Distributed Spectrum Management and Relay Selection in Interference-limited Cooperative Wireless Networks

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ABSTRACT

It is well known that the data transport capacity of a wireless network can be increased by leveraging the spatial and frequency diversity of the wireless transmission medium. This has motivated the recent surge of research in *cooperative* and *dynamic-spectrum-access* networks. Still, as of today, a key open research challenge is to design distributed control strategies to dynamically jointly assign (i) portions of the spectrum and (ii) cooperative relays to different traffic sessions to maximize the resulting network-wide data rate.

In this article, we make a significant contribution in this direction. First, we mathematically formulate the problem of joint spectrum management and relay selection for a set of sessions concurrently utilizing an interference-limited infrastructure-less wireless network. We then study distributed solutions to this (nonlinear and nonconvex) problem. The overall problem is separated into two subproblems, (i) spectrum management through power allocation with given relay selection strategy, and (ii) relay selection for a given spectral profile. Distributed solutions for each of the two subproblems are proposed, which are then analyzed based on notions from variational inequality (VI) theory. The distributed algorithms can be proven to converge, under certain conditions, to VI solutions, which are also Nash equilibrium (NE) solutions of the equivalent NE problems. A distributed algorithm based on iterative solution of the two subproblems is then designed. Performance and price of anarchy of the distributed algorithm are then studied by comparing it to the globally optimal solution obtained with a centralized algorithm. Simulation results show that the proposed distributed algorithm achieves performance that is within a few percentage points of the optimal solution.

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Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Distributed networks; G.1.6 [**Optimization**]: Constrained optimization

General Terms

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Keywords

Cooperative networks, spectrum management, relay selection, Nash equilibrium.

1. INTRODUCTION

The concept of *cooperative communications* has been proposed to achieve spatial diversity without requiring multiple transceiver antennas on a wireless device [1–3]. In cooperative communications, in their *virtual multiple-input single-output* (VMISO) variant, each node is equipped with a single antenna, and relies on the antennas of neighboring devices to achieve spatial diversity. Thanks to the broadcast nature of the wireless channel, signals transmitted by a source can be overheard by neighboring devices. Therefore, one (or multiple) relays can forward their received signals to the destination. Multiple copies of the original signal can then be received at the destination, which can combine them to decode the original message.

A vast and growing literature on information and communication theoretic results [4, 5] in cooperative communications is available. Readers are referred to [6, 7] and references therein for excellent surveys in this area. Problems addressed include the definition of algorithms to establish when and how to cooperate, and optimal cooperative transmission strategies. For example, [2, 8, 9] study in depth outage probability and capacity of an isolated communication link. However, only simple network topologies are studied, e.g., single source-destination pairs with single and fixed relay nodes, while network-wide interactions among multiple concurrent cooperative communication sessions are not considered. Distributed relay selection algorithms are also proposed based on non-cooperative game theory, e.g., auction theory [10] or Stackelberg games [11, 12]. However, typically a single channel and no interference among multiple concurrent communication links is assumed. Only recently, cooperative communications have been studied in conjunction with cognitive radio (CR) cellular systems [13–15] and ad hoc networks [16].

In this article, we look at the fundamental problem of designing algorithms to leverage the spatial and frequency diversity of the wireless channel by jointly allocating portions of the spectrum and cooperative relays to a set of concurrent data sessions to maximize the overall achievable data rate. Through our developments, we make the following contributions:

- *Cooperative Networks With Dynamic Spectrum Access*: We study the problem of joint spectrum management and relay assignment in dynamic-spectrum-access cooperative networks with decentralized control.

- *Effect of Cooperation and Dynamic Spectrum Access in Interference-Limited Networks*: Since cooperative and cognitive ad hoc networks are inherently interference-limited, and ideal orthogonal FDMA or TDMA channels can not be easily established without centralized control, we consider a general interference model. The results obtained can be applied to interference-free networks as a special case.

- *Distributed Algorithms*: We design and analyze distributed algorithms for spectrum assignment and relay selection, based on best-response local optimizations. Since the problem formulated in this paper has a rather complex (combinatorial, non-linear and non-convex) mathematical formulation, well-studied tools of game theory, e.g., contraction theory, cannot be applied because they require critical conditions on the joint utility function of all players. Therefore, we rely on notions from *variational inequality* (VI) theory (see [17] for a detailed survey of the theory and its applications to communications problems), through which we are able to prove convergence to Nash equilibrium (NE) under certain conditions and excellent performance in practice. We evaluate the price of anarchy of our distributed algorithm by comparing it to the optimal solution obtained through a newly designed centralized (optimal) algorithm.

The proposed algorithm can be directly applied to a scenario with multiple co-existing pre-established source-destination pairs. In addition, it can provide an upper bound to the performance of simpler centralized/distributed algorithms for spectrum management and relay assignment. Last, in a multi-hop ad hoc network, the proposed algorithm can be used to optimally control resource allocation for an independent set of transmissions with primary interference constraints (i.e., no transmitters and receivers in common) periodically scheduled by a separate scheduling algorithm, where idle nodes can be used as potential relays.

The rest of the paper is organized as follows. In Section 2 we discuss related work and in Section 3 we introduce system model and problem formulation. Then, we describe and analyze the distributed algorithm in Section 4. Finally, we present performance evaluation results in Section 5 and conclude the paper in Section 6.

