Multi-agent energy-efficient coverage path planning

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Problem Statement

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  - A number of drones are located near depot.
  - They need visit a set of nodes that geographically surround the depot.
  - Key constraints
    - Each node can be visited exactly once.
    - Each drone has limited battery.
- Energy Cost
  - The first part is based on the distance it travels.
  - The second one is based on the turns it takes.

Important notation

- $\mathcal{V}$: set of Nodes, $|\mathcal{V}| = V$
- $\mathcal{A}$: set of agents
- Node $0 \in \mathcal{V}$: depot
- c_{ij}: travelling distance cost from node i to node j
- q_{ijk}: turning cost between nodes $i, j, k \in \mathcal{V}$

EECPP Model

We represent the area to be surveyed as a set of grid cells and assume that a grid cell is covered if the drone visits its center, then we get a model as follows:

$$\min \sum_{a\in\mathcal{A}}[\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} c_{ij}x_{ij}^a + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}}\sum_{k\in\mathcal{V}} q_{ijk}I_{ijk}^a] \quad (1)$$

$$\text{s.t.} \sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{V}\backslash\{j\}} x_{ij}^a = 1,\ \forall j \in \mathcal{V}\backslash\{0\} \quad (2)$$

$$\sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{V}\backslash\{0\}} x_{i0}^a \leqslant |\mathcal{A}| \quad (3)$$

$$I_{ijk}^a \geqslant x_{ij}^a + x_{jk}^a - 1,\ \forall a \in \mathcal{A}, \forall i,j,k \in \mathcal{V} \quad (4)$$

$$I_{ijk}^a \leqslant x_{ij}^a,\ \forall a \in \mathcal{A}, \forall i,j,k \in \mathcal{V} \quad (5)$$

$$I_{ijk}^a \leqslant x_{jk}^a,\ \forall a \in \mathcal{A}, \forall i,j,k \in \mathcal{V} \quad (6)$$

$$x_{ij}^a \in \{0,1\},\ \forall i,j \in \mathcal{V} \text{ and } \forall a \in \mathcal{A} \quad (7)$$

- (1) is defined to minimize the total energy consumed across the drones.
- (2) requires that all nodes except node 0 are visited exactly once.
- (3) is defined that drones used must be less than the maximum number available.
- (4)-(6) by linear constraints it is forced that $I_{ijk}^a = x_{ij}^a \cdot x_{jk}^a$.

Additional Constraints

Battery capacity constraint

$$\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} c_{ij}x_{ij}^a + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}}\sum_{k\in\mathcal{V}} q_{ijk}I_{ijk}^a \leqslant C, \forall a \in \mathcal{A}$$

Any drone has limited flying path because of battery capacity constraint C(kJ).

Flow control

$$\sum_{i\in\mathcal{V}\backslash\{j\}} x_{ij}^a - \sum_{k\in\mathcal{V}\backslash\{j\}} x_{jk}^a = 0,\ \forall a \in \mathcal{A}, \forall j \in \mathcal{V}$$

It ensures that, after a drone visits a node, it departs from this node to the next one.

Subtour elimination

$$d_i^a + 1 \leqslant d_j^a + M(1 - x_{ij}^a),\ \forall i,j \in \mathcal{V} \text{ and } \forall a \in \mathcal{A}$$

d_i^a is the position of node i in drone a's flight path $(1, 2, \ldots, |\mathcal{V}|)$.

The maximum path length

$$d_i^a \leqslant M, \forall i \in \mathcal{V} \text{ and } \forall a \in \mathcal{A}$$

DP/SP Method

In our prior work[1], we showed the EECPP problem is NP-hard. To save time, we decide to try a new way. First, we use dynamic programming(DP) to determine the set of feasible paths l within battery capacity constraint. Second, with the idea of set partitioning(SP)[2], we use cplex to select best ones from the feasible labels to realize minimum cost. And the formulation will be as follows:

$$\min \sum_l C_l X_l \quad (8)$$

$$\text{s.t.} \sum_l a_{il} X_l = 1,\ \forall i \in \mathcal{V} \quad (9)$$

$$\sum_l X_l \leqslant |\mathcal{A}| \quad (10)$$

$$X_l \in \{0,1\} \quad (11)$$

- (8) describes objective function, which is to minimize the total cost of selected labels.
- (9) requires that all nodes except the depot are visited exactly once.
- (10) the maximum number of selected labels does not exceed the number of available drones.

