Flight Results of GPS Based Attitude Control on the REX II Spacecraft

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BIOGRAPHY

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ABSTRACT

The REX II spacecraft, funded by the US Air Force Space Test Program, was successfully deployed from a Pegasus XL rocket in March 1996. This gravity gradient stabilized spacecraft contains a Trimble TANS Vector GPS receiver that has been substantially modified for space use by Stanford University and the Goddard Space Flight Center. This mission marks the first successful extended real-time GPS based attitude determination application for spacecraft (the only other known real-time attitude determination experiment, Crista-SPAS, was of limited duration on a Space Shuttle mission). REX II is also the

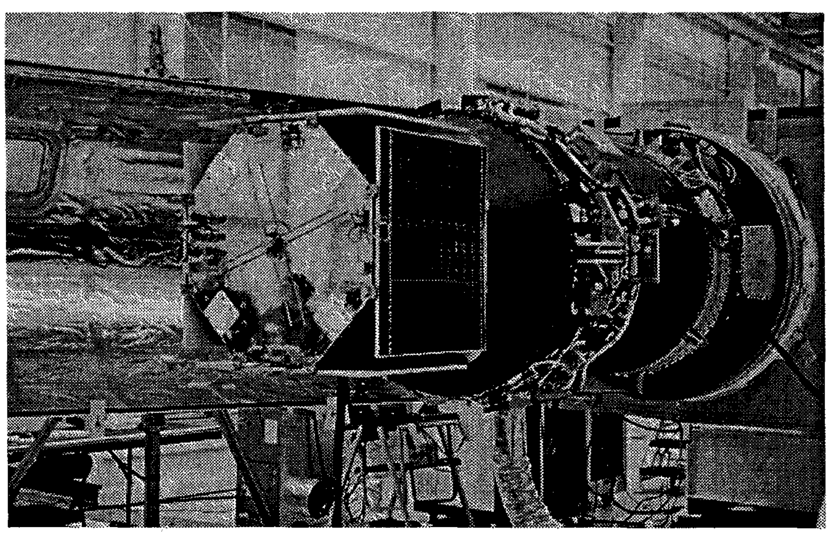

Figure 1. REX II Spacecraft At Integration.
The white rectangles on the near octagon are GPS patch antennas.

first known spacecraft attitude control application using GPS sensor inputs.

This paper presents results and lessons learned from the REX II spacecraft and other recent GPS space experiences. The REX II GPS solutions are compared to magnetometer solutions and dynamic simulations to assess the GPS attitude accuracy. These results are then contrasted with those from the RADCAL spacecraft, which was similar in design but performed GPS based attitude determination in ground based post-processing. Issues and lessons for future spacecraft that will employ GPS receivers on-orbit, such as solution availability and controller performance, are discussed.

INTRODUCTION

On March 8, 1996, the REX II spacecraft (Figures 1 and 2) was inserted into an 830 km polar orbit with a GPS receiver capable of performing real-time attitude determination. Prior to this date, experiments with GPS receivers had demonstrated the feasibility of on-orbit attitude determination on spacecraft in post-processing (RADCAL [1]) and real-time (Crista-SPAS [2]). These previous flights, although significant technical firsts, did not provide the practical demonstration of closed loop attitude control using GPS as the sensor for an extended real-time application. The REX II satellite marks the first known operational application of the GPS attitude sensor for closed loop control of a spacecraft.

EXPERIMENT PERSPECTIVE

The most analyzed GPS attitude data from an on-orbit receiver was taken from the Radar Calibration (RADCAL) spacecraft in 1993. Although this experiment demonstrated attitude determination of the satellite through ground based post-processing of GPS carrier phase measurements, the GPS subsystem was not fully calibrated prior to launch, requiring that the calibration parameters be estimated from the on-orbit data. Furthermore, the only other independent attitude sensor on this passively stabilized spacecraft was a magnetometer for which data was generally not available during times of GPS data collection. It has therefore been difficult to characterize the accuracy of

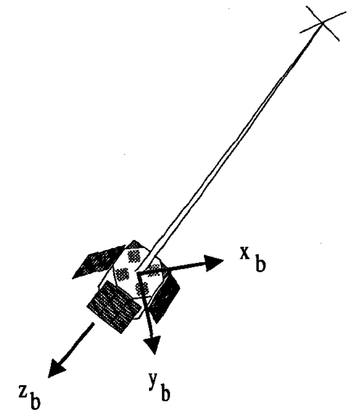

Figure 2. REX II On-Orbit Configuration

the GPS attitude measurements for this spacecraft. By performing dynamic filtering of the GPS solutions and studying the filter residuals, Melvin et al have estimated that the attitude determination accuracy is about 2 degrees per axis 3-σ for a 0.67 m antenna separation. [3] This is not an unreasonable level of performance for a high multipath, small antenna separation application where the true calibration parameters of the GPS subsystem are not well known.

