CSE 410 Fall 2025
Privacy-Enhancing Technologies

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Lecture 7: Protecting Data in Transit

Proteching Data in Transit

- We discussed many aspects of protecting data at rest
- Similar mechanisms are used for protecting data in transit
- The biggest missing piece is how to distribute keys for symmetric cryptography over the insecure internet
- This requires additional tools

Key Distribution Mechanisms

- Assume users communicate over an insecure network
- There are different possibilities for key distribution
  - they fundamentally differ based on whether the participants are within the same administrative domain
- When users are within the same adminstrative domain, we can rely on a trusted authority (TA)
  - the TA can verify user identities, issue certificates, transmit keys, etc.

Key Distribution Mechanisms

Major categories of key distribution are

- key agreement (a.k.a. key establishment or key exchange)
  - network users employ an interactive protocol to construct a session key
- key predistribution
  - a TA distributes keying information during the setup phase using a secure channel
  - a pair of users compute a key known only to them
- session key distribution
  - on request, an online TA chooses a session key and distributes it to two users
  - the TA communicates the new keys by encrypting them using previously distributed secret keys

Key Distribution Mechanisms

The difference between key distribution and key agreement:

- in key distribution, one party (a TA) chooses a key and transmits it to one or more parties
  - key transmission is performed in an encrypted form
- in key agreement, two or more parties jointly establish a secret key
  - communication is performed over a public channel
  - each participant contributes to the value of the resulting key
  - the key is not sent from one party to another

We'll talk about key agreement (key exchange) next

- this is achieved by means of public-key cryptography

Public-Key Cryptography

Public-key encryption

- a party creates a public-private key pair (pk, sk)
- the public key pk is used for encryption and is publicly available
- the private key sk is used for decryption only

$$\mathsf{Dec}_{sk}(\mathsf{Enc}_{pk}(m)) = m$$

- knowing the public key and the encryption algorithm only, it is computationally infeasible to find the secret key
- public-key crypto systems are also called asymmetric

Public-Key Cryptography

Digital signatures

- a party generated a public-private signing key pair (pk, sk)
- private key sk is used to sign a message
- public key pk is used to verify a signature on a message
- can be viewed as one-way message authentication

(Public-key) Key agreement or key distribution

- prior to the protocols the parties do not share a common secret
- after the protocol execution, they hold a key not known to any eavesdropper

Background

Before we proceed with the computation, we need to discuss the setup

Groups are a convenient way to represent sets and work with them

Background

A group G is a set of elements together with a binary operation $\circ$ such that

- the set is closed under the operation $\circ$, i.e., for every $a, b \in G$, $a \circ b$ is a unique element of G
- the associative law holds, i.e., for all $a, b, c \in G$, $a \circ (b \circ c) = (a \circ b) \circ c$
- the set has a unique identity element e such that $a \circ e = e \circ a = a$ for every $a \in G$
- every element has a unique inverse a^{-1} in G such that $a \circ a^{-1} = a^{-1} \circ a = e$

Groups

Size of a group

- a group is finite if it has only a finite number of elements
- a group is infinite if it has an infinite number of elements
- the number of elements of a finite group is called the order of the group

Groups

Example

- fix a positive integer m as the modulus
- the set of integers mod m relatively prime to m forms a group with multiplication modulo m as operation
- the identity element is 1
- it is called the multiplicative group modulo m
- what is the group order?

Groups

Example

- fix a positive integer m as the modulus
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- what is the group order?

Better example

- let the modulus m be prime
- what is the group order?

More on Groups

- If a is an element of a finite group with identity 1, then there is a unique smallest positive integer i with $a^i = 1$ (using multiplicative notation)
  - such i is called the order of a (different from the order of the group)
- The element a has infinite order is there is no positive integer i with $a^i = 1$
- A cyclic group is one that contains an element a whose powers a^i and a^{-i} make up the entire group
- An element a with such property is called a generator of the group

Cyclic Groups

Examples

- consider the multiplicative group modulo 7 $(\mathbb{Z}_7^*, \cdot)$
  - what is the order of 3?
  - what is the order of 2?
  - what is the order of 6?
- what is the multiplicative group generated by 3 modulo 11?

