Compressed-Sensing Dynamic MR Imaging with Partially Known Support

Dong Liang and Leslie Ying

***Abstract*— Compressed Sensing (CS) has recently been applied to dynamic MRI to improve the acquisition speed. Existing methods exploit the information that the dynamic images are sparse in the spatial and temporal-frequency (*y-f*) domain. In this paper, we propose to use the additional prior information in CS reconstruction that the support of *y-f* space is partially known from the motion pattern of dynamic MR images. The reconstruction is then formulated as a truncated ℓ_1 minimization problem. Experimental results show that the dynamic image reconstruction quality of the proposed method is superior to that of existing methods when the same number of measurements is used.**

***Index Terms*-**Compressed Sensing, Dynamic MRI, Partially Known Support, Truncated ℓ_1 Minimization

I. Introduction

DATA acquisition speed is a crucial factor in many dynamic MRI applications. It is desirable to reduce the amount of acquired data while preserving high spatiotemporal resolution. Many methods have been developed to achieve this objective. These methods can be divided into two categories. One exploits spatial correlations or/and temporal correlations, with RIGR [1], keyhole [2], view-sharing [3], UNFOLD [4], and *k-t* BLAST[5] as typical methods in this category. The other employs the recent compressed sensing (CS) theory [6, 7] to take advantage of the fact that the dynamic MR images are sparse in the spatial and/or temporal-frequency (*y-f*) domain. Methods in this category include *k-t* SPARSE [8], *k-t* FOCUSS [9, 10] and CS for dynamic MRI [11].

Recently, an extension of CS has been developed for signals with partially known support [12-16], where support is defined as the locations of non-zero elements in the sparse domain. Numerical and theoretical studies have show that CS with partially known support can reduce the required number of measurements for exact recovery. Vaswani and Lu [14, 15] have used this technique to reconstruct a series of dynamic MR images frame by frame, where the support information in the spatial-temporal (*y-t*) domain is obtained from the first frame with more measurements than the subsequent frames.

In this paper, we propose to improve the performance of dynamic MRI using CS with partially known support in the spatial and temporal-frequency (*y-f*) domain. The support information is known a priori based on the fact that the non-periodic motion is usually slowly varying and thus the support of the *y-f* domain should include the low temporal-frequency region. The signal at the known support is then excluded from the cost function during the minimization process. The proposed method is different from Ref. [11] in that [11] detects the locations of zero (instead of nonzero) elements in the *y-f* domain, which are then excluded from the cost function of the minimization. *In vivo* experimental results show that the use of support knowledge can improve the dynamic reconstruction quality of the conventional CS methods when the same number of measurements is acquired.

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Dong Liang and Leslie Ying are with the Department of Electrical Engineering and Computer Science, University of Wisconsin, Milwaukee, WI 53201 USA (e-mail: liangd, leiying@uwm.edu).

II. Dynamic MRI

Dynamic MRI is a technique to acquire a time series of images from a time-varying object at a high frame rate. In dynamic MRI, the acquired data is in the spatial-frequency and temporal (*k-t*) space. The *k*-space measurement at time *t* can be represented as

$$d(k,t)=\int m(y,t)e^{-j2\pi ky}dy\ , \tag{1}$$

where $m(y,t)$ is the image at time *t*, or as

$$d(k,t)=\iint \rho(y,f)e^{-j2\pi(ky+ft)}dydf\ , \tag{2}$$

where $\rho(y,f)$ is the Fourier transform of the image series $m(y,t)$ along time. Equation (2) can be written in matrix form as

$$\mathbf{F}\boldsymbol{\rho}=\mathbf{d}\ , \tag{3}$$

where $\mathbf{d}$ is the stacked data in *k-t* space, $\boldsymbol{\rho}$ is the image in the *y-f* domain to be reconstructed, and $\mathbf{F}$ denotes the 2D Fourier transform along the *k-t* direction. Most existing methods use (3) to reconstruct the image in the *y-f* domain.