2. RELATED WORK

Relay selection in cooperative wireless networks has been an important topic of research [10, 11, 11, 18–20]. Shi *et al.* [18] studied the relay selection problem in ad hoc networks and proposed an algorithm with attractive properties of both optimality guarantee and polynomial time complexity. Hou *et al.* [19] investigated the problem of joint flow routing and relay selection in multi-hop ad hoc networks, and proposed an optimal centralized algorithm with arbitrary predefined optimality precision based on the powerful branch-and-cut framework. Rossi *et al.* [20] studied the optimal cooperator selection in ad hoc networks based on Markov decision processes and the focused real time dynamic programming technique. Different from these works, we jointly study relay selection and spectrum management and focus on distributed algorithms.

Distributed relay selection based on game theory has also been an important research topic. For example, [10] studied the problem of distributed relay selection and relay power allocation in a single-relay network based on auction theory. In [11], the authors formulated the problem of distributed relay selection in a multiple-relay single-session network as a Stackelberg game. Zhang *et al.* [12] proposed an important framework for efficient resource management in cooperative cognitive radio network and formulated the problem of distributed relay selection and spectrum leasing as a Stackelberg game. In addition to [12], other excellent work on joint relay selection and spectrum management include [21, 22]. Zhao [21] investigated the power and spectrum allocation for cooperative relay in a three-node cognitive radio network. In [22], Ding *et al.* studied the cooperative diversity for three low-complexity relay selection strategies in spectrum-sharing networks. Different from the above work, we focus on interference-limited ad hoc networks with multiple concurrent sessions and multiple relay nodes.

Finally, the problem of distributed joint routing, relay selection and spectrum allocation in interference-limited ad hoc networks was investigated in [16], where distributed algorithms were proposed based on a "backpressure" framework. A centralized algorithm with optimality guarantee for joint relay selection and dynamic spectrum access in interference-limited video-streaming single-hop ad hoc networks was proposed in [23]. In this paper, we design a distributed algorithm to jointly allocate relays and spectrum in interference-limited infrastructure-less networks based on variational inequality theory and evaluate its performance by comparing it to an optimal centralized algorithm.

3. SYSTEM MODEL AND PROBLEM FORMULATION

A set $\mathcal{S}$ of communication sessions compete for spectrum resources. For each session, say $s \in \mathcal{S}$, a source-destination pair is identified. In this paper, each destination node is assumed to be reachable via one-hop by its source node while layer-3 routing in multi-hop networks will be investigated in our future work. Each source node can transmit to its destination node either using a direct link or through a cooperative relay. We refer to a link enhanced by a cooperative relay as a *cooperative link*. If a cooperative link is employed, the source node selects a node as relay from a set $\mathcal{R}$ of potential relay nodes. The available spectrum is divided into a set $\mathcal{F}$ of channels. Each channel is potentially shared among different sessions, i.e., each session can be seen as an interferer to any other sessions, and each user dynamically selects the best channels to access to maximize its own utility.

Assumptions. We make the following two assumptions:

- *Simultaneous access to a channel.* Multiple sessions are allowed to access a channel at any given time. Therefore, they will cause interference to one another. To mitigate the effect of interference, dynamic spectrum access is employed, i.e., each session dynamically selects which channels to use and allocates transmission power on each channel, based on the channel quality of the underlying wireless link and on the interference measured at the destination node.

- *Single relay selection.* For the sake of simplicity, we assume that each relay node can be selected by at most one session, and one session can select at most one relay node. It is worth mentioning, however, that the work in this paper can be easily extended to the case of multiple relay selection.

Cooperative Link Capacity Model. Denote the power allocation matrix for the source nodes as $\boldsymbol{P} = (P_s^f),\ s \in \mathcal{S}$, $f \in \mathcal{F}$, where P_s^f represents the transmission power for source node s on channel f. The power allocation matrix for the relay nodes is denoted with $\boldsymbol{Q} = (Q_r^f),\ r \in \mathcal{R},\ f \in \mathcal{F}$, where Q_r^f is the transmission power for relay node r on channel f. Further denote the relay selection matrix as $\boldsymbol{\alpha} = (\alpha_r^s),\ r \in \mathcal{R},\ s \in \mathcal{S}$, where $\alpha_r^s = 1$ if relay node r is selected by session s, and $\alpha_r^s = 0$ otherwise.

We let $C_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha})$ represent the capacity available to session s, which can be expressed as

$$C_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha}) = \underbrace{\left(1 - \sum_{r\in\mathcal{R}} \alpha_r^s\right) C_{dir}^s(\boldsymbol{P},\ \boldsymbol{Q})}_{\text{direct link}} + \underbrace{\sum_{r\in\mathcal{R}} \alpha_r^s C_{cop}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q})}_{\text{cooperative link}}, \quad (1)$$

where $C_{dir}^s(\boldsymbol{P},\ \boldsymbol{Q})$ represents the capacity available to session s if a direct link is used, and $C_{cop}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q})$ represents the capacity of cooperative link if relay r is selected by session s. For a direct link, $C_{dir}^s(\boldsymbol{P},\ \boldsymbol{Q})$ can be expressed as

$$C_{dir}^s(\boldsymbol{P},\ \boldsymbol{Q}) = B \sum_{f\in\mathcal{F}} \log_2\left(1 + \frac{G_{s2d}^{s,s} P_s^f}{\delta_{s,f}^2 + I_s^f}\right), \quad (2)$$

where B is the bandwidth of each channel, $\delta_{s,f}^2$ represents the power of additive white Gaussian noise (AWGN) at destination node s on channel f, $G_{s2d}^{s,s}$ represents the average channel gain from source node s to destination node s, and I_s^f represents the interference measured at destination node s on channel f.

