SPARSESENSE: APPLICATION OF COMPRESSED SENSING IN PARALLEL MRI

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ABSTRACT

Compressed sensing (CS) has recently drawn great attentions in the MRI research community. The most desirable property of CS in MRI application is that it allows sampling of k-space well below Nyquist sampling rate, while still being able to reconstruct the image if certain conditions are satisfied. Recent work has successfully applied CS to reduce scanning time in conventional Fourier imaging. In this paper, the application of CS to parallel imaging, a fast imaging technique, is investigated to achieve an even higher imaging speed. The sampling scheme for incoherence is discussed and reconstruction method using Begman iteration is proposed. Our experiments show that the combined method, named SparseSENSE, can achieve a reduction factor higher than the number of channels.

***Index Terms*—** SENSE, Compressed Sensing, Bregman iteration

1. INTRODUCTION

Conventional MRI using Fourier imaging is based on Shannon sampling theory. The large number of required samples that are sequentially sampled greatly limits the MR imaging speed. The new CS theory [1,2] allows exact recovery of a sparse signal from a set of samples that appears to be highly incomplete in Shannon sampling theory. This feature is very desirable in MRI for significant reduction in scan time without the requirement of improved gradient performance. Lustig et al [3,4] have applied CS to MRI to reduce the number of samples and have reported impressive results.

Improving on Fourier imaging, parallel MRI is an advanced fast imaging technique to reduce the number of samples using multiple channels for simultaneous data acquisition. Based on the generalized sampling theory, its maximum reduction in the number of samples is limited by the number of channels. In this paper, we apply CS to parallel MRI to achieve a reduction factor even higher than the number of channels. We randomly undersample the k-space along the phase encoding direction to ensure incoherent point spread functions required by CS. In reconstruction of the desired image from the reduced samples, we use Bregman iterations, in replace of the commonly used L_1 regularization method, to solve the constrained convex programming in CS accurately and efficiently. It has been shown that Bregman iteration is able to achieve a very accurate solution to the original constrained problem of CS, yet with a low computational and memory cost even when the equation is large-scale [5]. The feasibility of the proposed method has been validated by experiment results. The results show that the CS-based parallel MRI, named SparseSENSE, can achieve a reduction factor higher than the number of channels.

2. REVIEW OF THE EXISTING WORK

2.1 Compressed Sensing in MRI

In CS theory, a signal $\mathbf{x}$ with a sparse representation in the basis $\mathbf{\Psi}$, can be recovered from the compressive measurements $\mathbf{y} = \mathbf{\Phi x}$, if the $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are incoherent [6]. To recover the signal x, CS solves the convex programming

$$\text{Minimize } \|\Psi(x)\|_1 \quad \text{subject to } \mathbf{\Phi x} = \mathbf{y} \qquad (1)$$

In MRI, incoherence is satisfied when $\mathbf{\Psi}$ is the gradient matrix or the finer scales of a wavelet transform and $\mathbf{\Phi}$ is a random subset of rows from the discrete Fourier matrix [7].

However, there are two major issues with the practical implementation of CS theory in MRI. First, the random sampling of k-space in all dimensions is generally impractical as the k-space trajectories have to be relatively smooth due to hardware and physical considerations [3]. Instead, Lustig et al [3] designed practical sampling scheme for conventional Fourier imaging which randomly undersamples Cartesian grid along the phase-encoding directions only and fully samples the readouts. The level of incoherence was measured by the shape of the point spread function. In addition, the constrained problem in Eq. (1) is computationally and memory intensive when the equation is large-scale [5]. To simplify the calculation of the constrained optimization problem in Eq. (1), the unconstrained problem known as L_1-norm regularization is usually solved instead:

$$\arg\min_{\mathbf{x}}\{\|\mathbf{y} - \mathbf{\Phi x}\|_2^2 + \lambda\|\Psi(\mathbf{x})\|_1\}, \qquad (2)$$

where λ is the regularization parameter which manages the tradeoff between data consistency and the sparsity prior. In [3], both wavelet and total variation are used as the sparse representations to solve

$$\arg\min_{\mathbf{x}}\{\|\mathbf{y} - \mathbf{\Phi x}\|_2^2 + \lambda_1\|\mathbf{Wx}\|_1 + \lambda_2\mathrm{TV}(\mathbf{x})\} \qquad (3)$$

where $\mathbf{W}$ is the wavelet transform matrix, and $\mathrm{TV}(\cdot)$ is total variation [8]. Several numerical methods can be used to solve the L_1 regularization problem in Eq. (2), such as nonlinear conjugate gradients (CG) [9] and primal-dual interior point solver [10]. Most method reduce complexity by local linearization for the L_1-norm term.