As a new sampling theory, Compressed Sensing (CS) has been used to improve dynamic MRI performance. This is possible because most dynamic MR image sequences have a sparse/compressible representation in *y-f* space and Fourier encoding generates incoherent measurements. Several works have studied CS techniques in dynamic MRI. Among them, *k-t* FOCUSS [4, 5] is of great interest due to its low computational complexity and its ease to be adapted to solve the $\ell_p(p\le 1)$ minimization problem asymptotically. FOCUSS algorithm is used in *k-t* FOCUSS to asymptotically solve the ℓ_1 minimization problem by iteratively solving a reweighted ℓ_2 minimization problem. Specifically, the ℓ_2 minimization problem is to

$$\textbf{find}\quad \boldsymbol{\rho}=\mathbf{Dq} \tag{4}$$

where $\mathbf{q}$ is the solution to

$$\text{minimize } \|\mathbf{q}\|_2^2 \quad s.t. \quad \|\mathbf{d}-\mathbf{FDq}\|_2 \le \varepsilon, \tag{5}$$

$\mathbf{D}$ is a diagonal weighting matrix, and ε is the noise level. The optimal solution of the above ℓ_2 minimization problem with nonzero initialization has an analytical form:

$$\boldsymbol{\rho} = \overline{\boldsymbol{\rho}} + \mathbf{DD}^H\mathbf{F}^H\left(\mathbf{FDD}^H\mathbf{F}^H + \lambda\mathbf{I}\right)^{-1}\left(\mathbf{d}-\mathbf{F}\overline{\boldsymbol{\rho}}\right), \tag{6}$$

where $\overline{\boldsymbol{\rho}}$ is the temporal DC term in the *y-f* domain calculated by averaging the measurements along the temporal direction and λ is the regularization parameter.

To find the sparse solution $\boldsymbol{\rho}$ to (3), *k-t* FOCUSS iteratively solves the above ℓ_2 minimization by updating the weighting matrix $\mathbf{D}$. In the *k*-th iteration, the diagonal elements of the matrix $\mathbf{D}_k$ are the square root of the absolute value of the solution $\boldsymbol{\rho}_{k-1}$ in the previous iteration. The ℓ_2 minimization in the *k*-th iteration can be represented as

$$\text{minimize } \left\|\mathbf{D}_k^{-1}\boldsymbol{\rho}\right\|_2^2 \quad s.t. \quad \|\mathbf{d}-\mathbf{F}\boldsymbol{\rho}\|_2 \le \varepsilon. \tag{7}$$

Solutions to (7) asymptotically approach the solutions to ℓ_1 minimization with infinite iterations.

III. PROPOSED METHOD

A. Reconstruction Algorithm

In dynamic MRI, we observed that the non-periodic motion is usually slowly varying. It suggests that the temporal-frequency components of the dynamic images are significant only at low temporal-frequency region and thus the support of the signal $\boldsymbol{\rho}$ is partially known in the *y-f* domain. We denote the known support as T with size $|T|$, and the unknown support as Δ with size $|\Delta|$. The known support T should include a rectangular region at the center of the *y-f* domain. The region covers the entire y in the field of view, but only the central f for the temporal frequency. We define the width of the support T along f as $2u$+1, where u ($0 \le u \le \#\text{frame}/2-1$) is the distance from f=0. Specifically, given a point (i,j) in *y-f* space, we have

$$(i,j) \in \begin{cases} T & |j| \le u \\ \Delta & |j| > u \end{cases}. \tag{8}$$

The above knowledge on the support of the signal in the *y-f* domain can be used in CS reconstruction. It allows us to minimize the number of nonzeros outside the support T when searching for a sparse solution to $\mathbf{F}\boldsymbol{\rho} = \mathbf{d}$. Reconstruction of the signal $\boldsymbol{\rho}$ in the *y-f* domain with known support T can be formulated as the following truncated CS problem:

$$\text{minimize } \|\boldsymbol{\rho}_\Delta\|_0 \quad s.t. \quad \mathbf{F}\boldsymbol{\rho} = \mathbf{d}, \tag{9}$$

where $\boldsymbol{\rho}_\Delta$ denotes the signal outside the known support. This formulation favors a solution with more zeros outside the region of known support. In practice, considering the computational complexity and the presence of noise in measurements, we replace the ℓ_0 norm by the ℓ_1 norm and solve the following truncated ℓ_1 minimization problem [13-15]:

$$\text{minimize } \|\boldsymbol{\rho}_\Delta\|_1 \quad s.t. \quad \|\mathbf{d}-\mathbf{F}\boldsymbol{\rho}\|_2 \le \varepsilon \tag{10}$$

As a result, the sparsest solution not only satisfies the data consistency constraint, but also has nonzero values within the known support.

