PRAIRIE: Accelerating MR Parameter Mapping Using Kernel-Based Manifold Learning and Pre-Imaging

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INTRODUCTION: MR Parameter mapping provides useful quantitative biologic information for tissues property characterizations. However, the clinical application is usually limited by the relatively long acquisition time. Sparsity-based compressed sensing reconstruction methods have been proposed to accelerate the MR parameter mapping [1-6]. These methods usually use linear models which cannot characterize the nonlinearity in the relaxation curve. Recently nonlinear models have been proposed in the compressed sensing framework and shown success [7-9]. In this study, we propose a novel framework to utilize nonlinear models to sparsely represent the unknown image, named PRAIRIE. Different from the prior work with nonlinear models where the image series is reconstructed simultaneously, each image at a specific time point is assumed to lie in a low-dimensional manifold and is reconstructed individually. The low-dimensional manifold is learned from the training images generated by the parametric model. To reconstruct each image, among infinite number of solutions that satisfy the data consistent constraint, the one that is closest to the manifold is selected as the desired solution. The underlying optimization problem is solved using kernel trick [10] and split Bregman iteration algorithm. The proposed method was evaluated on a set of in-vivo brain T2 mapping data set and shown to superior to the conventional compressed sensing methods.

METHODS: We define a nonlinear mapping $\Phi : \Re^p \to \Im^q$ that maps the original data space $\Re^p$ to a much higher dimensional feature space $\Im^q$. Let $\mathbf{x}$ be the vectorized parameter-weighted image at the t-th echo time to be reconstructed. We assume that $\mathbf{x}$ is nonlinearly d-sparse, meaning it is can be represented by its first d largest coefficients in an orthonormal basis from the feature space $\Im^q$, i.e. $\Phi(\mathbf{x}) = \sum_{k=1}^{d} \beta_k v_k$ with β unknown and the basis function $\{v_k\}_{k=1}^{q}$ given by $\mathbf{V} = \sum_{l=1}^{L} \alpha_l \Phi(\mathbf{x}_l)$, where the coefficients α_l is obtained from L training images $\mathbf{x}_i$, $i=1,\ldots,L$ using kernel PCA, $\mathbf{K}\boldsymbol{\alpha} = \lambda \boldsymbol{\alpha}$ where $\mathbf{K}_{i,j} = k(\mathbf{x}_i, \mathbf{x}_j)$ and k is a kernel function.

Kernel-Based Manifold Learning: In the first step, to find the sparse coefficient $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_d]^T$ in the feature space, we solve $k(\mathbf{x}, \mathbf{f}_i) = \mathbf{G}\boldsymbol{\beta}$ (1). We assume a polynomial kernel $k(\mathbf{x}, \mathbf{f}_i) = (\mathbf{x}^T \mathbf{f}_i + a)^5 = (\mathbf{y}_i + a)^5$, where $\mathbf{y}_i$ is the ith k-space data points, $\{\mathbf{f}_i\}_{i=1}^{n}$ is the i-th row of the undersampled Fourier transform matrix $\mathbf{F}$, and $\mathbf{G} = \mathbf{M}\boldsymbol{\Lambda}$ with $\mathbf{M}_{i,j} = k(\mathbf{x}_i, \mathbf{f}_j)$ and $\boldsymbol{\Lambda}$'s rows formed by $\boldsymbol{\alpha}_l^T$, $l=1,2,\ldots,L$. The $\boldsymbol{\beta}$ can be computed by $\boldsymbol{\beta} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{M}$, and thereby we can obtain $\mathbf{z} = \sum_{k=1}^{d} \beta_k v_k$.

Pre-Imaging with Data Consistency and Sparse Constraints: In the second step, we need to find desired image $\mathbf{x}$, which is a pre-image of $\mathbf{z}$, that is $\Phi(\mathbf{x}) = \mathbf{z}$. To enforce that the pre-image satisfies the data consistency and transform-sparse constraints, we formulate the pre-imaging problem as finding $\mathbf{x}$ such that $\min_{\mathbf{x}} \|\mathbf{Fx} - \mathbf{y}\|_2^2 + \lambda_1 \|\mathbf{x}\|_{TV} + \lambda_2 \|\Phi(\mathbf{x}) - \mathbf{z}\|_2^2$ (2), where $\mathbf{F}$ is the undersampled Fourier matrix. If the kernel function $k(\mathbf{x}, \mathbf{f}_i) = f(\mathbf{x}^T \mathbf{f}_i)$ is invertible, then the desired image is given by the solution to the minimization problem $\min_{\mathbf{x}} \|\mathbf{Fx} - \mathbf{y}\|_2^2 + \lambda_1 \|\mathbf{x}\|_{TV} + \lambda_2 \left\| \mathbf{x} - \sum_{i=1}^{p} f^{-1}\left(\sum_{l=1}^{L} c_l k(\mathbf{x}_i, \mathbf{f}_i) \right) \mathbf{f}_i \right\|_F^2$ (3), where $c_l = \sum_{k=1}^{d} \beta_k \alpha_l^k$. The first term represents the data consistency, the second term exploits the sparsity constraint, and the third term enforces the solution to lie on the manifold spanned by the predefined feature space. The optimization problem is convex and is solved by split Bregman iteration algorithm.

