Compressed-sensing dynamic imaging with self-learned nonlinear dictionary

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INTRODUCTION: Dynamic MRI (dMRI) characterizes spatio-temporal structures of the subject under the consideration and exhibits high temporal correlation on top of the spatial correlation [1, 2]. A sub-Nyquist sampling paradigm, Compressed Sensing (CS) accelerates data acquisition process in MRI by exploiting such correlation and imposing sparsity constraint on certain transform domain [3-6]. Sparsity of the data in the transform domain is the key to the success of CS approaches. Most existing approaches use linear transformations such as Fourier [4-6], Wavelet [3], PCA [6] to sparsify the spatio-temporal signal, which often, might not be able to capture highly nonlinear characteristics in dMRI, As a result, the signal may not be as sparse, after such linear transformations. In this abstract, we introduce a nonlinear polynomial-kernel-based model to represent the signal sparsely. Based on the model, a novel compressed-sensing dMRI method with self-learned nonlinear dictionary (**NL-D**) is proposed. Simulation results show that the proposed method outperforms the conventional CS dMRI methods with linear transforms.

METHODS: The key idea of the proposed method is to represent the dynamic images more sparsely using nonlinear dictionaries such that the sparsity-constrained reconstruction is more accurate. The reconstruction problem can be formulated as solving $\mathbf{min_x}\|\boldsymbol{y}-\boldsymbol{F_u}\mathbf{x}\|_2^2+\eta_1\|\boldsymbol{\tau}\|_1{}^{(1)}$, where $\boldsymbol{y}$ is the acquired k-space data, $\boldsymbol{F_u}$is the undersampling Fourier operator, $\mathbf{x}$ the desired dynamic images and $\boldsymbol{\tau}$ the sparse coefficients with nonlinear dictionary. The optimization problem in (1) is solved using PreImage formulation [13] with polynomial kernel and iterative soft thresholding method [14]. The schematic of the proposed method is illustrated in the Fig. 1. The dynamic images are initialized by zero-filled Fourier reconstruction. **<u>Step 1: Nonlinear dictionary learning</u>.** First, the nonlinear dictionaries are learned from the training data obtained from the low resolution dynamic images using kernel principal component analysis (kPCA). Specifically, low-resolution images are obtained from a few central k-space lines. A set of T training signals $\boldsymbol{P_t}$ $(t=1,2,\ldots T)$, each representing the temporal variation at a particular spatial location, are formed from these low-resolution images. The training signals are projected from the original space to a high-dimensional feature space to calculate a $T\times T$ Kernel Matrix $\boldsymbol{K_P}$ whose elements are given by $K_{i,j}=k(\boldsymbol{P_i},\boldsymbol{P_j})$ **,** and then centered to generate $\boldsymbol{K_P^c}$ [8], where $k(.)$ is a kernel function. Here we use a polynomial kernel, $k(\boldsymbol{x_i},\boldsymbol{x_j})=(\langle \boldsymbol{x_i},\boldsymbol{x_j}\rangle+c)^d$ where, c is a constant and d is the order of polynomial. The linear PCA is performed in the feature space using $\boldsymbol{K_P^c}\ \boldsymbol{\alpha}=\lambda\ \boldsymbol{\alpha},$ and thus PC in feature space is represented as, $\mathbf{V}=\sum_{t=1}^{T}\boldsymbol{\alpha}_t\overline{\boldsymbol{\phi}}\ (\boldsymbol{P}_t)$, where, $\bar{\phi}:\chi\rightarrow H$ is the nonlinear map from the low dimensional input space χ to a high dimensional feature space H with centering. **<u>Step 2: Enforcing sparsity constraint</u>**. The temporal signal of the dynamic images (from previous iteration) for each spatial location is regarded as test data. The test data is projected onto the PC in the feature space from Step 1. For a test signal $\boldsymbol{x}$, the projection of $\overline{\boldsymbol{\phi}}\ (\boldsymbol{x})$ on to the k^{th} PC is computed using $\beta=(\boldsymbol{K}_{xp}^c)^T\boldsymbol{\alpha}^k$, where, $\boldsymbol{K}_{xp}^c$ is the centered kernel vector, $\boldsymbol{k}_{xp}=[k(P_1,x)\ \ k(P_2,x)\ldots k(P_T,x)]^T$. We assume $\overline{\boldsymbol{\phi}}\ (\boldsymbol{x})$ can be sparsely represented using only K largest PCs in feature space as, $\overline{\boldsymbol{\phi}}\ (\boldsymbol{x})\approx\sum_{k=1}^{K}\beta_k\boldsymbol{V}^k\ \approx\ \sum_{t=1}^{T}\overline{\boldsymbol{\gamma}}_t\ \overline{\boldsymbol{\phi}}(\boldsymbol{P}_t)$ and $\overline{\gamma_t}=\sum_{k=1}^{K}\beta_k\boldsymbol{\alpha}_t^k$. Therefore the $\boldsymbol{\tau}$ in Eq. (1) is defined as the vectorized projection of all test data over K principal components. After obtaining the sparse representation in the feature space, $\overline{\boldsymbol{\phi}}\ (\boldsymbol{x})$ needs to be projected back onto the original space (known as the pre-image problem). With odd order polynomial kernel, the pre-image is obtained as**:** $\boldsymbol{z}=\sum_{i=1}^{N}f_k^{-1}\left(\sum_{t=1}^{T}\boldsymbol{\gamma}_t\,k(\boldsymbol{P}_t,\boldsymbol{\xi}_i)\right)\boldsymbol{\xi}_\mathrm{i}\ ,^{(2)}$ where f_k^{-1} is inverse polynomial kernel function and $\{\boldsymbol{\xi}_1,\boldsymbol{\xi}_2,\ldots\boldsymbol{\xi}_N\}$ is any orthonormal basis of the input space. **<u>Step 3: Enforcing data consistent constraint</u>**. To enforce the reconstruction to be consistent with measured k-space data, at the sampled k-space locations, the k-space data is updated using a weighted combination of the values from pre-image and measurement: $\widehat{\boldsymbol{y}}=\frac{\boldsymbol{y}+\eta_2\boldsymbol{W}}{1+\eta_2},^{(3)}$, where $\mathbf{W}$ is the Fourier transform of the pre-image $\mathbf{z}$, and η_2 is the weight. The updated dynamic images are then obtained by Fourier transform of the updated k-space data. Steps 2 and 3 are alternated iteratively until convergence.

