threshold \( t \), where the exact operation depends on the distance metric (i.e., checking \( 1-t \leq d \leq 1 \) for cosine similarity and \( d < t \) for Euclidean distance); \( \text{dist}(B, \tilde{B}) \) denotes the distance computation.

DeepPrint representation requires 768 bytes of storage for 192 32-bit (single precision) floating-point values, but can be compressed to 200 bytes. This is accomplished in [18] by compressing a floating-point vector element to an 8-bit integer using min-max normalization as follows: Given DeepPrint floating-point vector \( B = (B[1], \ldots, B[192]) \), define \( h_B = \max_i \{B[i]\} \), \( t_B = \min_i \{B[i]\} \) and compute

\[
\tilde{B}[i] = \frac{255(B[i] - t_B)}{h_B - t_B}
\]

for \( i \in [1, 192] \). The compressed representation stores 192 8-bit integers \( \tilde{B}[i] \) and two 32-bit floating point values \( h_B \) and \( t_B \). Matching of two compressed representations is performed by decompressing the representations and computing the distance on floats. The compression has a minimal impact on the matching accuracy [18].

### 2.5 Vector Normalization in Adversarial Settings

Normalization of DeepPrint biometric representations is assumed to be performed as part of feature extraction after biometric sampling. Its presence has a direct impact on how the threshold \( t \) that determines a match of two biometric representations is chosen: scaling normalization will result in scaling the threshold \( t \) as well.

This is of interest for us because in the context of this work a biometric sample comes from a user who can act maliciously and construct a biometric representation that deviates from the expectations including normalization. Thus, it becomes important to enforce proper normalization of a biometric representation a user submits. If normalization is not enforced, a malicious user can succeed with authentication without a matching biometric by manipulating vector normalization. As a specific example, consider that squared Euclidean distance is used for distance computation and biometric vectors \( B \) are assumed to be unit-length normalized (i.e., \( \|B\| = \|B\|^2 = 1 \)). For two vectors \( B \) and \( \tilde{B} \), the squared Euclidean distance is \( \|B - \tilde{B}\|^2 \). By the triangle inequality, we have

\[
\|B - \tilde{B}\|^2 \leq (\|B\| + \|\tilde{B}\|)^2 = \|B\|^2 + 2\|B\|\|\tilde{B}\| + \|\tilde{B}\|^2
\]

Now if the distance between vectors \( B \) and \( \tilde{B} \) is compared to a predetermined threshold \( t \) and the client is at liberty to normalize both \( B \) and \( \tilde{B} \) to any value \( N \) they wish, then choosing \( N < \frac{1}{4}\sqrt{t} \) for both \( B \) and \( \tilde{B} \) will result in successful authentication independent of the actual vectors (i.e., in such cases, \( \|B - \tilde{B}\|^2 < t \) is always true). Even when the adversary tampers with (normalization of) one of the vectors, it is still possible to deviate from the intended authentication rules. For this reason, we enforce proper length normalization of all biometrics and include measures to verify that client submitted biometrics are of the correct form. While there may be input formats and distance metrics which prevent abuse when only the enrollment biometric is properly normalized, we conservatively enforce proper normalization at both enrollment and authentication time.

### 3 SOLUTIONS BASED ON GARBLED CIRCUIT EVALUATION

Recall that we consider two threat models: (i) semi-honest servers \( S_1 \) and \( S_2 \) and malicious client and C (ii) semi-honest \( S_1 \) and malicious and colluding \( S_2 \) and C. We label the first model as SH and the second as MAL. We start with our solution secure in the first model.
We start the description of our first solution with the expected functionalities for registration and authentication, which are listed in Figures 1 and 2, respectively.

At registration time, the client (which may be corrupt) supplies its biometric $B$, from which it generates two XOR shares $B_1$ and $B_2$. The ideal functionality performs the normalization check for $B$, the output of which is bit $b$, which is communicated to $S_1$. If the check succeeds, the ideal functionality outputs accept to all parties and shares $B_1$ and $B_2$ to $S_1$ and $S_2$, respectively. Otherwise, the parties receive reject and $S_1$ and $S_2$ receive empty string $\perp$ in place of shares.

During authentication, servers $S_1$ and $S_2$ contribute the shares $B_1$ and $B_2$ they received during registration, while the client contributes biometric $\tilde{B}$. The functionality performs two checks:

1. normalization check for $B$: $b_1 = (\sum_{i=1}^{m} (\tilde{B}[i])^2 \geq 1)$
2. comparison of the distance between enrollment and authentication biometrics $B$ and $\tilde{B}$ to threshold $t$: $b_2 = (\text{dist}(B_1 \oplus B_2, \tilde{B}) \sim t)$.