Different forwarding strategies can be employed for cooperative relaying, e.g., *amplify-and-forward* (AF) and *decode-and-forward* (DF) [2]. We assume that DF is used at each relay node, while AF will be addressed in our future work. Then, $C_{cop}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q})$ can be expressed as [2,18]

$$\begin{aligned} C_{cop}^{s,r}(\boldsymbol{P},\ \boldsymbol{Q}) &= \sum_{f\in\mathcal{F}} C_{cop}^{s,r,f}(\boldsymbol{P},\ \boldsymbol{Q}) \\ &= \frac{1}{2} \sum_{f\in\mathcal{F}} \min\left\{C_{s2r}^{s,r,f},\ C_{sr2d}^{s,r,f}\right\} \end{aligned} \quad (3)$$

where $C_{s2r}^{s,r,f}$ represents the capacity of link from source node s to relay node r on channel f, and $C_{sr2d}^{s,r,f}$ represents the capacity achieved through maximal ratio combining [2] on the two copies of the signal received by destination node s from source node s and relay node r on channel f. The coefficient $\frac{1}{2}$ in (3) indicates that the overall capacity for the cooperative link is averaged over two time-slots. Expressions for the two capacities are given by

$$C_{s2r}^{s,r,f}(\boldsymbol{P},\ \boldsymbol{Q}) = B \log_2\left(1 + \frac{G_{s2r}^{s,r} P_s^f}{\delta_{r,f}^2 + I_r^f}\right), \quad (4)$$

$$C_{sr2d}^{s,r,f}(\boldsymbol{P},\ \boldsymbol{Q}) = B \log_2\left(1 + \frac{G_{s2d}^{s,s} P_s^f + G_{r2d}^{r,s} Q_r^f}{\delta_{s,f}^2 + I_s^f}\right), \quad (5)$$

where $\delta_{r,f}^2$ represents gaussian noise power at relay node r on channel f, $G_{s2r}^{s,r}$ and $G_{r2d}^{r,s}$ represents average channel gain from source node s to relay node r, and from relay node r to destination node s, respectively, and I_r^f represents interference measured at relay node r on channel f.

Interference model. In (2), (4) and (5), the interference I_s^f and I_r^f depends on power allocation and relay selection at each individual node, but also on the network scheduling strategy of the whole network (i.e., the relative synchronization of transmission start times between different network communication links). To keep the model tractable, the interference at each receiver can be approximated in different ways. In the worst-case approximation, the assumption is that all source and active relay nodes cause interference in both time-slots. The average-based approximation considers instead the average effect of each interferer over the two time slots. Our experiments reveal that the average-based approximation models reality very well - in-depth validation of the average-based interference model is discussed in detail in Appendix A. With this model, I_s^f can be expressed as

$$\begin{aligned} I_s^f = \sum_{w\in\mathcal{S}_{-s}} \Bigg[&\left(1 - \sum_{r\in\mathcal{R}} \alpha_r^w\right) G_{s2d}^{w,s} P_w^f \\ &+ \frac{1}{2} \sum_{r\in\mathcal{R}} \alpha_r^w \left(G_{s2d}^{w,s} P_w^f + G_{r2d}^{r,s} Q_r^f\right)\Bigg], \end{aligned} \quad (6)$$

where $\mathcal{S}_{-s}$ represents the set of all sessions except session s, and $\mathcal{R}_{-r}$ represents the set of all relay nodes except for relay node r. I_r^f has a similar expression.

Problem formulation. We let U_s represent the utility function for session s and define it as

$$U_s = \log(C_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha})), \quad (7)$$

where $C_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha})$ is defined in (1) and a log-capacity utility function is considered to promote fairness among communication sessions. Then, the objective of our problem is to maximize a sum utility function of all communication sessions by selecting for each session: i) which channels to allocate, ii) transmission power to be used on each selected channel, iii) whether to use a direct link or cooperative link, and iv) which relay node to select, i.e.,

$$\underset{\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha}}{\text{maximize}} \quad U = \sum_{s \in \mathcal{S}} U_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha}) \tag{8}$$

$$\text{subject to} \quad \alpha_r^s \in \{0,\ 1\},\ \forall s \in \mathcal{S},\ \forall r \in \mathcal{R} \tag{9}$$

$$\sum_{r \in \mathcal{R}} \alpha_r^s \le 1,\ \forall s \in \mathcal{S} \tag{10}$$

$$\sum_{s \in \mathcal{S}} \alpha_r^s \le 1,\ \forall r \in \mathcal{R} \tag{11}$$

$$P_s^f \ge 0, \forall s \in \mathcal{S},\ \forall f \in \mathcal{F} \tag{12}$$

$$Q_r^f \ge 0, \forall r \in \mathcal{R},\ \forall f \in \mathcal{F} \tag{13}$$

$$\sum_{f \in \mathcal{F}} P_s^f \le P_{max}^s,\ \forall s \in \mathcal{S} \tag{14}$$

$$\sum_{f \in \mathcal{F}} Q_r^f \le Q_{max}^r,\ \forall r \in \mathcal{R}, \tag{15}$$

where U represents sum utility, and $U_s(\boldsymbol{P},\ \boldsymbol{Q},\ \boldsymbol{\alpha})$ is defined in (7) (also denoted as U_s for conciseness). The expressions in (9)-(11) impose constraints on relay selection (at most one relay per session and one session per relay), while (12)-(15) impose constraints on power allocation and power budget for each source and relay node. Here, P_{max}^s and Q_{max}^r represent the maximum transmission power of source node s and relay node r, respectively.