Comparison

With battery capacity $C = 16.0$ kJ, we do two comparisons: CPLEX and DP/SP method, One obstacle and no obstacle(both use DP/SP method). And get two tables as follows:

Scalability of CPLEX w.r.t our algorithm

	CPLEX		DP/SP (Proposed)	
Grid Size	Objective	Time(s)	Objective	Time(s)
2 x 4	13.983	2.505	13.983	0.516
3 x 3	20.389	76.424	20.389	1.710
2 x 5	20.289	1182.410	20.289	3.641
3 x 4	**26.117**[1]	7200	26.117	35.864
3 x 5	**38.010**[1]	7200	37.186	351.965
4 x 4	**39.905**[1]	7200	38.419	694.454
3 x 6	**76.811**[1]	7200	60.304	1247.346
4 x 5	**73.420**[1]	7200	54.062	4028.075
3 x 7	no result[2]	7200	Infeasible	3300.508
4 x 6	no result[2]	7200	no result[2]	7200

[1] Best incumbent solutions are shown in **bold**.
[2] "no result" indicates that no feasible solution was found within 2 hour.

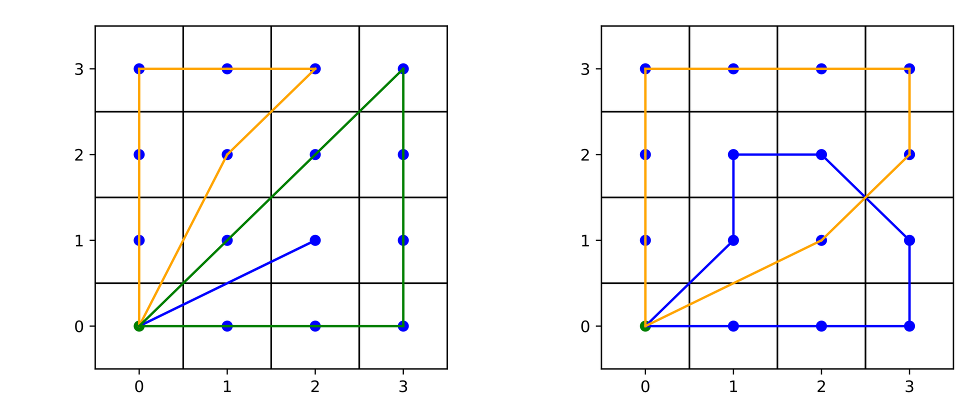
Scalability of One obstacle w.r.t no obstacle

	One obstacle	No obstacle
Grid Size	Time(s)	Time(s)
2 x 4	0.359	0.516
3 x 3	0.078	1.710
2 x 5	0.219	3.641
3 x 4	2.236	35.864
3 x 5	22.517	351.965
4 x 4	72.640	694.454
3 x 6	144.446	1247.346
4 x 5	577.771	4028.075
3 x 7	406.438	3300.508
4 x 6	2468.456	**7200**

[1] Best incumbent solutions are shown in **bold**.

Results

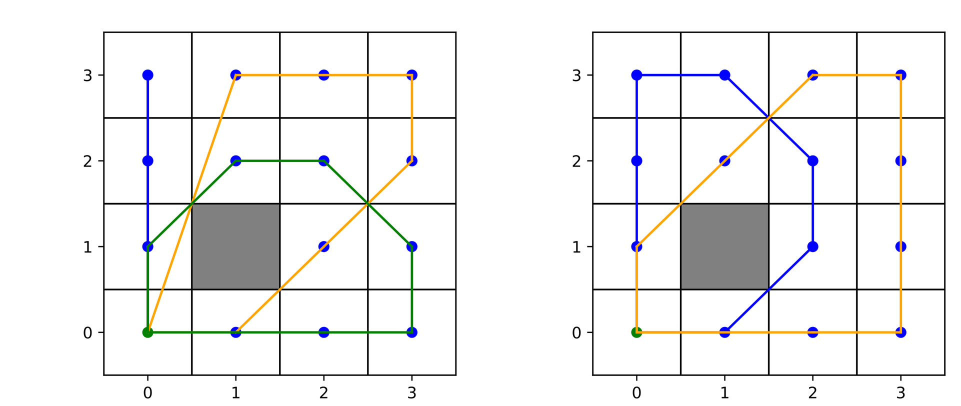
Trajectory results without obstacle using DP/SP Method



(a) Battery 16 (b) Battery 18

Figure: Experiment results using DP/SP Method for grid 4x4

Trajectory results with one obstacle using DP/SP Method



(a) Battery 16 (b) Battery 18

Figure: Experiment results using DP/SP Method for grid 4x4(obstacle at grid index 5)

Conclusion

- EECPP problem is extremely NP-hard.
- DP/SP(Proposed) w.r.t CPLEX.
  - For $C = 16.0$ kJ, when grid size is not greater than 3 x 4, our DP/SP method achieves optimal solution with CPLEX and uses less time.
  - As the grid size increases, the time for two methods both blows up quickly.
  - The obstacle can reduce complexity.

References

[1] J. Modares, F. Ghanei, N. Mastronarde, and K. Dantu, "Ub-anc planner: Energy efficient coverage path planning with multiple drones," in *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017, pp. 6182–6189.

[2] Y. Dumas, J. Desrosiers, and F. Soumis, "The pickup and delivery problem with time windows," *European journal of operational research*, vol. 54, no. 1, pp. 7–22, 1991.