The resulting bits $b_1, b_2$ are communicated to $S_1$ who then notifies the client of the accept (if both checks pass) or reject decision. Note that we could output a single bit $b_1 \land b_2$ to $S_1$ to indicate success, but it may be beneficial to differentiate between rejection based on the distance and rejection based on the normalization failure. The former may be the result of authentic user authentication failure, while the latter indicates malfeasance by the client.

The registration and authentication protocols in this model are given as Protocol 1, Reg-SH, and Protocol 2, Auth-SH, respectively. In Protocol 1, client $C$ samples a fresh biometric vector $B$ for enrollment, splits it into XOR shares $B_1$ and $B_2$, and sends the shares $B_1$ and $B_2$ to $S_1$ and $S_2$, respectively. The two servers engage in GC evaluation to determine whether or not the received biometric vector $B$ is unit-length normalized, with $S_1$ serving the role of the garbler and $S_2$ the role of the evaluator.

Instead of entering $B_1$ and $B_2$ as inputs into GC evaluation, the servers utilize $m$ instances of OTB$^2_\perp$ to enter $B_1 \oplus B_2$ directly using the first $m$ wires. The boolean operation of OTB allows for the computation of $B_1[i] \oplus B_2[i]$ outside the GC and is realized as follows. With regular OT, $S_1$ would supply labels $l_0$ and $l_1$, while $S_2$ would supply $B_2[i]$ and receive $l_{B_1}$ in our protocol, $S_1$ instead supplies labels $l_{B_1}$ and $l_{B_2}$. As a result, when $S_1$’s share $B_1[i] = 0$, the labels are supplied as usual. However, when $B_1[i] = 1$, the supplied labels are swapped relative to usual OT operation. The outcome is that the receiver obtains labels representing the XOR of share bits $B_1[i]$ and $B_2[i]$, or $B[i]$. We can use an OT extension in the implementation.

After circuit evaluation, $S_2$ obtains the output label $l_\perp$ that represents the outcome of the normalization check and indicates whether

---

**Figure 1: Ideal registration functionality with semi-honest servers.**

1. $\mathcal{F}_{\text{reg-SH}}$ receives input $B \in \{0,1\}^m$ from $C$.
2. $\mathcal{F}_{\text{reg-SH}}$ samples $r \leftarrow \{0,1\}^m$ and defines $B_1 = r$ and $B_2 = r \oplus B$.
3. $\mathcal{F}_{\text{reg-SH}}$ computes $b = (2^{\sum_{i=1}^{m} (B[i])^2} \geq 1)$.
4. $\mathcal{F}_{\text{reg-SH}}$ outputs $b$ to $S_1$.
5. If $b = 1$, then $\mathcal{F}_{\text{reg-SH}}$ outputs $B_1$ to $S_1$, $B_2$ to $S_2$ and accept to $C$ and $S_2$.
6. Otherwise, $\mathcal{F}_{\text{reg-SH}}$ outputs $\perp$ to $S_1$ and $S_2$ and reject to $C$ and $S_2$.

---

**Figure 2: Ideal authentication functionality with semi-honest servers.**

1. $\mathcal{F}_{\text{auth-SH}}$ receives $B_1 \in \{0,1\}^m$ from $S_1$, $B_2 \in \{0,1\}^m$ from $S_2$, and $\tilde{B} \in \{0,1\}^m$ from $C$.
2. $\mathcal{F}_{\text{auth-SH}}$ computes $(b_1, b_2) = (\text{dist}(B_1 \oplus B_2, \tilde{B}) \sim t, \sum_{i=1}^{m} (\tilde{B}[i])^2 \geq 1)$.
3. $\mathcal{F}_{\text{auth-SH}}$ outputs $(b_1, b_2)$ to $S_1$.
4. If $(b_1, b_2) = (1,1)$, then $\mathcal{F}_{\text{auth-SH}}$ outputs accept to $C$, otherwise $\mathcal{F}_{\text{auth-SH}}$ outputs reject to $C$.
5. $\mathcal{F}_{\text{auth-SH}}$ sends terminate to $S_2$. 

---

3.1 Malicious $C$, semi-honest $S_1$ and $S_2$

We start the description of our first solution with the expected functionalities for registration and authentication, which are listed in Figures 1 and 2, respectively.