Contributions of the paper. In the problem formulated in (1)-(15), the expressions defined in (2)-(6) are nonlinear (and non-convex) functions of the problem variables. Moreover, the relay selection variables α_r^s, $r \in \mathcal{R}, s \in S$, are constrained to take binary values (0 or 1). Therefore, the expression in (1) and consequently the objective function in (7) are both integral and non-convex. This causes the problem to be a *Mixed-Integer and Non-Convex Problem* (MINCoP), which is in general NP-hard (i.e., no existing algorithm can solve an arbitrary MINCoP in polynomial time). The paper makes the following contributions:

- *Distributed solution algorithms*: We propose distributed algorithms designed to dynamically control node behavior based on localized best-response strategies. The original problem is decomposed into two separate problems, namely, distributed relay selection with given power spectral profile, and distributed spectrum allocation for a given relay selection.
- *Convergence analysis*: We study the convergence of iterative algorithms based on iterative solutions of the two individual problems. We analyze the convergence and optimality of distributed algorithms for spectrum assignment and relay selection, based on notions from *variational inequality* (VI) theory.

4. DISTRIBUTED ALGORITHM

In this section, we propose a distributed algorithm for the problem formulated in Section 3 that is amenable to practical (distributed) implementation. The proposed distributed algorithm is designed to achieve the *Nash equilibrium* (NE) [24], which is a well-known concept from non-cooperative game theory often used as a tool for designing distributed algorithms in complex wireless communication systems [10–12]. There are two important characteristics of a NE solution, i) at any NE solution point, no user has incentives to deviate from the current transmission strategy unilaterally, and ii) each user's utility is maximized, given the transmission strategies of any other users. In this section, we study i) whether a NE solution point exists for our problem, ii) how to achieve such a NE solution point if it exits, and iii) the so-called price of anarchy, i.e., we compare the performance at NE solution point to the global optimal solution (obtained through a centralized algorithm).

Recall that our problem is a MINCoP and each user's utility function is neither linear nor convex. This imposes major challenges to the NE analysis. Due to the complex expression of the utility function, traditional mathematical tools for NE analysis, e.g., contraction mapping theory [25] are not applicable. We base our NE analysis on *variational inequality* (VI) theory [17, 26]. VI theory can be used to formulate and analyze problems that do not fit within the narrower scope of game theory. Second, and most important, based on VI theory, there are well-developed tools to analyze the convergence of distributed algorithms. Next, we first give a brief introduction of VI theory and of the relationship between VI and game theory. Then, we describe how to reformulate and analyze our problem using VI theory.

4.1 Basics of VI Theory

For the reader's convenience, we provide definitions for a *variational inequality problem* and *Nash equilibrium problem*, respectively. Readers are referred to [17] for a detailed introduction to the relationship between them and [26] for a comprehensive overview of VI theory.

Definition 1 (Variational Inequality Problem). *Given a closed and convex set $\mathcal{X} \in \mathbb{R}^n$ and a continuous mapping function $\boldsymbol{F} : \mathcal{X} \to \mathbb{R}^n$, the VI problem, denoted as* $\mathrm{VI}(\mathcal{X}, \boldsymbol{F})$, *consists of finding a vector $\boldsymbol{x}^* \in \mathcal{X}$ (called a solution of the VI) such that [26]*

$$(\boldsymbol{y} - \boldsymbol{x}^*)^T \boldsymbol{F}(\boldsymbol{x}^*) \le 0,\ \forall \boldsymbol{y} \in \mathcal{X}. \tag{16}$$

Definition 2 (Nash Equilibrium Problem). *Assume there are a Q players each controlling a variable $\boldsymbol{x}_i \in \mathcal{Q}_i$. Denote $\boldsymbol{x}$ as the vector of all variables $\boldsymbol{x} \triangleq (x_i, \cdots, x_Q)$, and let $\boldsymbol{x}_{-i} = (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_Q)$ represent the vector of all player variables except that of player i. Each player i is also associated with a utility function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i})$. Define the Cartesian product of all $\mathcal{Q}_i$ as $\mathcal{Q} \triangleq \prod_{i=1}^{Q} \mathcal{Q}_i$, the vector of utility functions as $\boldsymbol{f} = (f_1, \cdots, f_Q)$. Then, a Nash equilibrium problem, denoted as* $\mathrm{NE}(\mathcal{Q}, \boldsymbol{f})$, *consists of finding $\boldsymbol{x}^* = (\boldsymbol{x}_1^*, \cdots, \boldsymbol{x}_Q^*)$ (called Nash equilibrium solution), such that each player i's utility function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*)$ is maximized, i.e., [24]*

$$\boldsymbol{x}_i^* = \underset{\boldsymbol{x}_i \in \mathcal{Q}_i}{\arg\max} f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*),\ \forall i. \tag{17}$$

Given a Nash equilibrium problem $\mathrm{NE}(\mathcal{Q}, \boldsymbol{f})$, assume that for each player i, i) the strategy set $\mathcal{Q}_i$ is closed and convex, and ii) the utility function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i})$ is continuously differentiable with respect to $\boldsymbol{x}_i$ in $\mathcal{Q}_i$. Then, the Nash equilibrium problem $\mathrm{NE}(\mathcal{Q}, \boldsymbol{f})$ is equivalent to the $\mathrm{VI}(\mathcal{Q}, \boldsymbol{F})$, where $\boldsymbol{F} \triangleq (\nabla_{\boldsymbol{x}_i} f_i(\boldsymbol{x}))_{i=1}^{Q}$ [17]. Hence, to achieve a NE solution for $\mathrm{NE}(\mathcal{Q}, \boldsymbol{f})$, we only need to find a VI solution for $\mathrm{VI}(\mathcal{Q}, \boldsymbol{F})$.