The Crista-SPAS experiment, flown in November 1994, provided the first on-orbit demonstration of real-time attitude determination. This multi-experiment payload operated as a free-flyer for 2 days during the STS-66 Space Shuttle Mission. The spacecraft contained an accurate gyro reference, but the gyro coordinate frame alignment was not measured relative to the GPS attitude reference frame, which means that discrepancies between the two reference frames might account for slightly different measurements from the two systems. Over the course of the experiment the two sets of attitude solutions agreed to within 2 degrees, which is thought to be within the alignment tolerance of the two reference frames. [2]

The REX II spacecraft contains a magnetometer from which milligauss-level magnetic field measurements are simultaneously available with the GPS attitude solutions. Once again, it is difficult to measure the GPS solution accuracy without a more accurate reference sensor. In addition, the REX II telemetry system is constrained to operate with low data bandwidth over short ground passes (10 minutes), resulting in short time spans with full data or very low sampling rates that cover longer time spans. The lack of a single, long time span with full data complicates the dynamic analysis of the vehicle.

Despite these facts, the GPS solution performance may still be estimated in comparison with the independent sensor solution and the expected equations of motion. Furthermore, the magnetometer/GPS attitude sensor tandem is expected to be common on future LEO spacecraft, so it is worthwhile to consider the potential for combining these measurements into a single, more accurate estimate of the vehicle attitude. Finally, the suitability of this sensor for closed loop control will be assessed by examining GPS sensor data outages and controller performance.

MECHANICAL DESCRIPTION

A schematic of the REX II spacecraft is shown in Figure 2. It is mechanically similar to the predecessor RADCAL satellite, which was designed to be passively stabilized using a 6 m boom with gravity gradient torques and two magnetic hysteresis rods for damping. REX II is additionally actively controlled by electromagnetic coils and a pitch-axis reaction wheel which provides momentum bias. The control system requirement is to maintain a locally level orientation to within 5 degrees per axis. The vehicle attitude is expressed as a 3-2-1 yaw-pitch-roll Euler sequence from the locally level orientation shown in Figure 3. The control input may be provided by the GPS attitude sensor in a 'Normal' mode or the three-axis magnetometer in a backup mode.

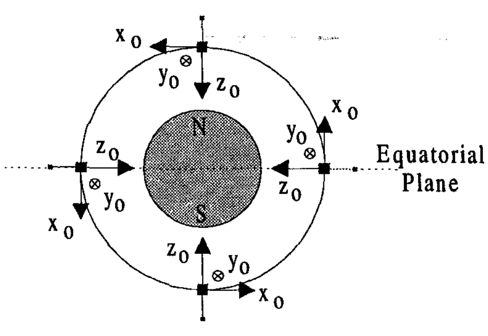

Figure 3. Locally Level Reference Attitude

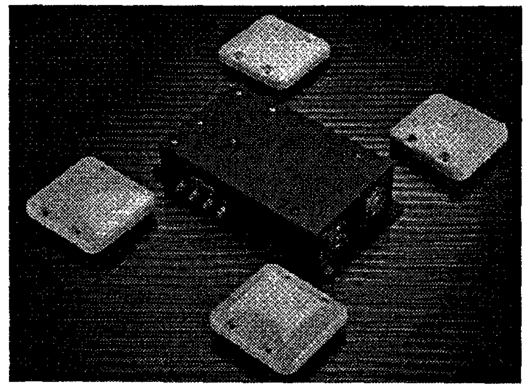

Figure 4. TANS Vector Receiver

The GPS receiver used was a Trimble TANS Vector that had software modifications performed by individuals at Stanford University and Goddard Space Flight Center. This receiver is a compact (6x13x24 cm) production unit that performs real-time attitude determination at 1 Hz based on differential carrier phase inputs from 4 antennas (see Figure 4). It is powered by voltages between 9 and 32 volts and consumes about 4 steady watts of power. The complete unit with antennas and preamplifiers has a mass of about 3 kg. Tracking and attitude determination algorithms inside the receiver firmware have been reprogrammed for Low Earth Orbit spacecraft dynamics.

