Toeplitz Random Encoding for Reduced Acquisition Using Compressed Sensing

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INTRODUCTION

Considerable attention has been paid to compressed sensing (CS) in the MRI community recently (1,2). CS theory allows exact recovery of a sparse signal from a highly incomplete set of samples (3,4), and thus has the potential for significant reduction in MRI scan time. While most existing work has focused on Fourier encoding, non-Fourier encoding has shown some promise (5,6). In this abstract, we design a pulse sequence to implement the Toeplitz random encoding method proposed earlier (6). The experimental results show that Toeplitz random encoding can be realized in practice as an alternative method for CS MRI.

THEORY

Block Toeplitz random matrices have been shown to be good compressed sensing matrices (7). To realize such an encoding scheme, a pseudo random RF pulse is generated and used for the first excitation. The subsequent excitations then use pulses with the same amplitude but linearly shifted by a fixed amount in phase from the previous one. A fixed gradient Gy is turned on with the RF pulse for each excitation. According to the MR physics of magnetization with small tip angles (8), each excitation profile should be $M(\mathbf{r}) = j\gamma M_0(\mathbf{r})\int_0^T B_1(t)e^{j\mathbf{k}(t)\cdot\mathbf{r}}$ [1], which depends on the Fourier transform of the random pulse $B_1(t)$. Each random RF pulse generates a random excitation profile in 1D along the y direction. The linear phase shift is designed such that the excitation profiles from consecutive excitations are spatially shifted by a single pixel along the y direction. After the RF excitation, the phase encoding Gy is turned off to realize the Toeplitz random encoding along y, but frequency encoding Gx is still on for Fourier encoding in x. For square field of view, the gradient Gy is designed to satisfy $Gx\Delta t_{AD} = Gy\Delta t_{RF}$ [2], where Δt_{AD} and Δt_{RF} are the A/D acquisition and RF excitation sampling period. Figure 1 shows the diagram of the pulse sequence. To reduce data acquisition, only the first few excitations are carried out and the reduced data are used to reconstruct the desired image using compressed sensing. Specifically, image $\boldsymbol{x}$ is reconstructed by solving the convex optimization problem: Minimize $\|\boldsymbol{\Psi}\mathbf{x}\|_1$ subject to $\|\boldsymbol{\Phi}\mathbf{x}-\mathbf{y}\|_2<\varepsilon$ [3], where $\boldsymbol{\Psi}$ is the sparsity basis, $\mathbf{y}$ is the acquired data after a 1D Fourier transform along the frequency encoding direction, and $\boldsymbol{\Phi}$ is the Teopliz random encoding matrix defined in Eq. [4]. The matrix $\mathbf{A}$ in Eq. [4] has a Toeplitz structure, where the independent elements $a_1,\ldots,a_n$ are the Fourier transform of the RF pulse in the first excitation. Due to the special property of Toeplitz matrices, the image reconstruction has a fast algorithm, whose complexity is about the same as compressed sensing for randomly sampled Fourier encoding.

$$\boldsymbol{\Phi} = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A} \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} a_1 & a_2 & & a_{n-1} & a_n \\ a_2 & a_3 & & a_n & a_1 \\ \vdots & & \ddots & & \\ a_k & a_{k+1} & \cdots & a_{k-2} & a_{k-1} \end{bmatrix} \quad [4]$$

METHOD AND RESULTS

A set of data was acquired from a 3T commercial MRI scanner (GE Healthcare, Waukesha, WI) with a single-channel coil using the proposed random encoding sequence (TE = 10ms, TR = 1200 ms, 3.2ms RF pulse, 20cm FOV, 64 × 64 matrix). Figure 2 shows the reconstructed images from the acquired data. L_1 norm was used as the sparse representation. The CS reconstruction from the reduced data is seen to be very close to the linear reconstruction from the fully sampled data.

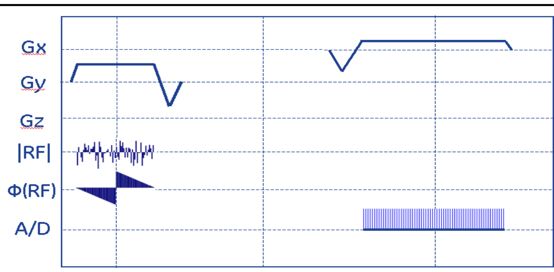

Figure 1. Timing diagram of the pulse sequence for Toeplitz random encoding.

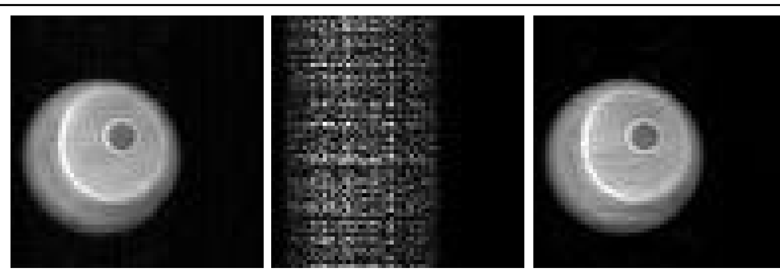

Figure 2. Reconstructed images from the random encoding data. Left image is the linear reconstruction from the fully sampled (64x64) data. Middle and right images are the minimum energy reconstruction and CS reconstruction from the reduced data (R = 1.5).

CONCLUSION

We have presented a random, non-Fourier encoding technique for application of CS in MRI. The experiments have shown promising results to accelerate imaging speed with good reconstruction quality. Future work will improve the pulse sequence and investigate images with different sizes and contents for further accelerations.

