Compressed Sensing MRI with Random B1 field

F. M. Sebert[1], Y. Zou[1], B. Liu[1], and L. Ying[1]

[1]University of Wisconsin - Milwaukee, Milwaukee, WI, United States

INTRODUCTION

Considerable attention has been paid to compressed sensing (CS) in the MRI community recently (1-3). CS theory allows exact recovery of a sparse signal from a highly incomplete set of samples in conventional sense, and thus has the potential for significant reduction in MRI scan time. According to the theory, images with a sparse representation can be recovered from randomly undersampled k-space data using nonlinear convex programming (1). Since completely random sampling trajectories are not feasible in MRI practice due to hardware and physiological constraints, most attention has been focused on the design of practical sampling schemes whose incoherence properties are close to those of random undersampling (3). In this abstract, we revisit the sufficient conditions for CS matrices, and show that if the receiver coil sensitivities or the RF excitation profiles can be designed to be spatially random, then the image can be recovered from uniformly undersampled k-space data. Our finding provides an alternative to the random sampling design for reducing the acquisition time using CS. In addition, it has the advantage that the number of samples needed is less than what is needed in the random sampling scheme.

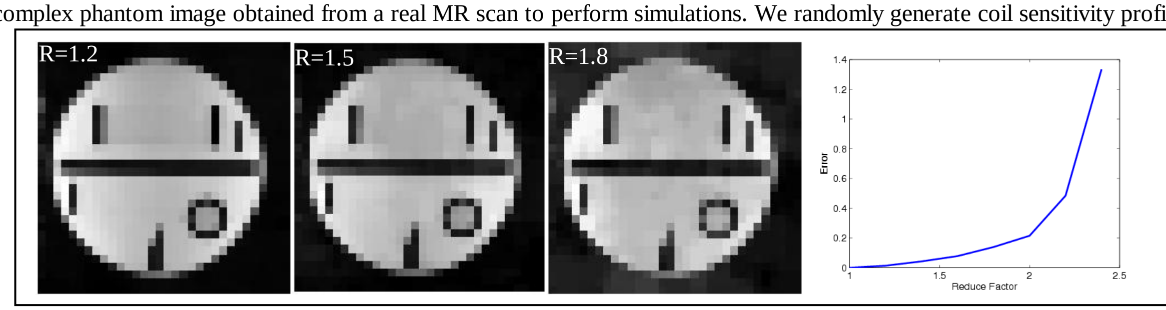
THEORY

In CS theory, it was established (4) that for a $\mathbf{\Phi}$ to be a good CS matrix, it is sufficient that it satisfies the following *Restricted Isometry Property* (RIP) condition: $(1-\delta_S)\|\mathbf{x}\|_2^2 \le \|\mathbf{\Phi x}\|_2^2 \le (1+\delta_S)\|\mathbf{x}\|_2^2$, where $\delta_S \in (0,1)$, [1]. A fundamental result by Candès and Tao (4) states that if the kxn (k<n) encoding matrix $\mathbf{\Phi}$ has RIP for any $\boldsymbol{x}$ with sparsity *S*, then the original signal $\boldsymbol{x}$ can be reconstructed exactly, with overwhelming probability, from very few samples (given by a vector **y**), by solving the convex optimization problem: Minimize $\|\mathbf{x}\|_1$ subject to $\mathbf{\Phi x} = \mathbf{y}$ [2], where $\|\mathbf{x}\|_1$ is the L_1 norm. We have proved that if the observation matrix is $\mathbf{\Phi} = \mathbf{AF\Psi}$, where $\mathbf{\Psi}$ is the sparsity basis, **F** is the Fourier transform matrix, and **A** is a block random Toeplitz matrix, then the encoding matrix $\mathbf{\Phi}$ is a good CS matrix (detailed proof appears else where). For the applications considered here, the bloack Toeplitz matrix has the form shown in [3], where k<n and the elements a_{ij} are drawn independently from certain probability distributions (5). Our result implies that this Toeplitz matrix $\mathbf{\Phi}$ satisfies RIP with probability $1-\exp(-ckn/S^2)$ for some constant c depending only on δ_S provided the number of samples provided the number of samples kn $\ge$ $c_2 \cdot S^3 \log(n/S)$ (see also (6)), for any **x** of size n^2 with sparsity of *S*. To interpret the sensing matrix in MR acquisition, $\mathbf{\Psi}x$ is the desired image, multiplication with **F** gives k-space data, and multiplication with a block Toeplitz matrix represents partial sampling of a two-dimensional convolution on uniform Cartesian grid. It is known that convolution in k-space is equivalent to pixel by pixel products in the image domain. Therefore, our result suggests that if the image has a sparse representation (e.g. wavelet basis) and the coil sensitivity or excitation profile is spatially random, then the image can be reconstructed from uniformly undersampled k-space data on a Cartesian grid. The degree of randomness in the sensitivity determines how "incoherent" the encoding operation is, and thus affects the minimum number of samples required for reconstruction. Although the coil sensitivity or excitation profile cannot be completely random due to practical constraints, their goodness can be evaluated by the level of incoherence in the encoding matrix $\mathbf{\Phi}$ (7). The fact that the bound of random Toeplitz block matrices is lower than that of random sampling in Fourier encoding suggests that the random profile design would require a smaller number of samples than the random Fourier sampling does.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_n} & \mathbf{A_{n-1}} & & \mathbf{A_2} & \mathbf{A_1} \\ \mathbf{A_1} & \mathbf{A_n} & & \mathbf{A_3} & \mathbf{A_2} \\ \vdots & & \ddots & & \\ \mathbf{A_{k-1}} & \mathbf{A_{k-2}} & \cdots & \mathbf{A_{k+1}} & \mathbf{A_k} \end{bmatrix}, \quad \mathbf{A_i} = \begin{bmatrix} a_{in} & a_{i(n-1)} & & a_{i2} & a_{i1} \\ a_{i1} & a_{in} & & a_{i3} & a_{i2} \\ \vdots & & \ddots & & \\ a_{i(n-1)} & a_{i(n-2)} & \cdots & a_{i1} & a_{in} \end{bmatrix} \quad [3]$$

METHOD AND RESULTS

We use a 32x32 (n=32) complex phantom image obtained from a real MR scan to perform simulations. We randomly generate coil sensitivity profiles, and then sample the resulting k-space data on a uniform Cartesian grid. Without loss of generality, undersampling was performed only along the phase encoding direction in our simulation, and for a reduction factor of R, k=round(32/R) central k-space lines are acquired. Figure 1 shows the reconstructed images by the nonlinear optimization program of [2] at different R's, and the average error curve as a function of R. Total variation was used as the sparse representation.



DISCUSSION AND CONCLUSION

We have presented an alternative to random sampling for application of CS in MRI. The method can reduce the acquisition up to a factor of 2 with good reconstruction quality. With multiple coils for transmission and receiving, the design of random sensitivity and excitation profiles should be more feasible. Our theoretical results also support the conclusion made in (8).

REFERENCES

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