The four GPS patch antennas were mounted on the top surface of the REX II main body in a coplanar, aligned configuration, as shown in Figure 5. All four antennas were keyed in the same direction to provide antenna phase center repeatability. The antenna separation was 0.67 m along the diagonal, and the gravity gradient

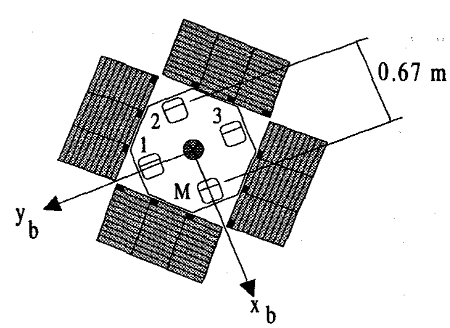

Figure 5. Antenna Placement on Spacecraft

boom extends out of the center of the main body. The M-2 and 1-3 axes were mechanically aligned to within 0.1 degrees of the spacecraft x and y body axes, respectively.

After spacecraft integration, the GPS subsystem was placed into a self-survey mode by allowing the antenna array to collect data from the GPS constellation for several hours in a fixed position on the ground. The receiver self-survey then solves for specific calibration parameters that are a function of the integrated system, namely the antenna phase center separation ('baseline') and electrical length ('line bias') terms associated with each differential carrier phase measurement. These calibration parameters are very important because without the self-survey test they may only be obtained by estimation from the on-orbit data, as was done for RADCAL. After the self-survey test was completed, the calibration parameters were verified by witnessing that the GPS subsystem successfully computed an attitude solution.

DYNAMIC EQUATIONS

The system equations of motion may be derived from the classical Euler equations:

$$\frac{d\boldsymbol{H}}{dt} + \boldsymbol{\omega} \times \boldsymbol{H} = \boldsymbol{T} \qquad (1)$$

where $\boldsymbol{H}$ is the angular momentum vector, $\boldsymbol{\omega}$ is the angular velocity vector and $\boldsymbol{T}$ is the external torque. Let:

$$H_x = I_x \omega_x \qquad (2a)$$

$$H_y = I_y \omega_y + h_y \qquad (2b)$$

$$H_z = I_z \omega_z \qquad (2c)$$

where I_x, I_y, and I_z are system moments of inertia and h_y is the angular momentum of a constant speed pitch momentum bias wheel. Representing the spacecraft attitude by small roll, pitch, and yaw angle (ϕ, θ, and ψ) deviations from the previously defined locally level attitude reference frame, the kinematic equations are given by:

$$\omega_x = \dot{\phi} - \omega_o \psi \qquad (3a)$$

$$\omega_y = \dot{\theta} - \omega_o \qquad (3b)$$

$$\omega_z = \dot{\psi} + \omega_o \phi \qquad (3c)$$

where ω_o is the orbit rate.

External torques that have been taken into account include gravity gradient, controller and those due to hysteresis rod magnetic moments. The gravity gradient torques for an axially symmetric ($I_x = I_y$) satellite are given by:

$$T_{ggx} = 3\omega_o^2 \left(I_z - I_y\right)\phi \qquad (4a)$$

$$T_{ggy} = 3\omega_o^2 (I_z - I_x)\theta \qquad (4b)$$

$$T_{ggz} = 0 \qquad (4c)$$

These equations may be combined to produce:

$$I_x\ddot{\phi} - \dot{\psi}\left[I_z\omega_o + h_y\right] + \phi\left[4\left(I_y - I_z\right)\omega_o^2 - h_y\omega_o\right] = T_x^C + T_x^H$$

$$I_y\ddot{\theta} + \theta\left[3(I_x - I_z)\omega_o^2\right] = T_y^C + T_y^H$$

$$I_z\ddot{\psi} + \dot{\phi}\left[I_z\omega_o + h_y\right] + \psi\left[-h_y\omega_o\right] = T_z^C + T_z^H$$

(5a,b,c)

which represents a special case (axial symmetry and one constant wheel speed only) of those

