# Informational model of the bid-ask spread with learning

# Assumptions

The market maker believes that the value (V) of an asset can be high (H) or low (L), and P(V=L) = δ and P(V=H) = 1- δ

The fractions of traders who are informed and uninformed are μ and 1- μ, respectively.

Any uninformed trader is equally likely to buy (B) or sell (S).

Informed traders will buy (B) when V=H and sell (S) when V=L.



# BID PRICE = E(V/S)

#  = H P(V=H/S) + L P(V=L/S)

#  = conditional expectation of V,

#  given Sell. (regret free price)

From [Bayes rule](https://corporatefinanceinstitute.com/resources/knowledge/other/bayes-theorem/),

 P(V=L)P(S/V=L)

P(V=L/S) = -----------------------------------------------

 P(V=L)P(S/V=L) + P(V=H)P(S/V=H)

Now note that

P(S/V=L) = informed will sell with p = 1 and uninformed will sell with p = ½

 = μ(1) + (1-μ)(1/2) = ½(1+μ)

P(S/V=H) = informed will not sell and uninformed will sell with p = ½

 = μ(0) + (1-μ)(1/2) = ½ (1-μ)

Hence P(V=L/S)

= δ½(1+μ)/[δ ½(1+μ) + (1-δ) ½ (1-μ)]

What is the conditional probability P(V=H/S)?

Since, P(V=H/S) = 1 - P(V=L/S) = 1 – δ½(1+μ)/[δ ½(1+μ) + (1- δ) ½ (1-μ)]

Thus, the market maker’s bid price = conditional expectation of V, given Sell

= E(V/S)

= H [1 – δ ½(1+μ)/{δ ½(1+μ) + (1- δ)½ (1-μ)}]

 + L [δ½(1+μ)/{δ ½(1+μ) + (1- δ)½ (1-μ)}]

= [H(1–δ)½(1-μ) + Lδ ½(1+μ)]/{δμ + ½ (1-μ)}

**ASK PRICE = E(V/B)**

 **= H P(V=H/B) + L P(V=L/B)**

 **= conditional expectation of V, given Buy. (regret free price)**

From Bayes rule,

 P(V=L) P(B/V=L)

P(V=L/B) = ------------------------------------------------

 P(V=L)P(B/V=L) + P(V=H)P(B/V=H)

Now note that

P(B/V=L) = informed will not buy and uninformed will buy with p = ½.

 = μ(0) + (1-μ)(1/2) = ½ (1-μ)

P(B/V =H) = informed will buy with p = 1 and uninformed will buy with p = ½.

 = μ (1) + (1-μ)(1/2) = ½ (1 + μ)

Hence P(V=L/B)

= δ½ (1-μ)/{δ½ (1-μ) + (1- δ)½ (1 + μ)}

What is the probability P(V=H/B)?

Since, P(V=H/B) = 1 - P(V = L/B) = 1 – δ½(1-μ)/{δ½ (1-μ) + (1- δ)½ (1 + μ)}.

Thus, the market maker’s ask price = conditional expectation of V, given Buy

= E(V/B)

= H [1 – δ½ (1-μ)/{δ½ (1-μ) + (1- δ)½ (1 + μ)}]

 + L [δ½ (1-μ)/{δ½ (1-μ) + (1- δ)½ (1 + μ)}].

 = [H(1 – δ)(1/2)(1+μ) + Lδ(1/2)(1-μ)]/

 [(1/2)(1+μ) – δμ]

The bid-ask spread = the ask price – the bid price = E(V/B) – E(V/S)

# Informational model of the bid-ask spread-Example

# Assumptions

The market maker believes that the value (V) of an asset is high (1) or low (0), and P(V=0) = δ = ½ and P(V=1) = ½.

Half of the traders are informed and half are uninformed.

Any uninformed trader is equally likely to buy (B) or sell (S).

Informed traders will buy (B) when V =1 and sell (S) when V = 0.

# BID PRICE = E (V/S)

#  = 1P(V=1/S) + 0P(V=0/S)

#  = conditional expectation of V, given Sell. (regret free price)

From Bayes rule,

P(V=0/S) = [P(V = 0)P(S/V=0)]/ [P(V=0)P(S/V=0) + P(V=1)P(S/V=1)]

Now note that

P(S/V=0) = informed will sell with p = 1 and uninformed will sell with p = ½

 = (1/2)(1) + (1/2)(1/2) = ¾.

P(S/V =1) = informed will not sell and uninformed will sell with p = ½

 = (1/2)(0) + (1/2)(1/2) = ¼.

Hence P(V=0/S) = [(1/2)(3/4)]/[(1/2)(3/4) + (1/2)(1/4)] = ¾ = posterior probability.

What is the conditional probability P(V=1/S)?

Since, P(V=1/S) = 1 - P(V = 0/S)

= 1 – ¾ = ¼.

Thus, the market maker’s bid price = conditional expectation of V, given Sell

= E(V/S) = 1 P(V=1/S) + 0 P(V=0/S)

= 1(1/4) + 0 (3/4) = ¼.

**ASK PRICE = E(V/B)**

**= 1P(V=1/B) + 0P(V=0/B)**

**= conditional expectation of V, given Buy. (regret free price)**

From Bayes rule,

P(V=0/B) = [P(V = 0) P(B/V=0)]/ [P(V=0)P(B/V=0) + P(V=1)P(B/V=1)]

Now note that

P(B/V=0) = informed will not buy and uninformed will buy with p = ½.

 = (1/2)(0) + (1/2)(1/2) = ¼.

P(B/V=1) = informed will buy with p = 1 and uninformed will buy with p = ½.

 = ½(1) + (1/2)(1/2) = ¾.

Hence P(V=0/B)

= [(1/2)(1/4)]/[(1/2)(1/4) + (1/2)(3/4)]

 = ¼ = posterior probability.

What is the probability P(V=1/B)?

Since, P(V=1/B) = 1 - P(V=0/B)

 = 1 – ¼ = 3/4.

Thus, the market maker’s ask price = conditional expectation of V, given Buy

= E(V/B) = 1 P(V=1/B) + 0 P(V=0/B)

= 1(3/4) + 0 (1/4) = ¾.

The bid-ask spread

= the ask price – the bid price

= E(V/B) – E(V/S) = ¾ - ¼ = ½.