In our solution, the client’s involvement is minimal and its task primarily consists of splitting its biometric into two XOR shares and communicating the respective shares to the servers $S_1$ and $S_2$. This will take place both at registration and authentication. At registration time, the servers perform the normalization check on the user’s private biometric using OTB and GBC. In this computation, $S_1$ acts as the garbler and $S_2$ as the evaluator. If the normalization check succeeds, the servers accept and store the biometric. The authentication phase proceeds similarly, where in addition to checking whether the submitted biometric meets the normalization criteria, the servers also compute the distance between the registered and newly received biometrics and determine if the distance is within the desired threshold.

When $S_2$ can be malicious (the second, stronger model), additional information is stored at registration time. In addition to storing shares of user biometric $B$, the servers obtain and check a one-way function of $B$ that allows the servers to verify correct share reconstruction within the garbled circuit without obtaining any information about $B$. That additional information is used during the authentication phase to ensure that $S_2$ did not tamper with its values, and we additionally employ stronger tools such as OT resilient to malicious behavior.

In the rest of the paper, we assume a fixed-length biometric representation of $m$ bits (representing $w$ elements of $B$). Notation $X \leftarrow X$ means that variable $X$ is sampled uniformly at random from the set $X$. When working with GCs, let $n$ denote the total number of wires, where the wires with the lowest indices correspond to the inputs and the wires with the highest indices correspond to the output. The parties hold security parameter $\kappa$ and agree on the realizations of the building blocks. All protocols assume the existence of secure channels between each pair of parties for sending sensitive information such as shares and keys.
Protocol 1 Registration Reg-SH

Input: \( C \) holds biometric \( B \).
Output: \( S_1 \) receives bit \( b \) and biometric share \( B_1 \); \( S_2 \) receives accept or reject and biometric share \( B_2 \); \( C \) receives accept or reject.

Common Input: Computational security parameter \( \kappa \).

Protocol steps:

1. \( C \) generates \( m \)-bit random value \( r \overset{\$}{\leftarrow} \{0,1\}^m \), sets \( B_1 = r \), computes \( B_2 = B_1 \oplus B \), and securely communicates \( B_1 \) to \( S_1 \) and \( B_2 \) to \( S_2 \). If the receiving server determines that \( B_1 \) or \( B_2 \) is not an \( m \)-bit string, it signals abort.

2. \( S_1 \) generates labels \( \ell_i^f \) for \( i \in [1,n] \) and \( j \in \{0,1\} \), computes garbled gates \( \mathcal{G}_f \) for the normalization check computation, and sends \( \mathcal{G}_f \) to \( S_2 \).

3. \( S_1 \) and \( S_2 \) engage in \( m \) instances of OTB\(^2 \) to communicate to \( S_2 \) labels \( \ell_i^{B_1[i]@B_2[i]} \) for \( i \in [1,m] \); \( S_1 \) enters labels \( \ell_i^{B_1[i]} \) and \( \ell_i^{B_2[i]@B_1[i]} \) into OT, \( S_2 \) enters bit \( B_2[i] \) and learns label \( \ell_i^{B_1[i]@B_2[i]} = \ell_i^{B_1[i]} \).

4. \( S_2 \) evaluates the circuit and sends the computed label of the output wire \( \ell_n^{B_1} \) to \( S_1 \).

5. If \( \ell_n^{B_1} = \ell_n^{B_2} \), \( S_1 \) signals rejection to \( C \) and \( S_2 \); \( S_1 \) and \( S_2 \) output \( B \).

6. Otherwise, \( S_1 \) signals acceptance to \( C \) and \( S_2 \); \( S_1 \) outputs \( B_1 \) and \( S_2 \) outputs \( B_2 \).

registration was successful. \( S_1 \) interprets the result and communicates the decision to the other parties.

Protocol 2 proceeds similar to Protocol 1. This time, the parties use \( 2m \) instances of OTB to communicate GC labels corresponding to inputs \( B = B_1 \oplus B_2 \) and \( B = B_1 \oplus B_2 \) to \( S_2 \). The output wires with indices \( n-1 \) and \( n \) correspond to the decision bits \( b_2 \) and \( b_1 \), respectively. If both checks succeed, the client obtains the accept decision and otherwise, it learns that the protocol did not succeed.

Our first security result is as follows:

**Theorem 1.** The sequence of Protocols 1 and 2 executed by participants \( S_1, S_2, C_{auth} \) is secure in the presence of semi-honest \( S_1 \) and \( S_2 \) and malicious \( C_{auth} \), according to Definition 2, given supplemental functionalities with security guarantees as discussed in Section 2.3.

**Theorem 2.** The sequence of Protocols 1 and 2, where Protocol 1 is executed by participants \( S_1, S_2, C_{auth} \), and Protocol 2 is executed by participants \( S_1, S_2, C_{imp} \), is secure in the presence of semi-honest \( S_1 \) and \( S_2 \) and malicious \( C_{auth} \) or \( C_{imp} \) according to Definition 2, given supplemental functionalities with security guarantees as discussed in Section 2.3.