It should be pointed out that, to take advantage of the well-developed theoretical results and of the existing distributed algorithms of the VI theory, a condition required by VI theory is that the mapping function $\boldsymbol{F}$ be at least component-wise strongly monotonic [26]. Recall that the utility function in our problem has a rather complex expression, which makes the monotonicity analysis very hard. To address this challenge, we decompose the problem formulated in Section 3 into two individual problems: i) distributed power allocation, for a given cooperative relaying strategy, and ii) distributed relay selection, for a given spectrum profile. Then, we study the problem of joint relay selection and power allocation based on the two individual problems.

4.2 Distributed Spectrum Management (DSM) by Power Allocation

4.2.1 Game Theory Formulation

A problem of DSM with given cooperative relaying strategy can be naturally formulated as a game, in which each communication session can be seen as a player, and each player tries to maximize its utility function defined in (7) by adjusting the transmission power over the available frequencies for its source node and corresponding relay node if cooperative relaying is employed. Hereafter, we use the terms of "communication session" and "player" interchangeably.

Assume relay node r is selected by session s, and denote the vector of power allocation for source node s and relay node r as $\boldsymbol{P}_s = (P_s^f),\ f \in \mathcal{F}$ and $\boldsymbol{Q}_r = (Q_r^f),\ f \in \mathcal{F}$, respectively. Then, any $\boldsymbol{P}_s$ or $\boldsymbol{Q}_r$ is feasible if it satisfies the constraints in (12) and (14), or the constraints in (13) and (15), respectively. We let $\mathcal{P}_s$ and $\mathcal{Q}_r$ represent the set of all feasible $\boldsymbol{P}_s$ and $\boldsymbol{Q}_r$, and denote the vector of power allocation for session s as $\boldsymbol{x}_s = (\boldsymbol{P}_s,\ \boldsymbol{Q}_r)$. Then, the set of transmission strategies for session s, denoted as $\mathcal{X}_s$, can be defined as the Cartesian product of $\mathcal{P}_s$ and $\mathcal{Q}_s$, i.e., $\mathcal{X}_s \triangleq \mathcal{P}_s \times \mathcal{Q}_r$. If session s uses only direct link, the set of transmission strategies simply reduces to $\mathcal{X}_s = \mathcal{P}_s$.

The utility function for communication session $s \in \mathcal{S}$ is defined in (7), where $C_s(\boldsymbol{P},\ \boldsymbol{Q})$ is defined in (2) if only direct link is used, and in (3) otherwise. Define $\boldsymbol{P}_{-s} = (\boldsymbol{P}_w),\ w \in \mathcal{S},\ w \neq s$, and $\boldsymbol{Q}_{-r} = (\boldsymbol{Q}_q),\ q \in \mathcal{R},\ q \neq r$, as the vectors of power allocation for all other source and relay nodes. Then, with given fixed $\boldsymbol{\alpha}$ and $\boldsymbol{x}_{-s} = (\boldsymbol{P}_{-s}, \boldsymbol{Q}_{-r})$, the utility function of session s can be rewritten as

$$\begin{aligned} U_s(\boldsymbol{x}_s,\ \boldsymbol{x}_{-s}) &= \log(C_s(\boldsymbol{x}_s,\ \boldsymbol{x}_{-s})) \\ &= \log(C_s(\boldsymbol{P}_s,\ \boldsymbol{Q}_r,\ \boldsymbol{P}_{-s},\ \boldsymbol{Q}_{-r})), \end{aligned} \quad (18)$$

where $C_s(\boldsymbol{P}_s,\ \boldsymbol{Q}_r,\ \boldsymbol{P}_{-s},\ \boldsymbol{Q}_{-r})$ is defined in the same way as $C_s(\boldsymbol{P},\ \boldsymbol{Q})$ with $\boldsymbol{P} = (\boldsymbol{P}_s, \boldsymbol{P}_{-s})$ and $\boldsymbol{Q} = (\boldsymbol{Q}_r, \boldsymbol{Q}_{-r})$.

According to Definition 2, the DSM problem can be modeled as a game $\mathrm{NE}(\mathcal{X}, \boldsymbol{U})$, where $\mathcal{X} = \prod_{s\in\mathcal{S}} \mathcal{X}_s$ and $\boldsymbol{U} = (U_s)_{s\in\mathcal{S}}$.

4.2.2 VI Formulation

Recall from Section 4.1 that a given NE problem is equivalent to a VI if the utility function for each player is defined in a closed and convex domain set and is continuously differentiable. In the DSM problem formulated above, the domain set of $\mathcal{X}$ is closed and convex, since it is defined by a set of linear constraints in (12)-(15). However, the utility function for each player is certainly not continuously differentiable. This is because if cooperative relaying is employed by a communication session, the capacity of the cooperative link is defined in (3), where the minimum operation leads to a non-smooth function. Hence, the resulting utility function is not continuously differentiable.

To facilitate the analysis, we can approximate the cooperative link capacity in (3) using a continuously differentiable function, denoted as $\widehat{C}_{cop}^{s,r,f}$, constructed based on ℓ_P-norm function as follows

$$\begin{aligned} \widehat{C}_{cop}^{s,r,f} &= \ell_P^{-1}((C_{s2r}^{s,r,f})^{-1},\ (C_{sr2d}^{s,r,f})^{-1}) \\ &= \left\{ \left[\left(\frac{1}{C_{s2r}^{s,r,f}} \right)^P + \left(\frac{1}{C_{sr2d}^{s,r,f}} \right)^P \right]^{\frac{1}{P}} \right\}^{-1} \end{aligned} \quad (19)$$

where $C_{s2r}^{s,r,f}$ and $C_{sr2d}^{s,r,f}$ are defined in (4) and (5), respectively. The ℓ_P-norm of a vector with large value of parameter P emphasizes the larger element in the vector [27], hence, it also emphasizes the element with the smallest inverse. The resulting approximation function is continuously differentiable in its domain, and the original function in (3) can be approximated with arbitrary precision by adjusting the value of parameter P. When $P \to \infty$, we have $\widehat{C}_{cop}^{s,r,f} \to C_{cop}^{s,r,f}$.