RESULTS: We evaluate the proposed method using T2 mapping, in which the signal is described by an exponential decay. However, the method can be generalized for any other parameter mapping models. A set of brain data was acquired on a 3T scanner (MAGNETOM Trio, SIEMENS, Germany) using a 12-channel head coil with a turbo spin echo sequence (matrix size = 192 × 192, FOV = 192 × 192 mm, slice thickness = 3.0 mm, ETL = 16, ΔTE = 8.8 ms, TR = 4000 ms, bandwidth = 362 Hz/pixel). The k-space obtained at the first echo time was not used due to its hypointense. To simulate the reduced acquisition retrospectively, the k-space data was randomly undersampled along the phase encoding direction at each echo time with a reduction factor of 4 and 8, respectively. Different sampling patterns were used at different TEs. The k-space at the second echo time was fully sampled and used to form an image $\boldsymbol{\rho}$. Similar to MR fingerprinting [11], the training images were generated by $\boldsymbol{\rho} e^{(-TE/T2)}$ with a number of possible T2 map based on the segmentation of different tissues. A total of 976 training images were generated for each echo

time. The T2 map obtained by pixel-wise fitting using the Levenberg-Marquardt algorithm was used as the gold standard for comparison between the proposed method and the conventional compressed sensing method with principle component analysis as the sparsifying transform (CS-PCA). Fig. 1 shows the reconstructed T2 maps and their corresponding pixel-wise error maps. The overall percentage errors were shown on the left top corner of the error maps. The T2 map obtained by PRAIRIE is seen to be much more accurate than that by the CS-PCA both visually and quantitatively.

CONCLUSION: In this study, a novel reconstruction method with nonlinear models is proposed to accelerate the MR parameter mapping. Our preliminary result demonstrated that the proposed method is able to accurately recover the T2 map at high reduction factors when the conventional compressed sensing methods with linear models fail.

Figure 1. Estimated T2 maps and pixel-wise error maps of the brain dataset using the reconstruction from fully sampled data, CS-PCA and the proposed PRAIRIE method with reduction factors of 4 and 8.

REFERENCES: [1] M. Doneva, MRM 2010. pp. 1114-1120. [2] J.V. Velikina, MRM 2013. pp. 1263-1273. [3] F.H. Petzschner, MRM 2011. pp. 706-716. [4] C. Huang, MRM 2012. pp.1355-1366. [5]W. Li, MRM 2012. pp. 1127-1134. [6] B. Zhao. MRM 2014. Preprint. [7]Y. Zhou. ISBI 2013, pp. 310-313. [8] Y. Wang. EMBC 2014. Preprint. [9] F.M. Johannes, ISMRM 2014. pp 746. [10] H.Qi. ICASSP 2011. pp. 3940-3943. [11] D. M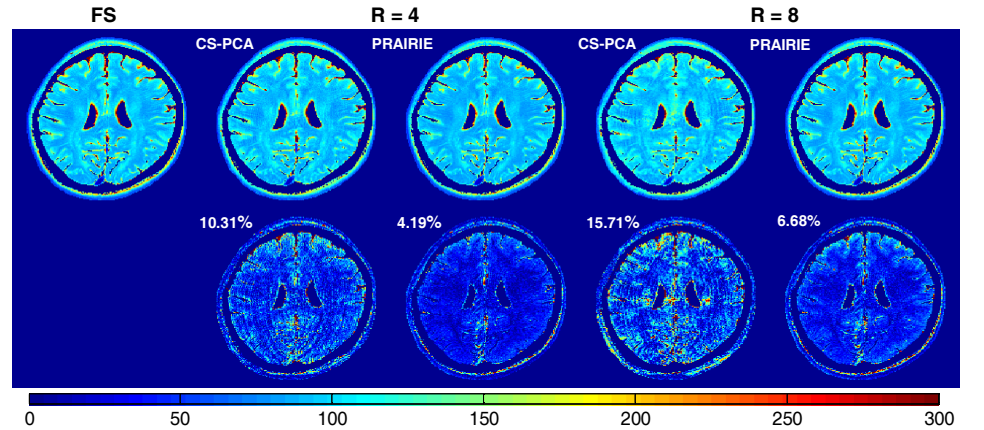a. Nature 495. pp. 187-192. 2013.