The proofs of all theorems can be found in the full version [13].

### 3.2 Malicious and colluding \( S_2 \) and \( C \)

**semi-honest \( S_1 \)**

We now consider a stronger threat model in which the helper server \( S_2 \) can act maliciously and collude with clients \( C \).

When \( S_2 \) is not guaranteed to follow the prescribed behavior, it can deviate from the prescribed computation during a protocol execution, but also modify the biometric share \( B_2 \) that it receives as part of registration when entering it in the authentication protocol. For that reason, we need to be able to detect this kind of misbehavior in addition to detecting client’s misbehavior when it does not use a normalized biometric. Deviations from the prescribed behavior during the protocol execution can be addressed by employing techniques resilient to malicious behavior, while changes to \( B_2 \) between protocol executions require a new solution.

Our solution to this problem is to modify the registration phase to enable \( S_1 \) to learn a function of \( S_2 \)’s share \( B_2 \), which is later used during the authentication to verify that the share that \( S_2 \) inputs matches \( S_1 \)’s verification token. We use a commitment scheme for this purpose: the client is instructed to compute a commitment c
Functionality $\mathcal{F}_{\text{auth-mal}}$

1. $\mathcal{F}_{\text{auth-mal}}$ receives $B_1 \in \{0,1\}^m$ and $c$ from $S_1$, $B_2 \in \{0,1\}^m$ and $\nu$ from $S_2$, and $\tilde{B} \in \{0,1\}^m$ from $C$.
2. $\mathcal{F}_{\text{auth-mal}}$ computes $(b_1, b_2, b_3) = (\text{dist}(B_1 \oplus B_2, \tilde{B})^2 \geq t, \sum_{i=1}^m (\tilde{B}[i])^2 \geq 1, \text{com}(B_1 \oplus B_2, \nu) = c)$.
3. $\mathcal{F}_{\text{auth-mal}}$ outputs $(b_1, b_2, b_3)$ to $S_1$.
4. If $(b_1, b_2, b_3) = (1, 1, 1)$, then $\mathcal{F}_{\text{auth-mal}}$ outputs reject to $C$ and terminate to $S_2$.
5. Otherwise if $b_1 = 0$ or $b_2 = 0$, then $\mathcal{F}_{\text{auth-mal}}$ outputs reject to $C$ and terminate to $S_2$.
6. Otherwise $\mathcal{F}_{\text{auth-mal}}$ signals abort.

Figure 4: Ideal functionality with malicious and colluding $S_2$ and $C$.

to its enrollment biometric $B$ and the commitment $c$ is given to $S_1$. The binding property of the commitment ensures that it is not feasible for $S_2$ (or $S_2$ in collusion with $C$) to later enter a different biometric $B' \neq B$ that matches commitment $c$. The random choices $\nu$ used in producing commitment $c = \text{com}(B, \nu)$ cannot be disclosed to $S_1$ because they permit the opening of the commitment (and thus disclosure of $B$) and for that reason, $\nu$ is known only to $S_2$.

The ideal functionality for registration in this stronger security model is given in Figure 3. In addition to producing shares $B_1$ and $B_2$ of enrollment biometric $B$, the computation includes two checks:

1. normalization check for $B$: $b_1 = (\sum_{i=1}^m (B[i])^2 \geq 1)$
2. check that commitment $c$ matches biometric $B$: $b_2 = (\text{com}(B, \nu) = c)$.

If registration is successful, $S_1$ obtains and stores $B_1$ and $c$, while $S_2$ obtains and stores $B_2$ and $\nu$.

At authentication time, the servers contribute their shares of $B$ and $\tilde{B}$ as before, but also the remaining values ($c$ and $\nu$) that they received at registration time. This time the authentication functionality computes three checks:

1. comparison of the distance between enrollment and authentication biometrics $B$ and $\tilde{B}$ to threshold $t$: $b_2 = (\text{dist}(B_1 \oplus B_2, \tilde{B})^2 \geq t)$
2. normalization check for $\tilde{B}$: $b_1 = (\sum_{i=1}^m (\tilde{B}[i])^2 \geq 1)$
3. check that commitment $c$ matches submitted biometric $B$: $b_3 = (\text{com}(B_1 \oplus B_2, \nu) = c)$.

$S_1$ receives these three bits, which allows it to determine the reason for failure (and address it outside the protocol). If at least one bit is 0, authentication fails. Figure 4 specifies the ideal functionality.