Based on the above approximation, the utility function for each player becomes continuously differentiable, and hence, we can rewrite the NE problem in Section 4.2.1 as a VI problem $\mathrm{VI}(\mathcal{X}, \boldsymbol{F})$, with $\boldsymbol{F} = (\nabla_{\boldsymbol{x_s}} U_s),\ s \in \mathcal{S}$.

4.2.3 VI Solution and Distributed Algorithm

After obtaining a VI formulation of the problem, we can study whether a VI solution (which is also a NE solution) exists for the problem, and if it exists, how to achieve it in a distributed fashion. Before developing a distributed algorithm, we give the following two lemmas about the domain set $\mathcal{X}$ and the utility function for each player $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$, respectively.

Lemma 1. *There exists at least one solution for* $\mathrm{VI}(\mathcal{X}, \boldsymbol{F})$.

PROOF. The domain set $\mathcal{X}$ is closed and convex, and the mapping function $\boldsymbol{F}$ is continuous. According to the existence theorem in [26], there exists at least one VI solution. □

Lemma 2. *Assume that relay node r is selected by communication session s for cooperative relaying. Then, the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ is a strongly concave function if it satisfies the condition that the signal-to-noise-plus-interference ratio (SINR) on each channel for the link between source node s and destination node s, source node s and relay node r, relay node r and destination node s, is greater than $e - 1$, where e is the base of natural logarithm.*

PROOF. Please see Appendix B for proof. □

The condition in Lemma 2 is a sufficient condition for the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ to be strongly concave, and requires that the SINR on each channel is not too low[1] for a

[1]A value of $e - 1 \approx 1.71828$ for SINR implies that the received signal power is comparable to the sum of noise and interference - which corresponds to very poor channel quality.

cooperative link. In practice, a communication session will never allocate much transmission power to a channel with very bad channel quality. Moreover, if all available channels have very bad quality, then even minimum quality of service (QoS) requirements cannot be guaranteed. In this case, the communication session should be denied access to the wireless network. Alternatively, the session should increase its maximum transmission power or select another routing path to avoid channels with poor quality. Therefore, with no loss of generality for all practical purposes, we assume that the condition in Lemma 2 can be satisfied.

Notice that, if a communication session uses only a direct link, we can prove that the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ is also a strongly concave function for any SINR level. The proof simply follows from the fact that logarithm of a concave and positive function is also a concave function [27]. Here, we omit details of the proof.

Next, we propose an iterative algorithm of dynamic spectrum management which is based on the local best-response of each communication session, and analyze the necessary and sufficient conditions for the algorithm to converge to a VI solution of $\text{VI}(\mathcal{X}, \boldsymbol{F})$.

We give the Gauss-Seidel implementation of the local best-response based algorithm in Algorithm 1, where S represents the number of communication sessions in $\mathcal{S}$. In Algorithm 1, a feasible initial power allocation means that the power level at a subchannel can not be too low. Otherwise, there would be no guarantee of convergence according to Lemma 2. An infeasible initial power allocation can be avoided by artificially increasing power values that are too low, while an infeasible power allocation during iterations in Algorithm 1 can be avoided by allocating zero power to a subchannel with poor quality (i.e., do not select poor subchannels for transmission).

It is worth pointing out that the Lemma 1 and Lemma 2 only provide necessary conditions for the Algorithm 1 to converge. In following Theorem 1 we present a sufficient condition, under which Algorithm 1 always converges to VI solution of $\text{VI}(\mathcal{X}, \boldsymbol{F})$.

Theorem 1. *Given the VI problem* $\text{VI}(\mathcal{X}, \boldsymbol{F})$ *formulated in Section 4.2.2, if Lemma 1 and Lemma 2 hold and any two sessions in* $\mathcal{S}$ *are located sufficiently far away from each other, then a Gauss-Seidel scheme based on the local best-response of each communication session converges to a VI solution.*

PROOF. In our VI problem, the domain set $\mathcal{X}$ is closed and convex. From Lemma 1, we have that $\text{VI}(\mathcal{X}, \boldsymbol{F})$ has at least one solution. From Lemma 2, we have that the utility function $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ is strongly concave for each player given fixed transmission strategies. Then, the strong concavity of $U_s(\boldsymbol{x}_s, \boldsymbol{x}_{-s})$ implies that $F_s = \nabla_{\boldsymbol{x}_s} U_s$ is strongly monotonic [26]. In the following, we present a sufficient condition for the most complicated case, i.e., when each session uses a cooperative relay, that can be derived based on the framework in [28]. The resulting sufficient condition can be easily extended to other cases.