B. Effect of Support Size

Intuitively, the more knowledge on the support of the signal in the *y-f* domain, the fewer measurements are needed for reconstruction. Two extreme cases are conventional CS where no support information is available and the case where all support about $\boldsymbol{\rho}$ is exactly known. For the former, we need measurements at least twice of the support size to recover any signal with that support size. While for the latter, we only need measurements equal to the support size to recover the signal. For truncated CS with known support T, it has been shown [13-15] that at lease $2|\Delta|$ measurements are needed for unknown support Δ and $|T|$ measurements for T when solving truncated ℓ_0 minimization problem. The total required number of measurements will be $|T|+2|\Delta|$. It agrees with our intuition that more knowledge about the support will reduce the required number of measurements for reconstruction. The sufficient conditions (depending on $|T|$) for perfect reconstruction using (9) and (10) have been derived theoretically and shown to be weaker than those of the conventional CS [13-16]. In practice, the support information is difficult to determine accurately. A small u would guarantee inclusion of the true support, but may also miss a good portion of the true support, which results in more measurements. On the other hand, a large u would include locations outside the true support and the wrong support information would result in reconstruction errors. This effect of u on the reconstruction quality is studied in the Experiments and Results section.

C. Robustness

The robustness of truncated ℓ_1 minimization to compressible signals and noisy measurements has been studied in [13, 15]. An upper bound is given for the reconstruction error as

$$\|\boldsymbol{\rho}^* - \boldsymbol{\rho}\|_2 \le c_3 \frac{\|\mathbf{r}-\mathbf{r}^{|\Delta|}\|_1}{\sqrt{|\Delta|}} + c_4\varepsilon \tag{11}$$

where $\boldsymbol{\rho}^*$ is the solution to (10), c_3 and c_4 are some constants, and $\mathbf{r} = \boldsymbol{\rho} - \boldsymbol{\rho}_T$ is the residual only containing the elements in Δ. This bound is proportional to the noise level ε and the approximation error between the residual and its closest $|\Delta|$-sparse signal $\mathbf{r}^{|\Delta|}$.

D. Implementation Issues

In our implementation, (10) was rewritten as an equivalent unconstrained ℓ_1 minimization problem. The signal in the *y-f* domain can be reconstructed by

$$\arg\min_{\boldsymbol{\rho}}\left\{\left\|\mathbf{d}-\mathbf{F}\boldsymbol{\rho}\right\|_2^2+\lambda\left\|\mathbf{W}\boldsymbol{\rho}\right\|_1\right\}, \tag{12}$$

where $\mathbf{W}$ is a weighting matrix whose value is 1 if the corresponding element of $\boldsymbol{\rho}$ in the *y-f* domain belongs to the unknown support, and 0 otherwise. The regularization parameter λ is selected by solving (12) with different values and choosing one so that $\left\|\mathbf{d}-\mathbf{F}\boldsymbol{\rho}\right\|_2 \approx \varepsilon$. The *k-t* FOCUSS algorithm [9, 10] was modified to solve (12). Similar to all other CS methods in MRI, the variable-density random sampling is used with denser sampling near the center of the phase encoding direction.

IV. EXPERIMENTS AND RESULTS

The feasibility of the proposed method was validated on a set of *in vivo* data, acquired on a 1.5 T Philips scanner. The steady-state free precession (SSFP) sequence was used with a flip angle of 50 degree and TR = 3.45msec. The fully acquired *k-t* measurements are with size of 256x256x25. The field of view (FOV) is 345mm x 270mm and the slice thickness is 10 mm. The heart rate was 66 bpm and the retrospective cardiac gating was used.