As can be seen from the figure, the ideal functionality is written to differentiate between two authentication failure modes: communicating a reject decision to the client and sending an abort signal. The reason is that when the last check fails ($b_3 = 0$), we know that the failure is due to $S_2$’s misbehavior and the client receives a message that the operation did not go through (as opposed to successfully finished with a negative result). It is also possible for other checks to fail due to $S_2$’s misbehavior, but they can also be a result of the client submitting a biometric which is not normalized or not within the desired distance from the enrollment biometric.

The registration protocol for this setting is called Reg-MAL and is given as Protocol 3. It proceeds by the client generating shares and a commitment, and the servers verifying that they received consistent values and properly normalized input. Similar to the normalization check, the commitment check takes place within the garbled circuit. For concreteness, let $|\nu| = k_1$, $|c| = k_2$, and the inputs being entered into the GC evaluation as $B$, $\nu$, and $c$. Secret-shared $B$ is entered into GC evaluation via OTB as before, while $\nu$ is entered using conventional OT. We have to resort to a maliciously secure variant of OT to guarantee correct execution. Recall that GC evaluation itself is resilient to malicious behavior. At the end of GC evaluation, $S_2$ obtains the output labels $\ell_{\nu}^{(i)}$ and $\ell_{c}^{(i)}$ that it communicates to $S_1$. Note that $S_2$ cannot tamper with them prior to sending. If the received labels correspond to bits 1, $S_1$ announces successful completion. If at least one of the labels is invalid, $S_1$ aborts. Otherwise, it sends a reject signal.

Authentication is termed Auth-MAL and is given as Protocol 4. The changes to the previous authentication protocol include: (i) the addition of commitment inputs ($c$, $\nu$), (ii) the use of OT for entering $\nu$, (iii) changes to the circuit to perform commitment verification, (iv) the use of maliciously secure OT, and (v) different handling of the results of function evaluation by $S_1$. We assume that the circuit wires are allocated to the inputs in the following order: $B$ (m bits),

<table>
<thead>
<tr>
<th>Protocol 3 Registration Reg-MAL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> $C$ holds biometric $B$.</td>
</tr>
<tr>
<td><strong>Output:</strong> $S_1$ receives bits $b_1$ and $b_2$, biometric share $B_1$, and verification token $c$; $S_2$ receives accept or reject, biometric share $B_2$, and verification supplement $\nu$; $C$ receives accept or reject.</td>
</tr>
<tr>
<td><strong>Common Input:</strong> Computational security parameter $\kappa$ and statistical security parameter $\rho$.</td>
</tr>
<tr>
<td><strong>Protocol steps:</strong></td>
</tr>
<tr>
<td>(1) $C$ generates $m$-bit random value $r \leftarrow {0,1}^m$, sets $B_1 = r$ and $B_2 = B_1 \oplus B$, and computes $c = \text{com}(B, \nu)$ using freshly generated randomness $\nu$.</td>
</tr>
<tr>
<td>(2) $C$ securely communicates $(B_1, c)$ to $S_1$ and $(B_2, \nu)$ to $S_2$. If any communicated value is malformed, the corresponding server signals abort.</td>
</tr>
<tr>
<td>(3) $S_1$ generates labels $\ell_{\nu}^{(i)}$ and garbled gates $\mathcal{G}<em>\nu$ for the normalization and commitment checks and sends $\mathcal{G}</em>\nu$ to $S_2$.</td>
</tr>
<tr>
<td>(4) $S_1$ and $S_2$ engage in $m$ instances of maliciously secure OTB to communicate to $S_2$ labels $\ell_{B_1}^{(i)} = B_1[i] \oplus B[i]$ for $i \in [1, m]$, as in prior protocols and $k_1$ instances of conventional maliciously secure OT to communicate to $S_2$ labels $\ell_{c}^{(i)}$ for $i \in [1, k_1]$.</td>
</tr>
<tr>
<td>$S_1$ also sends labels $\ell_{\nu}^{(i)}$ for $i \in [1, k_2]$ to $S_2$.</td>
</tr>
<tr>
<td>(5) $S_2$ evaluates the circuit and communicates the output labels $\ell_{\nu}^{(i)}$ and $\ell_{c}^{(i)}$ to $S_1$.</td>
</tr>
<tr>
<td>(6) $S_1$ performs the following:</td>
</tr>
<tr>
<td>(a) If $\ell_{\nu}^{(i)} = \ell_{\nu}^{(i)}$ and $\ell_{c}^{(i)} = \ell_{c}^{(i)}$, $S_1$ broadcasts accept. $S_1$ stores $(B_1, c)$ and $S_2$ stores $(B_2, \nu)$.</td>
</tr>
<tr>
<td>(b) Otherwise, $S_1$ sends reject to all parties.</td>
</tr>
</tbody>
</table>
Protocol 4 Authentication Auth-MAL
Input: C holds biometric $\hat{B}$, $S_1$ holds biometric share $B_1$ and verification token $c$; $S_2$ holds biometric share $B_2$ and verification supplement $v$.
Output: $S_1$ receives bits $b_1, b_2$, and $b_3$; $C$ receives accept or reject.
Common Input: Computational security parameter $\kappa$, statistical security parameter $\rho$, and threshold $t$.
Protocol steps:

1. $C$ generates $m$-bit random value $\hat{B}_2 \leftarrow \{0, 1\}^m$, sets $\hat{B}_1 = B_2 \oplus \hat{B}$, and sends $\hat{B}_1$ to $S_1$ and $\hat{B}_2$ to $S_2$. If any received value is malformed, then the corresponding server signals abort.
2. $S_1$ generates labels $t_1^i$, computes garbled gates $G_f$ for the over-the-threshold distance computation, normalization check, and commitment verification, and sends $G_f$ to $S_2$.
3. $S_1$ and $S_2$ engage in $2m$ instances of maliciously secure OTB to communicate to $S_2$ labels $\hat{B}_1 \oplus \hat{B}_2$ and $\hat{B}_1 \oplus \hat{B}_2$ for $i \in \{1, m\}$ and $\kappa_1$ instances of conventional maliciously secure OT to communicate to $S_2$ labels $t_1^i$, for $i \in \{1, \kappa_1\}$.
4. $S_1$ also sends labels $t_1^i$ for $i \in \{1, \kappa_2\}$ to $S_2$.
5. $S_1$ evaluates the circuit and sends the computed output labels $t_1^i, t_1^i, a_{l_2}$, and $a_{l_2}$ to $S_2$.
6. $S_1$ performs the following:
   a. If $b_2 = t_1^i \oplus \hat{B}_2$, $t_1^i \oplus \hat{B}_2$, and $t_1^i \oplus \hat{B}_2$, $S_1$ sends accept to $C$ and terminate to $S_2$.
   b. If $b_2 = t_1^i \oplus \hat{B}_2$, $t_1^i \oplus \hat{B}_2$, and $t_1^i \oplus \hat{B}_2$, $S_1$ sends reject to $C$ and terminate to $S_2$.
   c. Otherwise, $S_1$ signals abort to $C$ and $S_2$.

$\hat{B}$ (m bits), $\nu$ ($\kappa_1$ bits), and $c$ ($\kappa_2$ bits). As before, the circuit size is denoted by $n$, while the output wires this time are $n - 2, n - 1, n$.

Authentication is successful when all output bits are 1 (i.e., all three checks pass). Any malformed output labels and the failure of the commitment check point to $S_2$’s misbehavior and result in abort, while failures of the normalization check and a large difference between $\hat{B}$ and $\hat{B}$ can be due to $C$ or $S_2$ and result in reject.

**Theorem 3.** The sequence of Protocols 3 and 4 executed by participants $S_1, S_2, C_{auth}$ is secure in the presence of semi-honest $S_1$, and malicious and colluding $S_2$ and $C_{auth}$, according to Definition 2, given supplemental functionalities with security guarantees as discussed in Section 2.3.

**Theorem 4.** The sequence of Protocols 3 and 4, where Protocol 3 is executed by participants $S_1, S_2, C_{auth}$, and Protocol 4 is executed by participants $S_1, S_2, C_{imp}$, is secure in the presence of semi-honest $S_1$, and malicious and colluding $S_2$ and (Cauth, or Cimp), according to Definition 2, given supplemental functionalities with security guarantees as discussed in Section 2.3.

4 IMPLEMENTATION AND EVALUATION

4.1 Working with Compressed DeepPrint Representation

The use of GCs permits implementing any desired functionality and we realize DeepPrint’s matching using compressed representation to lower the cost of the computation. Recall that the main benefit of compressed DeepPrint representation is to lower its storage cost, as the value is uncompressed size during the matching. However, in the context of this work, shorter biliteral representation and the use of integer instead of floating-point values can aid efficiency of the computation itself. Thus, we would like to compute as much as possible using the compressed form in a manner which is not lossy with respect to this compression heuristic.