2.2 Parallel MRI

Parallel MRI [11-13] is a new technique to improve on conventional Fourier encoding for fast imaging. In parallel imaging, the k-space data are acquired from multiple channels simultaneously, but sampled with a rate lower than the Nyquist sampling rate such that the data acquisition time is reduced. The imaging equation for the k-space data acquired at the *l*th receiver coil can be expressed as

$$d_l(\vec{k}_m) = \int \rho(\vec{r}) s_l(\vec{r}) e^{-i2\pi \vec{k}_m \vec{r}} d\vec{r} \tag{4}$$

where $\rho(\vec{r})$ is the spin density of the desired object, $d_l(\vec{k}_m)$ is the data measured at the *k*-space location $\vec{k}_m$ by the *l*th receiver channel whose sensitivity profile is $s_l(\vec{r})$. The imaging equation is written in matrix form as

$$\mathbf{Ef} = \mathbf{d}, \tag{5}$$

where $\mathbf{d}$ is the vector formed from all *k*-space data acquired at all channels, and $\mathbf{f}$ is the unknown vector formed from the desired image to be reconstructed, both with a lexicographical column ordering of the two-dimensional array components. The encoding matrix $\mathbf{E}$ consists of the product of Fourier encoding with undersampled *k*-space and channel-specific sensitivity modulation over the image, i.e.

$$\mathbf{E}_{\{l,m\},n} = e^{-i2\pi \vec{k}_m \cdot \vec{r}_n} s_l(\vec{r}_n), \tag{6}$$

According to the generalized sampling theorem [14], the maximum reduction factor is equal to the number of channels if $\mathbf{E}$ is of full rank and $\mathbf{d}$ is free of noise. Because these conditions are rarely guaranteed in practice, the reduction factor is usually lower. In presence of noise in MR data measurement, SENSE algorithm [11] is popularly used to solve the matrix equation:

$$\min_{\mathbf{f}} \| \mathbf{d} - \mathbf{Ef} \|. \tag{7}$$

If $\mathbf{E}$ is ill-conditioned (close to rank deficient), regularization is usually used.

3. PROPOSED METHOD

We combine CS with parallel imaging to achieve an even higher reduction in data samples. The combined method should satisfy the incoherence requirement of CS, and solve a constrained optimization problem accurately and efficiently.

3.1 Sampling Scheme

In Fourier imaging, the random sampling can be denoted by element-wise multiplication with a vector whose components are either 0 or 1 distributed randomly. This multiplication in Fourier domain is equivalent to convolution with a random point spread function in image domain. The degree of randomness in the point spread function is used to measure level of incoherence in [3]. In parallel imaging, the point spread function depends not only on the sampling pattern, but also on the channel sensitivities. In addition to random sampling, the extra degrees of freedom from the sensitivity functions can be utilized to improve the incoherence for better performance. CS-based parallel imaging may have an even greater potential for high reduction factors when incoherence is also considered in coil design.

Due to the practical constraints in our study, we use commercially available phased array coils with deterministic sensitivities. Similarly to [3], we randomly generate some sampling patterns that undersample the k-space along the phase encoding direction, and evaluate the goodness of the patterns based on the shape of the point spread functions. The sampling pattern resulting in noise-like random point spread functions is chosen for data acquisition.

3.2 Reconstruction algorithm

In noise-free CS parallel MRI, the image is reconstructed by the constrained nonlinear convex program:

$$\text{Minimize } \|\mathbf{\Psi}(\mathbf{f})\|_1 \quad \text{subject to } \mathbf{Ef} = \mathbf{d} \tag{8}$$

In [3], the above program is approximated by the unconstrained L_1 regularization problem

$$\arg\min_{\mathbf{f}} \{\|\mathbf{d} - \mathbf{Ef}\|_2^2 + \lambda \|\Psi(\mathbf{f})\|_1\} \tag{9}$$

for simplified computation. However, the solution never equals that of Eq. (8) unless the solution is trivial. Here we use Bregman iteration on top of the L_1 regularization. It has been shown that Bregman iteration is able to achieve an accurate solution to the original constrained problem in (8) with a very small multiple of the computational complexity of (9) [5].

To solve (8), in each Bregman iteration, the solution is obtained for the regularization problem [15, 16]

$$\mathbf{f}_k = \arg\min_{\mathbf{f}} \{\|\mathbf{Ef} - \mathbf{d}\|_2^2 + \lambda D_\Psi(\mathbf{f}, \mathbf{f}_{k-1})\}, \tag{10}$$

where $D_\Psi(\mathbf{f}, \mathbf{f}_{k-1})$ is the Bregman distance based on a convex functional $\Psi(\cdot)$ between point $\mathbf{f}$ and $\mathbf{f}_{k-1}$:

$$D_\Psi(\mathbf{f}, \mathbf{f}_{k-1}) = \| \Psi(\mathbf{f}) \|_1 - \| \Psi(\mathbf{f}_{k-1}) \|_1 - \langle \mathbf{f} - \mathbf{f}_{k-1}, \partial \| \Psi(\mathbf{f}_{k-1}) \|_1 \rangle \tag{11}$$

where $\langle \cdot, \cdot \rangle$ denotes inner product and $\partial(\| \Psi(\mathbf{f}_{k-1}) \|_1)$ is subgradient of $\| \Psi(\mathbf{f}_{k-1}) \|_1$ at the point $\mathbf{f}_{k-1}$. The Bregman distance is an indication of the increase in $\| \Psi(\mathbf{f}) \|_1$ over $\| \Psi(\mathbf{f}_{k-1}) \|_1$ above linear growth with slope $\partial \| \Psi(\mathbf{f}_{k-1}) \|_1$. It has been shown that the sequence $\mathbf{Ef}_k$ monotonically converges to the acquired data $\mathbf{d}$ such that the constraint in (8) holds [16]. We name the proposed Bregman iterative nonlinear reconstruction method for randomly undersampled parallel MRI SparseSENSE.

3.3 Implementation

The minimization problem in Eq. (10) seems complicated, but is actually equivalent to [16]

$$\mathbf{f}_k = \arg\min_{\mathbf{f}} \{ \| \mathbf{v}_{k-1} + \mathbf{d} - \mathbf{Ef} \| + \lambda \| \Psi(\mathbf{f}) \|_1 \}, \qquad (11)$$

where $\mathbf{v}_k = \mathbf{v}_{k-1} + \mathbf{d} - \mathbf{Ef}_k$ for $k > 0, \mathbf{v}_0 = 0$. It is seen that the above formulation is the same as an L_1 regularization except that $\mathbf{d}$ is replaced by $\mathbf{v}_k$. Among many algorithms for the nonlinear L_1 regularization, we use lagged diffusivity fixed-point algorithm, whose details can be found in [17]. When the regularization parameter λ is chosen properly, only several Bregman iterations would be sufficient for an accurate solution of Eq. (8).

There are several popular choices for the sparse representation including gradient and wavelet transform. Here, we select $\| \Psi(\cdot) \|_1$ to be the total variation such that

$$\|\Psi(\mathbf{f})\|_1 = \sum \sqrt{|\nabla_x \mathbf{f}_r|^2 + |\nabla_x \mathbf{f}_i|^2 + |\nabla_y \mathbf{f}_r|^2 + |\nabla_y \mathbf{f}_i|^2} \qquad (12)$$

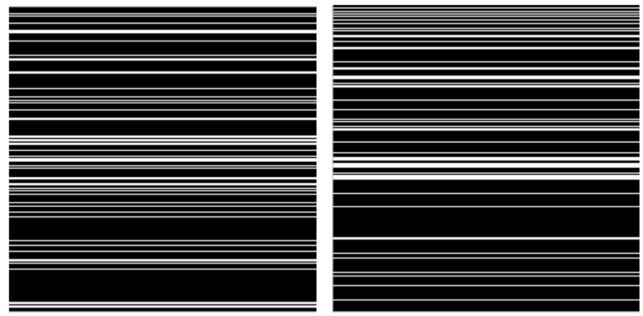
where f_r and f_i are real and imaginary part of the complex MR image, and ∇_x and ∇_y denote the gradient along x and y respectively. In addition, to speed up computation, NUFFT [18], a min-max optimal method for nonuniform fast Fourier transform, has been used for the variable-density Cartesian trajectory.

4. EXPERIMENTAL RESULTS

We show representative results from both phantom and in vivo data sets. To demonstrate the feasibility of the proposed method to achieve a reduction factor higher than the number of channels, the results here are for the noise-free case only. In present of noise, parallel imaging may suffer from ill-conditioning problem, which requires additional regularization and results in a lower achievable reduction factor. In both experiments, we use the image and sensitivity functions generated from real data sets to simulate the acquired k-space data without noise.