A group generator is often denoted by g

Discrete Logarithm Problem

Discrete logarithm: simple version

- given an appropriate group with generator g and $h = g^x$, it is difficult for someone to compute x
- x is called the discrete logarithm of h to the base g

Discrete Logarithm Problem

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Discrete logarithm: more elaborate version

- we are given a cyclic group G of order q and its generator g
- for each $h \in G$ there is a unique x such that $g^x = h$
- such x is called the discrete logarithm of h with respect to g, written as $x = \log_g h$
- many properties of regular logarithms apply
  - $\log_g 1 = 0$
  - $\log_g(h_1 \cdot h_2) = (\log_g h_1 + \log_g h_2) \bmod q$

Discrete Logarithm Problem

The discrete logarithm problem

- in a cyclic group G with given generator g, compute unique $\log_g h$ for a random element $h \in G$

We want the discrete logarithm problem to be hard (relative its setup)

When is it hard?

- most often a multiplicative group modulo prime p, $\mathbb{Z}_p^*$, or its subgroup is used
- the order of the group must contain a large prime
- the choice of parameters is driven by known algorithms for solving the discrete logarithm problem

Key Agreement Schemes

- The best-known key exchange protocol is due to Diffie and Hellman
- Alice and Bob want to establish a shared key
  - the common parameters are (G, q, g)
  - Alice chooses a random number a from $\mathbb{Z}_q$, computes g^a, and sends g^a to Bob
  - Bob chooses a random number b from $\mathbb{Z}_q$, computes g^b, and sends g^b to Alice
  - Alice computes the shared key as $(g^b)^a = g^{ab}$
  - Bob computes the shared key as $(g^a)^b = g^{ab}$

Diffie-Hellman Key Exchange

Diffie-Hellman key exchange properties

- Alice and Bob compute the same key
- it is computationally difficult for someone else (without a or b) to compute their key
- the security property holds only against a passive attacker
  - a passive adversary simply monitors network communication
- a more powerful adversary can do severe damage

Threat Models

- Since the network is insecure, we need to protect against attackers
  - the adversary might be one of the users in the network
- An active adversary can:
  - prevent a message from being delivered
  - modify messages being transmitted on the network
  - try to masquerade as another user in the network
- Adversary's goal might be:
  - fool someone into accepting an invalid key as valid
  - learn some information about the key being established
  - use another user's identity to establish a shared key with someone

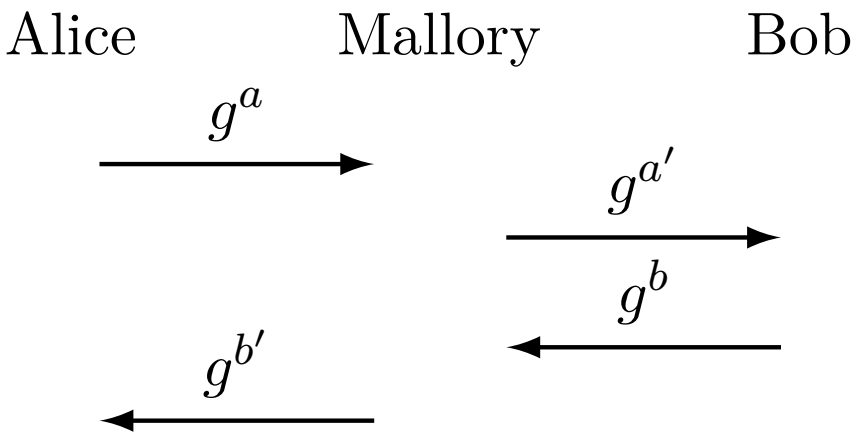
Diffie-Hellman Key Exchange

Security properties of Diffie-Hellman key exchange fall apart in the presense of an active adversary

- the attack is called a man-in-the-middle attack
- Mallory will intercept messages between Alice and Bob and substitute her own
- Alice establishes a shared key with Mallory and Bob also establishes a shared key with Mallory

Diffie-Hellman Key Exchange

Man-in-the-middle attack on Diffie-Hellman key exchange



- Alice shares the key $g^{ab'}$ with Mallory
- Bob shares the key $g^{a'b}$ with Mallory
- Alice and Bob do not share any key
- what is Mallory capable of doing?

Diffie-Hellman Key Exchange

Alice and Bob need to make sure they are exchanging messages with each other

- there is a need for authentication
- preceding this protocol with an authentication scheme is not guaranteed to solve the problem
  - after they authenticate, the same attack can be carried out

We need a protocol that authenticates the participants at the same time the key is being established

- such a protocol is called an authenticated key agreement scheme
- it should simultaneously guarantee secure mutual authentication and secure key computation

Authenticated Diffie-Hellman Key Exchange

Authenticated Diffie-Hellman key exchange uses

- digital signatures
- certificates

We need to discuss those topics next

Digital Signatures

- A digital signature scheme is a method of signing messages stored in electronic form and verifying signatures
- Digital signatures can be used in very similar ways conventional signatures are used
  - paying by a credit card and signing the bill
  - signing a contract
  - signing a letter
- Unlike conventional signatures, we have that
  - digital signatures are not physically attached to messages
  - we cannot compare a digital signature to the original signature

Digital Signatures

Digital signatures allow us to achieve the following security objectives:

- authentication
- integrity
- non-repudiation
  - this is the main difference between signatures and MACs

What security property do we want from a digital signature scheme? How does it relate to that of MACs?