To this end, suppose that we are comparing two DeepPrint representations $X$ and $Y$ consisting of $w$ (=192) elements. Recall that the compressed representation of $X$ uses $h_X, f_X$ together with 8-bit integers $\bar{X}[i]$ defined in equation 1. We also define $\Delta_X = (h_X - f_X)/255 > 0$, represent a compressed biometric as $\bar{X} \equiv (\Delta_X, f_X, (\bar{X}[i]))$, and define its decompressed 32-bit (single precision) floating point biometric as $\tilde{X} = (\Delta_X | (\bar{X}[i] + f_X]$ when using cosine similarity comparing normalized vectors which have been compressed and subsequently decompressed, as is done in [18], it suffices to compare the dot product $\tilde{X} \cdot \tilde{Y}$ against a threshold value. With this in mind, we have

$$\tilde{X} \cdot \tilde{Y} = \sum_{i=1}^{w} \bar{X}[i] \bar{Y}[i] = \sum_{i=1}^{w} (\bar{X}[i] \Delta_X + f_X) (\bar{X}[i] \Delta_Y + f_Y)$$

$$= \left(\Delta_X \Delta_Y \sum_{i=1}^{w} \bar{X}[i] \bar{Y}[i] + \Delta_X \sum_{i=1}^{w} \bar{Y}[i] \right)$$

$$+ \Delta_Y \sum_{i=1}^{w} \bar{X}[i] \bar{Y}[i] + w f_X f_Y$$

In the final line of this equation, there are $w$ 8-bit multiplications, which are much cheaper than floating-point or even 32-bit integer operations. The summations then require $8 + \lceil \log_2(w) \rceil$ bits to represent the $\sum_{i=1}^{w} \bar{X}[i]$ and $\sum_{i=1}^{w} \bar{Y}[i]$ terms, and $16 + \lceil \log_2(w) \rceil$ bits for the $\sum_{i=1}^{w} \bar{X}[i] \bar{Y}[i]$ term. Once the computation is performed on short values, we convert the sums to floating-point representation and compute the remaining operations using regular floating-point arithmetic. This adds 3 conversions, 8 single-precision floating-point multiplications and 32-bit floating-point additions.

Conversion to a floating-point value involves locating the index of the most significant non-zero bit, shifting the mantissa by this value, and adjusting the exponent by that value as well. Other operations such as floating-point addition involve shifting by an oblivious value as well.

Euclidean distance can be computed over compressed values as

$$\sum_{i=1}^{w} (\bar{x}_i - \bar{y}_i)^2 = \sum_{i=1}^{w} (\bar{X}[i] \Delta_X - \bar{Y}[i] \Delta_Y + f_X - f_Y)^2$$

$$= \left(\Delta_X^2 \sum_{i=1}^{w} \bar{X}[i]^2 \right) + \left(\Delta_Y^2 \sum_{i=1}^{w} \bar{Y}[i]^2 \right)$$

$$- 2 \Delta_X \Delta_Y \sum_{i=1}^{w} \bar{X}[i] \bar{Y}[i] + \left(2 (f_X - f_Y) \Delta_X \sum_{i=1}^{w} \bar{X}[i] \right)$$

$$- 2 (f_X - f_Y) \Delta_Y \sum_{i=1}^{w} \bar{Y}[i] + w (f_X - f_Y)^2$$

$$= \left(\Delta_X^2 \sum_{i=1}^{w} \bar{X}[i]^2 \right) + \left(\Delta_Y^2 \sum_{i=1}^{w} \bar{Y}[i]^2 \right)$$

$$- 2 \Delta_X \Delta_Y \sum_{i=1}^{w} \bar{X}[i] \bar{Y}[i] + w (f_X - f_Y)^2$$
which shows an increase in the number of short integer, floating-point, and integer to floating-point conversion operations.

These operations need to be implemented within GCs. Our GC implementation was built to maximize efficiency starting from the low level such as the use of constant publicly known values, optimizing AND, OR, and XOR gates when at least one input is constant, by flagging each constant wire as special and systematically eliminating the need to evaluate gates whenever possible. Such optimization can, for example, cut in half the number of gates needed to evaluate multiplication when compared to the built-in multiplication in JustGarble [9] (specifically, by not processing gates to add bits known to be zero).

The above together with efficient shifting by an oblivious value allowed us to generate very efficient floating-point circuits. We are aware of only one other work [32] that built garbled circuits for floating-point operations, and our circuits compare very favorably. In particular, our single-precision floating-point addition uses 2030 gates vs. 7052 gates in that work; our multiplication has 3690 gates vs. 7701 gates in that work, and comparison is efficient at 300 gates (not tested in [32]). There can be differences in the treatment of exceptions (e.g., we treat infinity as non-a-number NaN which improves performance), but we expect that even with full IEEE 754 standard treatment, our circuits will compare favorably.