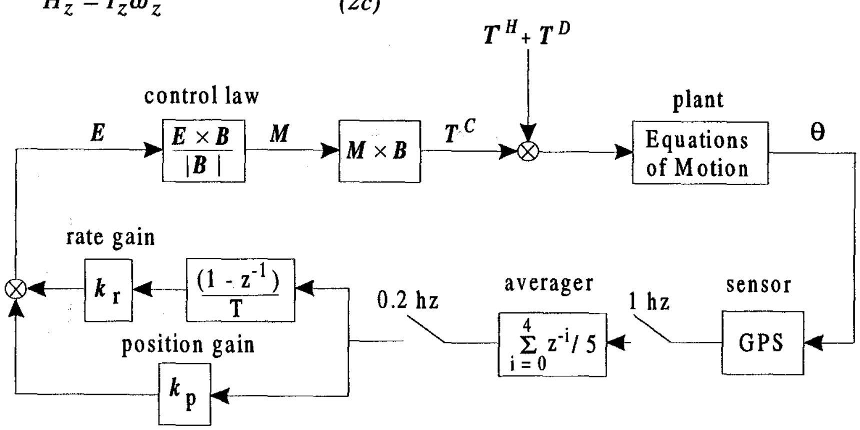

Figure 6. Controller Block Diagram (3-Axis)

residual within 5 degrees is considered to be quite reasonable since many disturbance torques have been omitted, the dynamic model is not exact, and there are measurement errors associated with the simulation initial conditions.

MAGNETOMETER ANALYSIS

A three-axis magnetometer measures the geomagnetic field in body coordinates and can then, using the position of the spacecraft, be compared to a tenth order spherical harmonic model of the Earth's magnetic field at that position. Because these observations and references are vectors, only two directions of attitude can be determined at any given point in time (i.e. all but the rotation about the observed vector). However, with some knowledge of the spacecraft dynamics, usually via gyroscope measurements, either batch or filter solutions may use information from past measurements to obtain full attitude solution.

In this case, REX II did not fly with gyroscopes. Further, due to the data set limitations (either too short for filter convergence, or sampled very far apart), uncertainties in the field model, and potential magnetic activity near the magnetometer, an attitude reference that may be used as 'truth' to benchmark GPS accuracy is all but impossible. However, at worst, using the magnetometer data and some knowledge of the kinematics and dynamic behavior, a sanity check of the GPS attitude behavior is feasible.

An estimation scheme developed by Crassidis and Markley [6] was adopted for the REX II data. This method involves predictive filtering specifically designed for spacecraft without gyroscopes or other rate measuring devices and has been successfully tested with flight data from the Solar Anomalous and Magnetic Particle Explorer (SAMPEX) spacecraft. The feature that most obviously separates this filtering scheme from a traditional Kalman filter is that it does not assume that the model error, or process noise, is Gaussian and unbiased, but it instead determines the model error during the estimation process such that the model and the corrections together form an accurate representation of the system behavior. This is achieved by simultaneously solving system optimality conditions and an output error covariance constraint. Given the unusually awkward data characteristics caused by the low telemetry bandwidth, more reasonable attitude solutions were achieved using this approach than with a traditional Kalman filter.

MAGNETOMETER ANALYSIS RESULTS

Magnetometer data, along with GPS position, time, and attitude information was collected from the REX II spacecraft during a ground pass. This "short" span of data lasted 25 minutes (less than 1/3 of an orbit) at a relatively high sample period of 20 seconds. Magnetometer-only attitudes were derived using this predictive filter, with the initial attitude provided from the GPS solution.

The first plot in Figure 11 shows the three-axis difference between the GPS attitude point solutions and the magnetic field-only filter attitudes as the filter converges. The error in the reference geomagnetic field model, used in the magnetometer-only attitude computation, could be as much as 2 degrees, so that the difference in the two attitudes appears to be converging to a value within that tolerance. Further, this attitude difference looks to be smaller than that for the simulated data above.

The second plot in Figure 11 shows, for the same data span, the magnetic field measurement residual. Here, the magnetometer measurement was transformed into inertial space by both the magnetometer-only filtered attitude (the solid line) and GPS attitude point solutions (the dashed line). These values are then compared to

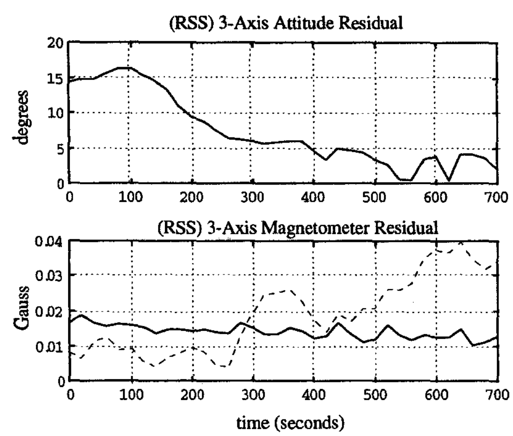

Figure 11. Attitude and Magnetometer Flight Residuals. Lower figure dashed line is GPS point attitude solutions, solid line is magnetometer filtered attitudes.

presented for an Earth-Referenced Spacecraft as shown in Wertz [4]. T^C is the control torque and T^H is the hysteresis rod torque.