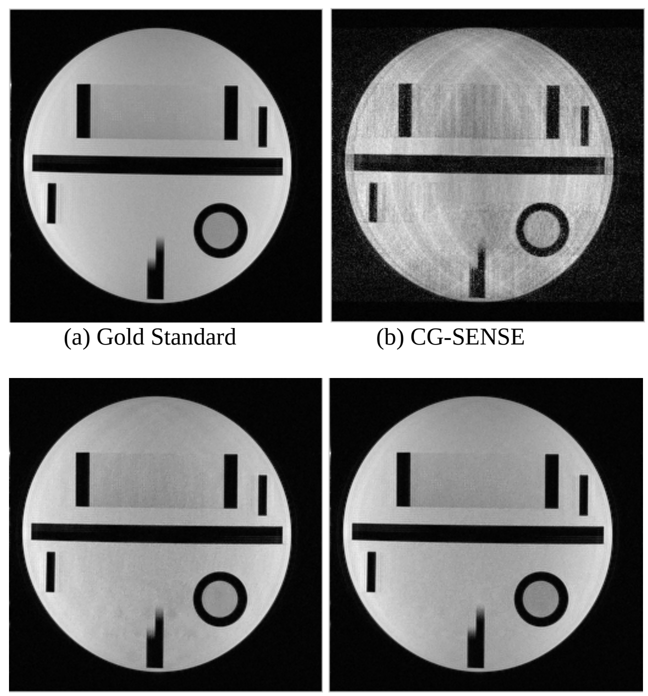
Phantom data were collected on a Hitachi Airis Elite 0.3T permanent magnet scanner with a four-channel head coil and a single slice spin echo sequence (TE/TR = 40/1000ms, 8.4KHZ bw, 256x256 matrix size, FOV = 220 mm2). 52 out of 256 k-space lines in phase-encoding direction were randomly selected to simulate a reduction factor 5. The selected sampling pattern that results in noise-like random point spread functions is shown in Fig 1(a).

The Bregman iterative regularization algorithm was implemented in MATLAB. Fig. 2 (a) shows the sum-of-square reconstruction from the fully sampled data as the gold standard for comparison, (b) shows the linear conjugate gradient (CG) SENSE reconstruction [8] after 40 iterations, (c) TV regularized SENSE reconstruction using Eq. (9) after 90 iterations, and (d) shows the reconstruction using the proposed SparseSENSE after 9 Bregman iterations, each with 10 inner-TV-based iterations for Eq. (10). The SparseSENSE reconstruction shows only very few discernable artifacts compared to the gold standard.



(a) (b)

Fig. 1 Sampling patterns for (a) phantom and (b) brain data.



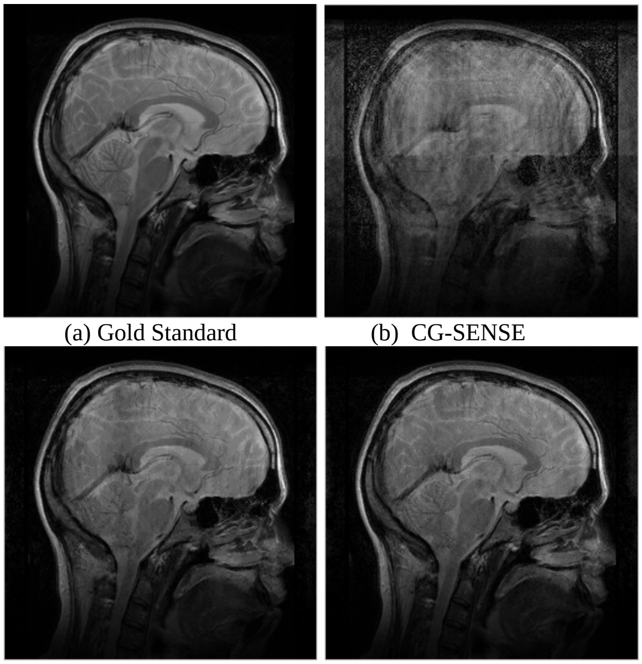
(a) Gold Standard (b) CG-SENSE

(c) TV regularization (d) Proposed SparseSENSE

Fig. 2 Phantom experiment results.

A set of brain data was collected on a 1.5T SIEMENS Avanto system with a 4-channel head coil. Similarly, 52 out of 256 k-space lines in phase-encoding direction were randomly selected for reconstruction. The sampling pattern is shown in Fig. 1 (b). Similarly, the gold standard, the reconstruction using CG-SENSE, TV regularization, and the proposed SparseSENSE are shown in Fig.3 (a), (b), (c), and (d), respectively.

The running time for 9 Bregman iterations is about 10 minutes on a HP xw8400 workstation (2.33GHz CPU and 4GB RAM), which is about the same as that for 90 iterations of TV regularization.



(a) Gold Standard (b) CG-SENSE

(c) TV regularization (d) Proposed SparseSENSE

Fig 3. In vivo experiment results

5. DISCUSSION AND CONCLUSION

The proposed method demonstrates the feasibility of combining CS with parallel MRI. In data acquisition, we use the practical pseudo random sampling scheme, and in image reconstruction, we use the Bregman iterative regularization method to solve the constrained optimization problem. Our parallel imaging experiments show that the proposed method is able to achieve a reduction factor higher than the number of channels in absent of noise. In present of noise, the reduction factor may be lower when the parallel imaging equation is ill-conditioned. In addition, the computational cost of Bregman iterations is about the same as that of the linear CG-SENSE reconstruction method.

In our implementation, we minimize the total variation under the data consistent constraint. Our future work would explore other sparse representations such as wavelet for possible improvement on reduction factors.

6. ACKNOWLEDGMENTS

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