### 4.2 Experimental Evaluation

Our implementation uses the GC instantiation from [9] with the free-XOR and row reduction optimizations. The OT extension is from [5] for the protocols with semi-honest $S_2$, and from [6] for the protocols with malicious $S_2$. Both of them build on an optimized version of the semi-honest protocol from [23] to generate the base OTs. Commitments are formed using SHA-256 with $k_2 = 256$ and $|\nu| = k_1 = 128$ random bits included as entropy supplement, $m = 1600$ and reported performance is averaged over 100 runs. The implementation is available as open source from https://github.com/applied-crypto-lab/biom-auth.

The following machines were used to run the experiments:

- An AMD Ryzen5-3600 6-core processor machine operating at 3.6 GHz, running openSUSE Leap 15.3.
- Identical computers with Intel Xeon E5-2620v4 8-core processors operating at 2.1 GHz, running Ubuntu 20.04.3 LTS.

All communication used TCP sockets with the following setup:

- Each party on a different Xeon machine on a LAN.
- The two servers on Xeon machines on a LAN, with the client on the Ryzen machine connecting via VPN from the internet.

Network latency and throughput were measured by transmitting buffers of size $4^i$ bytes for $i \in [0, 12]$ bidirectionally. Round trip time (RTT) is taken to be the average of transmitting $\leq 256$ bytes (representing one packet). Throughput is taken to be $(2 \cdot 8 \cdot \text{bufsize}) / \text{latency} / \text{time(sec)}$ as a buffer of size bufsize is sent twice in this round trip test. We obtain RTT of 0.345 ms and throughput of 946 Mb/sec for Xeon LAN and RTT of 45.9 ms and throughput of 20.4 Mb/sec for Ryzen to Xeon over internet. We use encrypted channels for sensitive information such as biometric shares, GC labels, and protocol outcomes (while garbled tables and OTB communication are not encrypted).

Table 1 reports performance of authentication protocols. The online time and communication correspond to all work with the exception of garbling and transmitting the garbled table and labels from $S_1$ to $S_2$, which can be precomputed and constitutes offline work. Note that online communication is independent of the distance metric, while offline runtime is independent of the network setup (as the connection between the servers does not change). The GC size is sub-divided into gates that involve communication (AND and OR) and those that do not (XOR and NOT). The category “Other” of online time includes communication time, other local computation time, and down time. The (online) communication time is relatively insignificant for LAN tests but dominates the mixed internet test times.

The client computation time is not included, but it is minimal, showing that the solution is well suited for constrained devices. In particular, client’s computation took on average 0.17 ms with additional 0.05 ms for data transmission to the socket, after which the client awaits a response. In addition, the client only transmits 400B (and receives 1B), while the remaining communication is between the servers.

We test squared Euclidean distance and cosine similarity, and find the latter to be slightly more efficient as it uses fewer operations. Although we further optimize the squaring operation in Euclidean distance computation, this is not enough to offset the difference. Note that Euclidean distance can reuse some operations from the normalization check.

We can compare performance of our solutions with that of other constructions that treat biometric-based authentication and consider at least the client to be malicious [2, 17, 21, 22, 34]. All of them require the clients to store keys on their devices and all with the exception of [21] consider only semi-honest servers (and thus are closer to our first threat model). Among these, [34] has performance on the order of seconds or larger and is not suitable for real-time authentication. [22] takes about a second for a computation that leaks the distance to the server in the semi-honest server model. [17] does not provide sufficient information to determine the time, but it is lower-bounded by several hundred ms. With [2], authentication takes 71ms on 128-byte vectors and 102ms on 256-byte vectors without taking communication into account, both of which are higher than our 192-element times are. This protocol does not leak the distance to the server, but does not achieve security (the client notifies the server of authentication outcome). Lastly, in [21] authentication (client and server work) takes over 150ms with much shorter 64-byte templates and over 175ms with 128-byte templates without taking network communication into account and disclosing the distance to the server. Once again, this is slower than performance of all of our protocols.

### 5 CONCLUSIONS

In this work, we treat the topic of privacy-preserving biometric-based authentication that permits users to authenticate with biometric data in a such a way that users do not have to maintain any additional secrets and the authentication server does not learn information about user biometrics. We build solutions using a number of cryptographic techniques such as garbled circuit evaluation, a new variant of oblivious transfer, and a commitment scheme that
rely on a helper server. An interesting aspect of our work is that the standard security definitions adopted in secure multi-party computation literature were not sufficient to demonstrate security in our application and we extend them to accommodate computation consisting of multiple phases where the set of participants might change from one phase to another. We consider two different security models, both of which model users as malicious and differ in the assumptions on the servers. We formally prove all of our constructions to be secure in the respective models and implement them to demonstrate that they have practical performance.

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