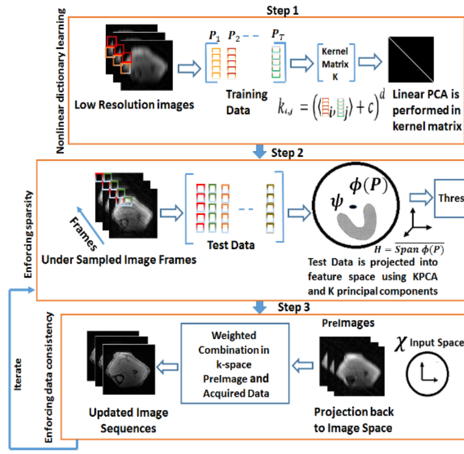

Fig. 1 Schematic of the proposed method

RESULTS: We used ASL perfusion data on a calf muscle to evaluate the proposed method. *Data acquisition parameters*: TR/TE = 2.8/1.2*ms*, flip angle = 5^o, FOV= $160\times 112\ mm^2$ and matrix size = $112\times 100\times 20$ (#FE × # PE × # frames). 1-D random under-sampling pattern along PE was used frame by frame with a net reduction factor of *3*. *Simulation Parameters:* Low-resolution images obtained from 5 central k space lines were used to create a training data set. Polynomial kernel with *c=1* and $d=3$ was used. Number of training signals *T = 1000*, number of PCs (K) = 80. Soft thresholding values η_1 and η_2 were tuned appropriately. Fig. 2 shows the reconstruction and error patterns using the proposed method and conventional CS method. We can clearly see from the error pattern that the proposed method outperforms the conventional CS-PCA method. Aliasing artifacts are observed in the conventional CS-PCA reconstruction method. Fig. 3 compares the average temporal curves of a selected ROI (Fig.2). The proposed method follows the reference curve more precisely than the conventional approach.

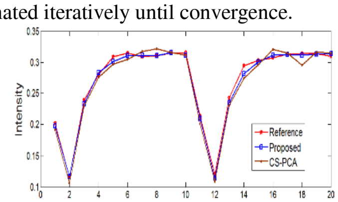

Fig 3. Average Intensity of ROI

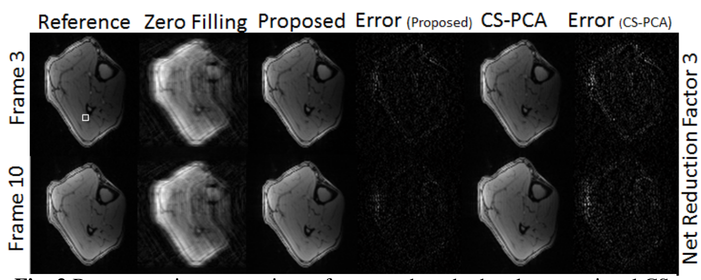

Fig. 2 Reconstruction comparison of proposed method and conventional CS-PCA. Error is magnified 10x times

DISCUSSION AND CONCLUSION: We have developed a novel CS method to accelerate dynamic MRI using self-learned nonlinear dictionary. Simulation results show that the proposed method outperforms the conventional CS methods. The proposed method maintains the spatial structural information as well as the kinetic information of dMRI. Selection of training signals and various simulation parameters are interesting challenges to be explored in future endeavors.

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