The control law, shown in Figure 6, may be computed in terms of Euler Angles as:

$$E_x = k_{rx}(\dot{\phi} - \omega_o \psi) + k_{px}\phi \qquad (6a)$$

$$E_y = k_{ry}\dot{\theta} + k_{py}\theta \qquad (6b)$$

$$E_z = k_{rz}(\dot{\psi} + \omega_o \phi) + k_{pz}\psi \qquad (6c)$$

where E_x, E_y, and E_z are the control error signals, k_p and k_r are the position and rate gains, and in this case ϕ, θ, and ψ and their derivatives are the sensed Euler angles and Euler rates as provided by GPS. The Euler angles are computed as a five second time average of the 1 Hz GPS solutions to reduce sample noise, and the rates are derived rates from the last two averaged Euler angles (they are not an independent measurement).

The commanded magnetic moments are determined from the computed control errors:

$$M_x = (E_y B_z - E_z B_y)/|B| \qquad (7a)$$

$$M_y = (E_z B_x - E_x B_z)/|B| \qquad (7b)$$

$$M_z = (E_x B_y - E_y B_x)/|B| \qquad (7c)$$

And the controller torques are the cross product of the magnetic moment and the magnetic field:

$$T_x^C = M_y B_z - M_z B_y \qquad (8a)$$

$$T_y^C = M_z B_x - M_x B_z \qquad (8b)$$

$$T_z^C = M_x B_y - M_y B_x \qquad (8c)$$

HYSTERESIS RODS

Hysteresis rod magnetic moments are multi-valued functions of the ambient magnetic field component parallel to the axes of the rods. The moments are bounded by so-called "B/H" curve major loops which are functions of the rod material and length to diameter ratio. The magnetic moments at a particular time depend on past history and thus cannot be written as explicit functions. The mathematical model described in reference [5] was used to estimate the performance of the rods in a computer simulation of the spacecraft dynamics. Variable parameters in the model were selected to fit available tabulated empirical data for the REX II spacecraft. The model was devised during the development of the Passive, Aerodynamically-Stabilized, Magnetically-Damped Satellite (PAMS) which was released from the Orbiter Endeavour during the STS-77 Mission in May 1996. Its validity was demonstrated by observations of the on-orbit performance of that spacecraft.

SIMULATION RESULTS

The REX II vehicle attitude was simulated for 24 hours using the above dynamic equations with initial conditions provided from the GPS attitude sensor. The simulation was run with and without the effects of the hysteresis rods included in the dynamic model. The sensor data from the spacecraft is shown in Figure 7, and the simulation results without and with hysteresis rods are shown in Figures 8 and 9, respectively.

The most interesting result from the simulation is that the fit of the simulation without the effect of the hysteresis rods included (Figure 8) is much better than that with the effect included (Figure 9). Large oscillations in all axes are observed when the hysteresis effect is included that are not observed in the GPS attitude measurements. This effect of the hysteresis rods is counter intuitive since they are usually