If relay node r, with $r \in \mathcal{R}$, is selected by session s, with $s \in \mathcal{S}$, then the gradient vector of session s with respect to $\boldsymbol{x}_s$ can be written as $J_{\boldsymbol{x}_s}(U_s) = \left(\left(\frac{\partial U_s}{\partial P_s^f} \right)_{f=1}^F, \left(\frac{\partial U_s}{\partial Q_r^f} \right)_{f=1}^F \right)$, where U_s is utility function of session s defined in (7), $\boldsymbol{x}_s = ((P_s^f)_{f=1}^F, (Q_r^f)_{f=1}^F)$ is the power allocation vector of session s, and F represents the number of sub-channels in set $\mathcal{F}$. Similarly, we represent the gradient vector of session g, with $g \in \mathcal{S}$, as $J_{\boldsymbol{x}_g}(U_g) = \left(\left(\frac{\partial U_g}{\partial P_g^f} \right)_{f=1}^F, \left(\frac{\partial U_g}{\partial Q_t^f} \right)_{f=1}^F \right)$, where t is the relay node selected by session g. Further, denote the Jacobi matrix of $J_{\boldsymbol{x}_s}(U_s)$ and $J_{\boldsymbol{x}_g}(U_g)$ with respect to x_s as $J_{\boldsymbol{x}_s \boldsymbol{x}_s}(U_s)$ and $J_{\boldsymbol{x}_g \boldsymbol{x}_s}(U_g)$, respectively. Here, $J_{\boldsymbol{x}_s \boldsymbol{x}_s}(U_s)$ is namely the Hessian matrix session s. Then, we can define a matrix $[\gamma]_{ij}$ as follows

$$[\gamma]_{sg} \triangleq \begin{cases} \alpha_s^{\min}, & \text{if } s = g, \\ -\beta_{sg}^{\max}, & \text{otherwise,} \end{cases} \tag{20}$$

where $\alpha_s^{\min} \triangleq \inf_{x \in \mathcal{X}} \lambda_{\text{least}}(J_{\boldsymbol{x}_s \boldsymbol{x}_s}(U_s))$ and $\beta_{sg}^{\max} \triangleq \sup_{x \in \mathcal{X}} \| J_{\boldsymbol{x}_g \boldsymbol{x}_s}(U_g) \|$, with $\lambda_{\text{least}}(\boldsymbol{A})$ representing the eigenvalue of $\boldsymbol{A}$ with the smallest absolute value. Then, based on the properties of the P-matrix [29], to guarantee the convergence of the proposed distributed algorithm, we only need to show that the matrix γ_{sg} defined in (20) is a P-matrix [28]. It can be shown that if any two session in $\mathcal{S}$ are located sufficient away from each other, γ_{sg} is a P-matrix. On the contrary, γ_{sg} is not a P-matrix if there exists two or more sessions that are close to each other. □

Algorithm 1 : Gauss-Seidel best-response algorithm for DSM

Step 1: Initialize to any feasible power allocation $\boldsymbol{x}^{(0)} = (\boldsymbol{x}_s^{(0)})_{s \in \mathcal{S}}$ that satisfies the constraints in (12)-(15), and set iteration index $n = 0$.

Step 2: For $s = 1, \cdots, S$, calculate $\boldsymbol{x}_s^{(n+1)}$ by solving

$$\begin{aligned} \underset{\boldsymbol{x}_s}{\text{maximize}} \quad & U_s(\boldsymbol{x}_1^{(n+1)}, \cdots, \boldsymbol{x}_{s-1}^{(n+1)}, \boldsymbol{x}_s, \boldsymbol{x}_{s+1}^{(n)} \cdots, \boldsymbol{x}_S^{(n)}) \\ \text{subject to} \quad & \boldsymbol{x}_s \in \mathcal{X}_s \end{aligned} \tag{21}$$

Step 3: Set $\boldsymbol{x}^{(n+1)} = (\boldsymbol{x}_s^{(n+1)})_{s=1}^S$ and set $n \leftarrow n + 1$.

Step 4: If $\boldsymbol{x}^{(n+1)}$ is a VI solution of the problem in (16), terminate, and go to step 2 otherwise.

4.3 Distributed Relay Selection (DRS)

4.3.1 Game Theoretic Formulation

The problem of relay selection with given fixed spectrum profile can also be formulated as a game, in which each communication session selects its best relay node in a competitive fashion to maximize its own utility function in (7). Recall that in Section 3 we assumed that only one single relay is selected by a communication session. Therefore, competition occurs if a relay node is the best relay for more than one session. Moreover, as shown in (6), the transmission strategy of a session, i.e., using only direct link or cooperative link, also affects the interference caused by the session to the other sessions.

Denote the vector of relay selection variables for communication session $s \in \mathcal{S}$ as $\boldsymbol{\alpha}_s = (\alpha_r^s)$, $r \in \mathcal{R}$, and the vector of relay selection variables for all other communication sessions except s as $\boldsymbol{\alpha}_{-s} = (\alpha_r^w)$, $r \in \mathcal{R}$, $w \in \mathcal{S}$, $w \neq s$. Then, $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$. We let Φ_s represent the set of all possible $\boldsymbol{\alpha}_s$, and Φ represent the set of all possible $\boldsymbol{\alpha}$. Given a fixed spectrum profile and relay selection strategies for all other communications, the utility function for session s in (7) can be rewritten as $U_s(\boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{-s})$. Denote the vector of all utility functions as $\boldsymbol{U} = (U_s),\ s \in \mathcal{S}$. Then, the Nash equilibrium problem of relay selection can be formulated as $\mathrm{NE}(\varPhi, \boldsymbol{U})$.