All methods were implemented in MATLAB (Mathworks, Natick, MA). The image sequences reconstructed from the full *k-t* data were used as the reference for comparison. To simulate reduction factors of 4, 6 and 8, a variable-density random sampling pattern was generated with 8 phase encoding lines being fully sampled at the centre of *k*-space. Both the proposed and *k-t* FOCUSS methods were used to reconstruct the desired image using asymptotic ℓ_1 minimization. All images were normalized and shown on the same scale.

Figure 1 compares one frame of the dynamic sequence reconstructed using the proposed method and *k-t* FOCUSS with that of the reference sequence. A zoomed region is also shown to reveal more detail. In comparison, the proposed method presents fewer undersampling artifacts along the phase encoding direction and preserves more details than *k-t* FOCUSS does. The superiority of the proposed method can also be demonstrated by the difference images shown in Fig. 2. The larger values at the edge of the object in the difference images of *k-t* FOCCUS indicate more loss of resolution, while the ripples indicate more undersampling artifacts.

To quantify the improvement of proposed method over *k-t* FOCUSS, the normalized MSE between the reconstructed and reference frames were calculated and plotted in Fig. 3. Clearly, the proposed method outperforms *k-t* FOCUSS in all frames in terms of MSE.

We also studied the effect of the width parameter u on the reconstruction quality for the proposed method. The frame-by

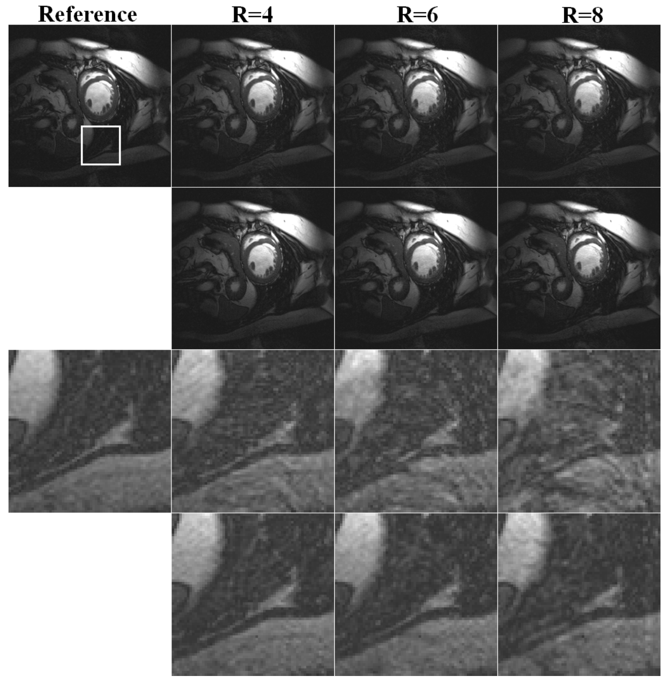

Fig. 1 One frame of the dynamic sequence reconstructed using *k-t* FOCUSS (1st and 3rd rows) and the proposed method (2nd and 4th rows). 1st and 2nd rows are the full field of view and 3rd and 4th rows are the zoomed regions. Reduction factors of 4, 6 and 8 were used.

-frame normalized MSE were calculated for different widths (1, 3, 7, 11, 15 and 19) and different reduction factors (4, 6 and 8). These MSEs were plots in Fig. 4. We can see that the proposed method performs the best at a moderate width (7 or 11) where the support information is the most inclusive and accurate. A larger width (15 and 19) has wrong support information, while a smaller width (1 and 3) includes only a portion of the true support. A moderate support width is preferred to balance the tradeoff. The choice of width was found to be effective when selecting a moderate width (slightly less than the half of frame number).