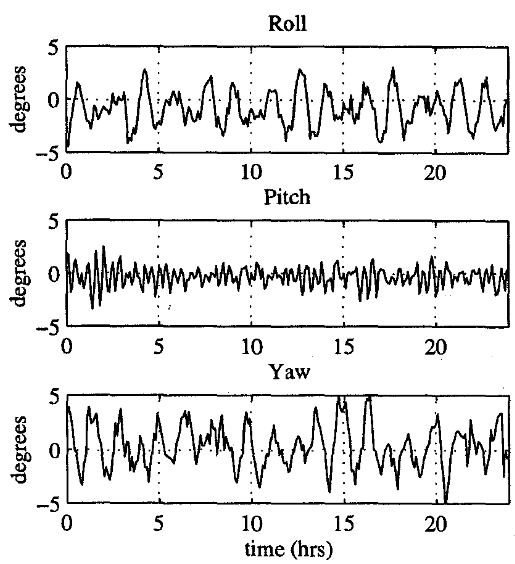

Figure 7. REX II GPS Attitude Measurements, 24 hours

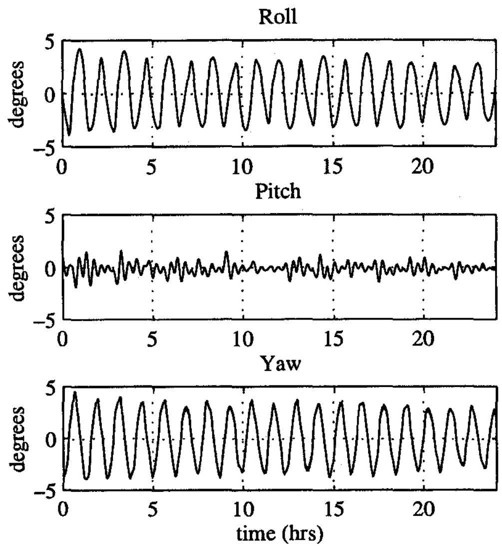

Figure 8. Simulated REX II Attitude Without Hysteresis Rods

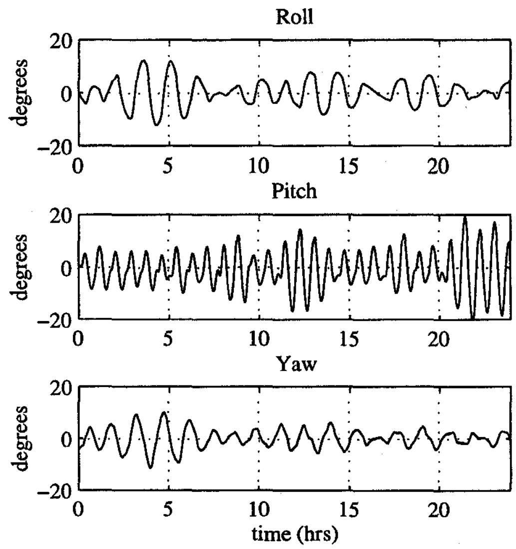

Figure 9. Simulated REX II Attitude With Hysteresis Rods

employed on spacecraft to passively damp out oscillations.

One explanation for the large amplitude motion is an interaction of the hysteresis torques with the control system, especially in the pitch axis, which is the momentum wheel axis. When the hysteresis model is removed, the large amplitude motion is not evident in the simulation. The conclusions are that either the hysteresis rods are not modeled properly, or they are not behaving in a manner according to the model.

Since the hysteresis rod model has been successfully employed on other spacecraft, and the simulated magnetic field solution, using a tenth order spherical harmonic model of the Earth's magnetic field, compares well to the magnetometer data (Figure 10), the preliminary conclusion is that the rods are not producing the expected theoretical effect on the spacecraft, which in this case is actually better for the overall control performance. The spacecraft designers have stated that it is possible that the hysteresis rods may have become compromised during spacecraft assembly, launch, or deployment such that the material is no longer as effective.

Figure 10 shows the residual between the simulation and the GPS attitude and magnetic field measurements during the 24 hour time span to have a mean error of 4.6 degrees without the hysteresis rods and 8.8 degrees with the hysteresis rods included. It is important to stress that the 5 degree error does not represent a measure of sensor accuracy but instead provides general confirmation of the dynamic equations. Only a reasonable agreement between the dynamic model and the sensor results is sought to demonstrate general sensor validity. An attitude

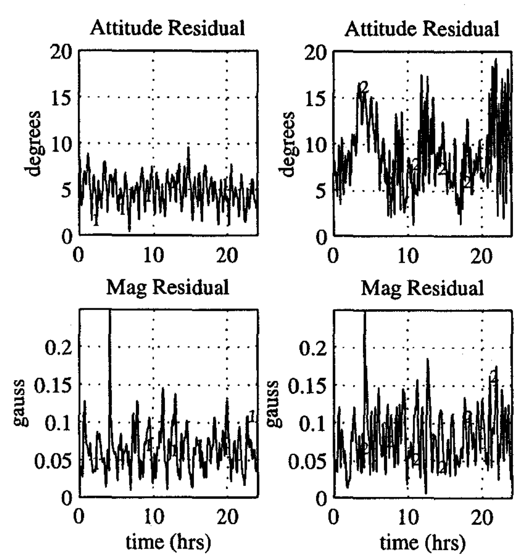

Figure 10. 3-Axis Attitude and Magnetometer Simulation Residuals, Without (1) and With (2) Hysteresis Rods Included

the reference geomagnetic field model; the three-axis differences are presented in gauss. The final magnetometer residual (0.013 gauss) is about a 1.8 degree angular residual, and the GPS point solution at that time is about 4.8 degrees. These residuals also appear to be within values predicted by the simulations.

