Accelerating SENSE using distributed compressed sensing

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INTRODUCTION:

With the emergence of compressed sensing (CS) theory (1-2), its combination with parallel magnetic resonance imaging (pMRI) is of great interests. SparseSENSE has been proposed independently (3-6) which applies CS to SENSE (7) with random sampling as a regularization method. However, there are a couple of issues with the method. First, the incoherence condition required in CS cannot be guaranteed in SparseSENSE because the encoding matrix is channel sensitivity dependent and can vary from scan to scan. In addition, the imaging equation of SparseSENSE is usually overdetermined, which does not fit in the underdetermined CS framework. In this abstract, we propose a novel method to address these issues in applying CS to SENSE. The method first reconstructs a set of aliased images from all channels simultaneously using distributed CS (DCS), which is an extension of CS for multi-signal ensembles that exploit both intra- and inter-signal correlation structures (8). And the Cartesian SENSE reconstruction is then applied to the aliased images for the final desired image. The use of DCS brings in further acceleration than CS when taking advantage of the fact that the aliased images from all channels share the same sparse support in the sparsifying transform domain.

THEORY AND METHOD:

The sensitivity encoding in pMRI can be decomposed to two sequential linear operations when the sampling position is on a uniform Cartesian grid. The first one is the sensitivity modulation, where the original full FOV image, weighted by different sensitivities from all channels, is folded to generate a set of aliased images with reduced FOV. The second one is Fourier transform of aliased images $\mathbf{F}\mathbf{f}^{A}=\mathbf{d}$ [1], where **F** the Fourier operator, $\mathbf{f}^{A}=[\mathbf{f}_1^{A},\mathbf{f}_2^{A},\cdots,\mathbf{f}_L^{A}]$ with size of $N\times L$ and $\mathbf{f}_l^{A}$ the aliased image of the *l*-th coil with reduced FOV, $\mathbf{d}=[d_1,d_2,\cdots,d_L]$ and $\mathbf{d}_l$ the reduced *k*-space data from the *l*-th coil. Since Eq. [1] is the same as the conventional Fourier encoding formulation, the same random sampling scheme as that for SparseMRI (9) can be employed to further undersample the phase encoding lines that are already reduced for the aliased images with reduced FOV. Here in reconstruction, DCS is used instead of CS to take advantage of the fact that all aliased images share the same sparse support in the sparsifying transform domain. With DCS, all aliased images are reconstructed simultaneously by a row-l_0 minimization: $\min\ \left(\#\ of\ nonzero\ rows\ in\ \mathbf{f}^{A}\right)$ s.t. $\mathbf{F}^{u}\mathbf{f}^{A}=\mathbf{d}^{u}$ [2], where $\mathbf{F}^{u}$ the random subset of the rows of the Fourier encoding matrix and $\mathbf{d}^{u}=[d_1^{u},d_2^{u},\cdots,d_L^{u}]$ is the randomly undersampled reduced *k*-space data from all channels. Because solving Eq. [2] is NP-hard, the objective function is replaced with a closely related convex function as $\min_{\mathbf{f}^{A}}\ \|\mathbf{C}_1\|_2+\|\mathbf{C}_2\|_2+\cdots+\|\mathbf{C}_N\|_2$ s.t. $\mathbf{F}^{u}\mathbf{f}^{A}=\mathbf{d}^{u}$ [3] where $\mathbf{C}_i$ is the *i*-th row of $\mathbf{f}^{A}$ (10). This objective function is equivalent to applying the l_2 norm to rows (to promote nonsparsity) and then applying the l_1 norm to the resulting vector (to promote sparsity). It suggests the aliased images should all be sparse under the same transform, but all images should have non-zero entries at the same support in the transform domain. SPGL1 (11) was used to solve this convex relaxation problem. With the aliased images from all channels, the desired full FOV image **f** can be reconstructed pixel by pixel using the image domain basic SENSE method (12). It is easy to see that the net reduction factor *R* of proposed method is equal to the product of the reduction factor R_1 in DCS reconstruction and the reduction factor R_2 in Cartesian SENSE, i.e., $R = R_1 \times R_2$.

Figure 1: Phantom images reconstructed using proposed method(A), SparseSENSE (B), SM-SoS (C), and SENSE (D) from a set of eight-channel scanned data with different reduction factors.

RESULTS AND DISCUSSION:

A T1-weighted phantom scan was performed using a 2D spin echo sequence on a 3T commercial scanner (GE Healthcare, Waukesha, WI) with an 8-channel torso coil (TE/TR = 11/300 ms, 18cm FOV, 8 slices, 256 × 256 matrix). The full *k*-space data were acquired and randomly undersampled to simulate a reduction factor of 4, 6, 8 and 12. The central 32 fully sampled phase encodings were used to estimate the channel sensitivity profiles. The reconstructions of proposed method (A), SparseSENSE (B), SparseMRI for reconstructing full FOV image of each channel followed by the sum-of-square (SM-SoS) (C), and SENSE (D) are shown in Fig. 1for comparison The method and the reduction factor are shown on the top left and right corner of each image. In addition, the corresponding "comb" region in the phantom was zoomed to reveal details. All the algorithms were implemented in MATLAB (MathWorks, Natick, WA). For a moderate reduction factor (R = 4), both the proposed method and SparseSENSE are able to reconstruct an image that is visually almost the same as the reference SoS image. (The SoS image is not shown in Fig. 1 because this similarity.) In contrast, SM-SoS has visible artifacts and SENSE has visibly more noise. As the reduction factor becomes larger (R = 6 and 8), the proposed method reconstruction is seen to be less blurry with more details than the SparseSENSE reconstruction, which is better shown in the zoomed "comb" region. At an even larger reduction factor (R = 12), both methods fail to reconstruct the phantom image faithfully.

CONCLUSION:

A novel method is proposed to accelerate the conventional SENSE using DCS. The results show that the proposed method is able to achieve higher reduction factors than the existing methods.

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