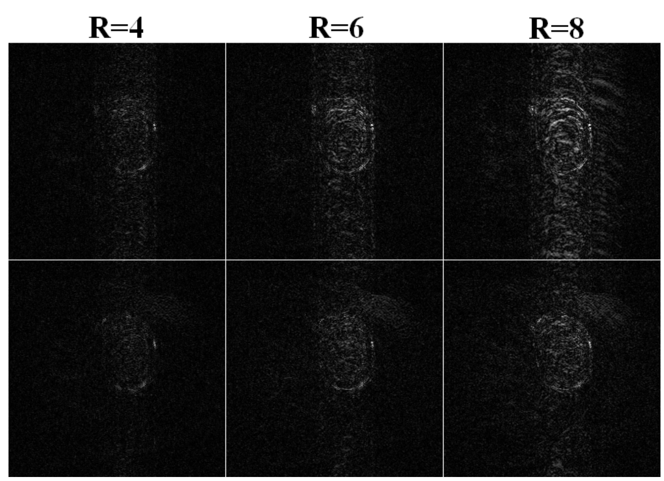

Fig. 2. The error images of the reconstructions in Fig. 1 with respect to the reference. Top row is *k-t* FOCUSS and bottom row is the proposed method.

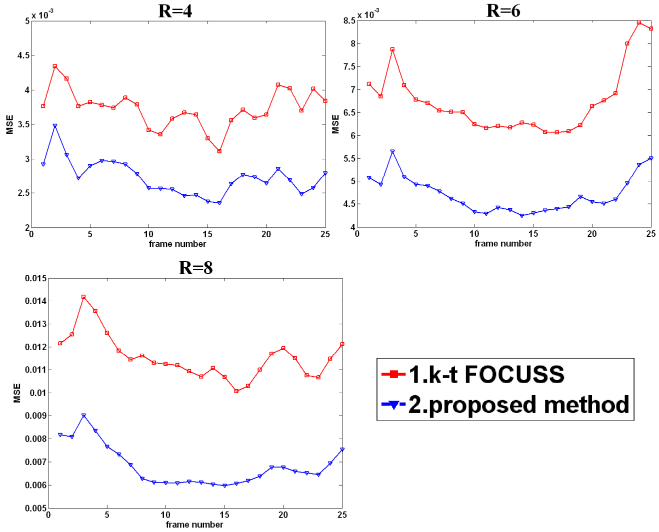

Fig. 3. The frame-by-frame plots of the normalized MSE for *k-t* FOCUSS and proposed method with reduction factors of 4, 6 and 8.

V. CONCLUSION

In this paper, the support knowledge of the dynamic MRI sequence in the *y-f* domain is used to improve the CS reconstruction quality in dynamic MRI. The experiment results show that the proposed method is able to suppress more artifacts and preserve more details than the conventional CS in dynamic MRI. Although this study is based on single-coil Cartesian acquisition, the proposed method can be easily extended to multi-coil and non-Cartesian acquisitions. Future work will incorporate support information in $\ell_p \left(p<1\right)$ minimization to reduce measurements and include iterative procedure to update the support information more accurately.

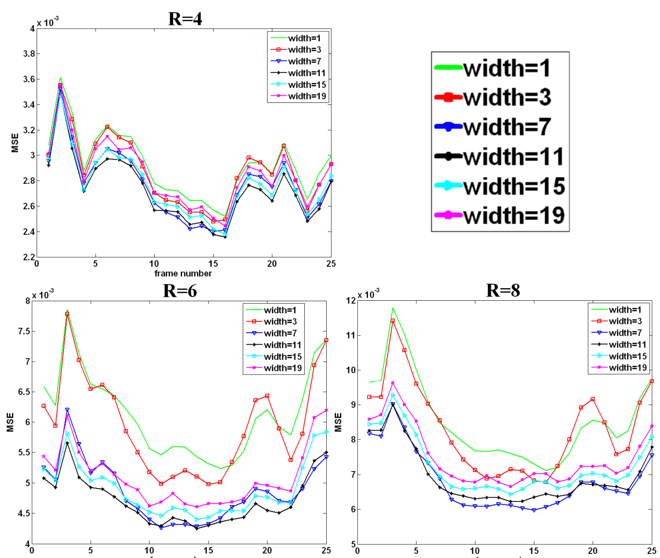

Fig. 4. The frame-by-frame plots of the normalized MSE of the proposed method for different widths and reduction factors.

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