A "long" span of data from the REX II spacecraft was also considered. These data, while 24 hours long, were sampled at the very low rate of 6 minutes/sample. The filtering scheme requires integration across the data gaps, and unfortunately, 6 minutes is too long for the integrator to operate without a measurement update. In an effort to bridge the gap, a spline fit interpolation scheme was attempted. Unfortunately, a coning motion is evident along the y axis at a higher frequency than the 6 minute sampling reveals. In order to illustrate this aliasing effect, the "short" data span is plotted in Figure 12. The circles show 6 minute sampling, the dotted lines show the spline fit, and the solid lines show the measurements from the magnetometer (20 second sampling time). The high frequency activity is clear in the y direction

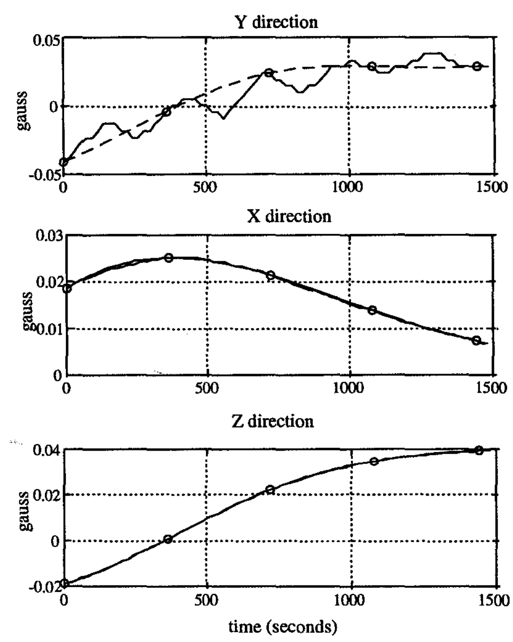

Figure 12. Magnetometer Data from the "Short" Data Span (solid lines) as Compared to a 6 Minute Subsampling (circles) and Spline Fit (dashed line)

plots, as is the 0.004 gauss quantization of the data. The stability of the z and x directions is also apparent in that the spline fit data lie virtually on top of the measured data.

While the y axis higher frequency disturbance can be modeled, as in the simulations above, getting the phasing correct would require some trial and error guesswork, with the out of phase data imposing an even larger error than the spline fit data illustrated below. Therefore, as a further quick check, the spline fit scheme illustrated in Figure 12 was used in the estimation filter. The results showed relatively large residuals, but still within the 0.03 gauss error seen in the y axis in Figure 12. Further effort did not seem warranted, as the "short" span converged to within expected error tolerances.

In summary, the GPS and magnetometer attitudes for REX II match to within 2 degrees for the 25 minutes of data available for processing. This agreement is consistent with the dynamic simulations discussed earlier.

COMBINED GPS/MAGNETOMETER ATTITUDE DETERMINATION

The estimator allows data from more than one sensor, each weighted independently, to be included as measurements. Therefore, the GPS attitude solutions were included as measurements along with the magnetometer data from the "short" span to determined attitudes.

Figure 12 shows the angles between the combined attitude transformed reference geomagnetic field vector and the measured magnetic field. As it turned out, the two measurement sources were nearly equally weighted to achieve the best solution (lowest residuals).

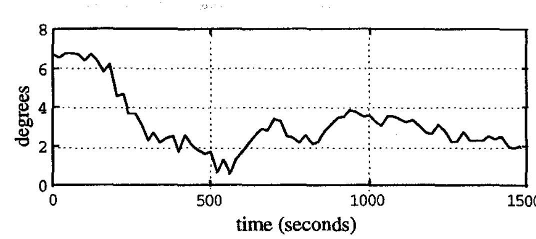

Figure 13. GPS and Magnetometer Combined Filter - Magnetic Field Residual Angles

The combined filter seems to be converging to a residual of about 2 degrees. As a comparison, the GPS only attitude residual angles for the "long" data span had a mean of 4.4 degrees, while the "short" span had 3.6 degrees.

CONTROL SENSOR PERFORMANCE

Having bounded the sensor accuracy with the magnetic attitude determination accuracy (approximately 2 degrees), it remains to characterize the GPS attitude performance as a control sensor on the REX II satellite.

Figure 7 shows the GPS attitude measurements over a 24 hour span. The controller performance is within the specification of 5 degrees per axis with GPS as the sole input attitude sensor. The three-axis error during this time span has a mean value of 2.66 degrees with a standard deviation of 1.16 degrees. A simulation of the control laws given the same initial conditions of the first GPS measurement with perfect sensing afterwards (see Figure 8) yields a controller performance with a three-axis mean attitude error of 3.30 degrees with a standard deviation of 0.49 degrees. This result demonstrates that the controller performance is a function of the control behavior and not the sensor inputs. In other words, input sensor accuracy is not adversely affecting the controller performance. This basically means that the GPS sensor is "up to the task" as a control input sensor in this application.