Digital Signatures

It is meaningful to consider the following attack models

- key-only attack
- known message attack
- chosen message attack

Adversarial goals might be

- total break
- selective forgery
- existential forgery

As with MACs, we want existiential unforgeability under an adaptive chosen message attack

Digital Signatures

A digital signature scheme consists of three algorithms:

- key generation: given a security parameter κ, creates a public-private key pair (pk, sk)
- signing algorithm takes a messages m and uses private signing key sk and output a signature σ
- signature verification algorithm takes a message m, a signature on it σ, and the signer's public key pk and outputs a yes/no answer

Signature Algorithms

- RSA signature scheme
  - relies on the difficulty of factoring large composite moduli and hashing
- ElGamal signature scheme
  - was published in 1985 and works in groups where the discrete logarithm problem is hard
- Schnorr signature scheme
  - modifies ElGamal signature scheme to sign a digest of a message in a subgroup modulo p
- Digital Signature Algorithm (DSA)
  - a signature standard adopted by NIST

Digital Signature Standard (DSS)

Digital Signature Standard (DSS) or Digital Signature Algorithm (DSA) was adopted as a standard in 1994

- its design was influenced by prior ElGamal and Schnorr signature schemes
- it assumes the difficulty of the discrete logarithm problem
- no formal security proof exists

Digital Signature Standard (DSS)

- DSS was published in 1994 as FIPS PUB 186
  - specified to hash the message using SHA-1 before signing
  - produced a 320-bit signature on a 160-bit hash
- later variants such as FIPS PUB 186-4 (2013) supported different sizes
  - DSA could be used with a 1024-, 2048-, or 3072-bit modulus
  - the signature size is 320, 448, or 512 bits (2 values of the same bitlength as q)
- the current version FIPS PUB 186-5 uses only elliptic curve variants

Digital Signature Security

Thorough evaluation of security of a signature scheme is crucial

- often a message can be encrypted and decrypted once and long-term security for the key is not required
- signatures can be used on legal documents and may need to be verified many years after signing
- choose the key length to be secure against future computing speeds

There is fundamental difference between short-lived session keys and long-term public signing keys

Digital Signature Algorithm

The setup requires a group where the discrete logarithm problem is hard

- choose a large prime p
- there is another large prime q that divides $p - 1$
- the bitlength of p and q must be from predefined size pairs
- g is a generator of subgroup of $\mathbb{Z}_p^*$ of order q
- we obtain a setup for the group (p, q, g)

Digital Signature Algorithm

Key generation

- let (p, q, g) be a group setup for the discrete log problem to be hard
- let $H : \{0,1\}^* \to \mathbb{Z}_q$ be a cryptographic hash function
- choose secret $x \in \mathbb{Z}_q$
- compute $h = g^x \pmod{p}$
- the public key is $pk = (H, p, q, g, h)$
- the private key is $sk = x$

Digital Signature Algorithm

Signing

- given a message $m \in \{0,1\}^*$, public key $pk = (H, p, q, g)$, and secret key $sk = x$
- choose $y \in \mathbb{Z}_q^*$ uniformly at random
- compute the signature $\sigma(m) = (\sigma_1, \sigma_2)$, where

$$\begin{aligned} \sigma_1 &= (g^y \bmod p) \bmod q \quad \text{and} \\ \sigma_2 &= (H(m) + x\sigma_1)y^{-1} \bmod q \end{aligned}$$

- if $\sigma_1 = 0$ or $\sigma_2 = 0$, a new value of y should be chosen

Digital Signature Algorithm

Signature verification

- given a message $m \in \{0,1\}^*$, signature $\sigma(m) = (\sigma_1, \sigma_2)$ and $pk = (H, p, q, g, h)$
- compute

$$\begin{aligned} e_1 &= H(m)\sigma_2^{-1} \bmod q \\ e_2 &= \sigma_1\sigma_2^{-1} \bmod q \end{aligned}$$

- then test $(g^{e_1}h^{e_2} \bmod p) \bmod q \stackrel{?}{=} \sigma_1$
- output 1 (valid) iff verification succeeds

Digital Signature Algorithm

Performance is favorable for public-key cryptography

Security stood the test of time