REFERENCES

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Introduction

- Considerable attention has been paid to compressed sensing (CS) in the MRI community recently [1,2]. CS theory allows exact recovery of a sparse signal from a highly incomplete set of samples [3,4], and thus has the potential for significant reduction in MRI scan time. While most existing work has focused on Fourier encoding, non-Fourier encoding has shown some promise [5,6].

- In this abstract, we design a pulse sequence to implement the Toeplitz random encoding method proposed earlier [6]. The experimental results show that Toeplitz random encoding can be realized in practice as an alternative method for CS MRI.

Theory

- Toeplitz Random matrix have been proved to be a good CS matrix [7].
- The objective is to implement such an encoding matrix in MR systems.

Data acquisition

- According to the MR physics of magnetization $B_1(t)$ with small tip angles [8], each excitation profile should be $M(\mathbf{r}) = j\gamma M_0(\mathbf{r})\int_0^T B_1(t)e^{jk(t)\mathbf{r}}$ (1), which depends on the Fourier transform of the random pulse.
- Random RF pulses are used to generate random excitation along y.
- All excitations use RF pulses with the same amplitude but linearly shifted by a fixed amount in phase from the previous one.
- A fixed gradient is turned on Gy with the RF pulse for each excitation.
- A 180° sinc pulse on RF is turned at the half TE, and a gradient with two crushers is turned on Gz at the half TE.
- The conventional phase encoding is turned off.
- The frequency encoding is still on for Fourier encoding along x.
- The gradient Gy and gradient Gx satisfy $G_x\Delta t_{AD} = G_y\Delta t_{RF}$ (2), for square field of view, where Δt_{AD} and Δt_{RF} are the A/D acquisition and RF excitation sampling period.

Image reconstruction

- The reduced data are acquired to reconstruct the desired image using CS.
- Image f is reconstructed by solving the convex optimization problem:

$$\min_f \|\Psi f\|_1 \quad s.t. \quad \|\Phi f - d\|_2 < \varepsilon \qquad (3)$$

where Ψ is the sparsity basis, d is the acquired data after a 1D Fourier transform along the frequency encoding direction, and Φ is the Teopliz random encoding matrix defined below. The matrix A has a Teopliz structure, where the independent elements $a_1,\ldots,a_n$ are the Fourier transform of the RF pulse in the first excitation.

$$\Phi = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A} \end{bmatrix} \quad \text{where } \mathbf{A} = \begin{bmatrix} a_1 & a_2 & & a_{n-1} & a_n \\ a_2 & a_3 & & a_n & a_1 \\ \vdots & & \ddots & & \\ a_n & a_1 & \cdots & a_{n-2} & a_{n-1} \end{bmatrix}$$

Method and Results

- Phantom images were acquired from a 3T commercial MRI scanner (GE Healthcare, Waukesha, WI).
- TE = 30ms, TR = 1000 ms, 6.4ms RF pulse, 16cm FOV, 64 × 64 matrix.
- Identity transform and finite difference both were used as the sparse representation.
- Figure 2 compares:
  - CS reconstruction from the reduced data (R=1.5) acquired through Toeplitz random encoding sequence in Figure 1;
  - CS reconstruction from the reduced data (R=1.5) acquired through Fourier encoding with 1D variable density random sampling in [1];
  - Fourier reconstruction from the fully sampled data through Fourier encoding with the conventional spin echo pulse sequence.

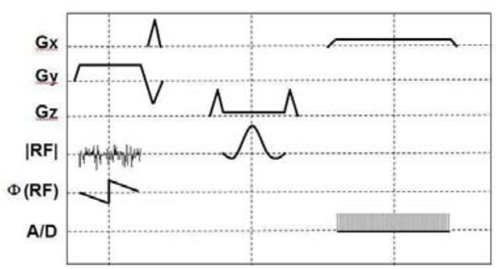

Figure 1. Timing diagram of Toeplitz pulse sequence

Raw data | Data after 1D FT | Toeplitz Random (R=1.5) | Random Fourier (R=1.5) | Full Fourier

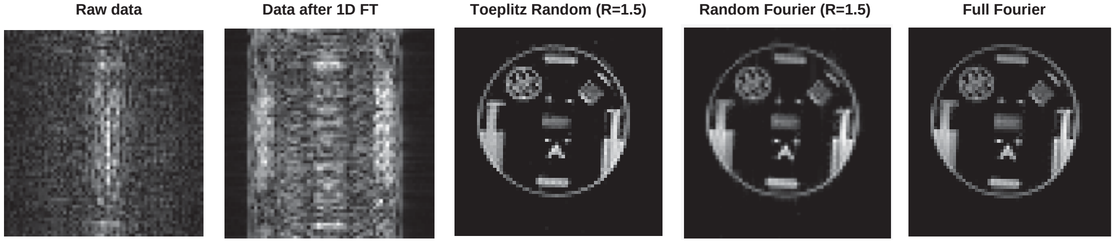

Figure 2. From left to right shows the raw data, the intermediate data after 1D Fourier transform along x direction, the CS reconstructions of Toeplitz random encoding (R=1.5) and randomly sampled Fourier encoding (R=1.5), and the Fourier reconstruction from the fully sampled data.

Discussions

- We have presented a random, non-Fourier encoding technique for application of CS in MRI.
- The experiments have shown promising results to accelerate imaging speed with good reconstruction quality.
- Due to the special property of Teoplitz matrices, the image reconstruction has a fast algorithm, whose complexity is about the same as randomly sampled Fourier encoding.

References

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