An important topic to discuss regarding GPS sensor performance is the subject of attitude solution availability. GPS attitude outages may be caused by a number of physical effects, such as carrier phase measurement noise, solution observability, and integrity checks. With the current six-channel TANS Vector, these outages will routinely occur during normal operations for short periods of time. Using all available data taken to date on the REX II mission, Freesland, et al, have observed these outages at a consistent rate of 1.6 times per orbit. [7]

The GPS attitude dropouts are sorted by latitude and duration in Figure 14. The dropouts are nearly evenly distributed at all latitudes for this polar orbit. More than 75% of all reported dropouts last between 5 and 10 minutes. This fact is consistent with the hypothesis that most outages are geometry related, as the orbit geometry changes significantly in about the same time. Since these outages may be expected to routinely occur once or twice per orbit, a successful attitude control system that employs GPS must be designed to operate in the presence of these outages. For a low bandwidth controller with small actuator power, as in the case of REX II, the controller performance is essentially unaffected by the attitude outages. With tighter pointing requirements and higher controller bandwidth, an alternative scheme must be adopted, such as an additional sensor input, perhaps a magnetometer, that may be used in place of the GPS input during any outage.

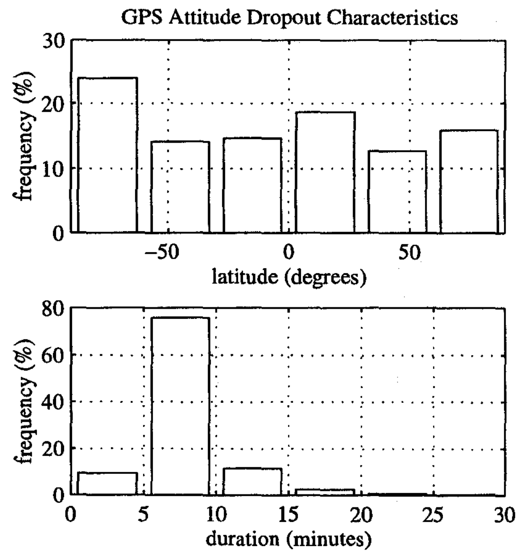

Figure 14. GPS Attitude Dropouts As Functions of Latitude and Duration

For sub-degree accuracy pointing applications, the GPS/magnetometer sensor tandem is a practical combination. Magnetometers are readily available on most LEO spacecraft, and they are cost effective and relatively simple to implement. Magnetic field measurements are always available, unlike sun sensor measurements, which are unavailable during times of Earth shadowing of the spacecraft. As previously mentioned, magnetometer based attitude determination is generally not as accurate as GPS based attitude determination; but a properly designed estimation filter will gracefully degrade from GPS based attitude accuracy over several minutes, so that the solution will in general be better than the magnetometer-alone attitude accuracy during the time of the outage.

Estimation filters may be readily augmented to accept multiple sensor inputs. In fact, the estimation method used in this paper to combine GPS and magnetometer measurements is an acceptable filter for this application. Vehicle dynamics may also be added to further improve the estimation accuracy if these are thought to be sufficiently well known.

CONCLUSION

This paper has discussed the REX II spacecraft in perspective with previous on-orbit GPS experiments. REX II is the first operational application using real-time GPS carrier phase-based attitude determination as a sensor input for closed loop attitude control of a spacecraft. The controller was found to have a mean three-axis error of 2.66 degrees with a standard deviation of 1.16 degrees, which is within the controller specification of 5 degrees per axis.

The GPS attitude solutions were validated to within 5 degrees by the use of a dynamic simulation and to within 2 degrees by an independent magnetic field measurement. The dynamic model indicates better agreement with the observed GPS solutions if the hysteresis rods are not included, leading to the suggestion that the hysteresis rods may have become less effective at some point during the assembly, launch, or deployment of the spacecraft. The magnetometer validation of the GPS attitude solutions agrees to within the measurement accuracy of the magnetic field attitude determination method (about 2 degrees). GPS accuracy of better than 1 degree is expected, but cannot be reliably proven with this sensor complement.

GPS solutions and magnetic field measurements were combined into a single estimate of the vehicle attitude. This combined sensor output estimator is considered to be more practical for sub-degree controller performance because it provides an acceptable attitude measurement even during periods of GPS attitude sensor outage, which has been shown to occur routinely with current GPS receiver hardware during normal spacecraft operations.

ACKNOWLEDGMENTS

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