Each individual domain set $\varPhi_s$ is described by constraints in (9), and (11), while the overall domain set $\varPhi$ is described by constraints in (9), (10) and (11). Since all communication sessions are coupled through constraint (10), $\varPhi$ can not be written in the form of the Cartesian product of $\varPhi_s$. Given $\boldsymbol{\alpha}_{-s}$, the set $\varPhi_s$ is a function of $\boldsymbol{\alpha}_{-s}$, i.e., $\varPhi_s = \varPhi_s(\boldsymbol{\alpha}_{-s})$. Hence, the Nash equilibrium problem $\mathrm{NE}(\varPhi, \boldsymbol{U})$ is not a standard NE problem as defined in Definition 2 with respect to the domain set. In this case, the $\mathrm{NE}(\varPhi, \boldsymbol{U})$ is called a generalized Nash equilibrium (GNE) problem, denoted as $\mathrm{GNE}(\varPhi, \boldsymbol{U})$. We propose a penalization-based algorithm to transform the $\mathrm{GNE}(\varPhi, \boldsymbol{U})$ into a series of standard NE problems, which can then be analyzed and solved using the existing VI theory.

4.3.2 VI Reformulation

In the formulated GNE problem, the domain set $\varPhi$ is closed and convex. Moreover, with given spectrum profile and fixed relay selection strategy for all communication sessions except s, the capacity for session s in (2) and (3) is fixed. Hence, $U_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$ becomes a function of $\boldsymbol{\alpha}_s$ only. To cast the GNE problem into a VI problem to make the theoretical analysis easier, we still need to relax the integer problem (due to the binary relay selection variables) to a continuous one. To this end, in the following discussion we relax the binary requirement and let each $\alpha_r^s,\ r \in \mathcal{R},\ s \in \mathcal{S}$ be real. Effects of the relaxation will be analyzed later together with the proposed distributed algorithm. Then, the utility function $U_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$ becomes continuously differentiable, and $\mathrm{GNE}(\varPhi, \boldsymbol{U})$ can be reformulated as a VI problem with mapping function $\boldsymbol{F} = (F_s)_{s\in\mathcal{S}}$, where $F_s = \nabla_{\boldsymbol{\alpha}_s} U_s$. Corresponding to a GNE, the VI problem with coupled domain set $\varPhi$ is called a quasi-VI (QVI) problem, denoted as $\mathrm{QVI}(\varPhi, \boldsymbol{F})$. Before developing distributed relay selection algorithm based on the QVI reformulation of the GNE problem, we first give following lemma.

Lemma 3. *There exists at least one VI solution (also Nash Equlibrium) for* $\mathrm{QVI}(\varPhi, \boldsymbol{F})$.

Proof. For a QVI problem with closed and convex domain set and continuous utility function, there exists at least one VI solution solving the QVI problem [26]. □

4.3.3 Distributed DRS Algorithm

Each communication session, say s, locally decides its optimal relay selection strategy $\boldsymbol{\alpha}_s$ for a given $\boldsymbol{\alpha}_{-s}$. Since the interference measured at each destination and corresponding relay nodes are affected by the relay selection strategies of all other sessions, an update of relay selection for any communication session will trigger update of relay selection for all other sessions. More importantly, the set of possible relay selection strategies for each communication session also changes, i.e., $\boldsymbol{\alpha}_s$ is a function of $\boldsymbol{\alpha}_{-s}$.

A natural way to address this case with coupled domain sets is to set a price for each relay node, and make each communication session pay a price to it [17]. Then, each relay node updates its price based on the relay selection strategies of all sessions. If more than one session selects the same relay node, then the price for the relay node is increased. Otherwise, the relay node keeps its price unchanged. However, a main concern of the price-based algorithm is how to guarantee that the algorithm converges to a VI solution, which is also a NE solution for the NE problem. We propose to design the price-based algorithm using a penalized version of the original utility function for each communication session such that the resulting algorithm can be proven to converge to a VI solution of the DRS problem.

The proposed algorithm converges to a VI solution iteratively. At iteration k, communication session s has a penalized version of the utility function $U_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$, denoted as $\widehat{U}_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$, as follows

$$\widehat{U}_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s}) = U_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s}) \underbrace{-\frac{1}{2\rho_k}\sum_{r\in\mathcal{R}}\left(\max\left(0, u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}}\alpha_r^s - 1\right)\right)\right)^2}_{\text{Penalization}}. \quad (22)$$

In (22), each player's utility is penalized by subtracting a value, which is zero if the constraint in (10) is not violated, and is positive otherwise. $\{\rho_k\},\ k = 0, 1, \cdots$, is a sequence of positive scalars and satisfies $\rho_k < \rho_{k+1}$ and $\rho_k \to \infty$ as $k \to \infty$. $\{\boldsymbol{u}_k\},\ k = 1, 2, \cdots$, is a bounded sequence of vectors with $\boldsymbol{u}_k = (u_k^r),\ r \in \mathcal{R}$. We will see later that u_k^r is used as the price for relay node r at iteration k, while ρ_k is employed as the stepsize based on which each relay node updates its price.

Based on the penalized function $\widehat{U}_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$, we can construct a new VI problem $\mathrm{VI}(\widehat{\varPhi},\ \widehat{\boldsymbol{F}})$, where $\widehat{\varPhi}$ is the Cartesian product of each individual domain set $\varPhi_s,\ s \in \mathcal{S}$, and $\widehat{\boldsymbol{F}} = (\nabla_{\alpha_s}\widehat{U}_s),\ s \in \mathcal{S}$. Moreover, we have that Lemma 4 holds true for each $\nabla_{\alpha_s}\widehat{U}_s$.

Lemma 4. $\nabla_{\alpha_s}\widehat{U}_s$ *is a strongly monotonic function of* $\boldsymbol{\alpha}_s$ *for each* $s \in \mathcal{S}$.

Proof. We only need to show that the penalized utility function $\widehat{U}_s$ is a strongly concave function [26]. Then, it is sufficient to show that on the right hand-side of (22), both $U_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s})$ and the penalization item are strongly concave functions of $\boldsymbol{\alpha}_s$. This can be proven based on the definition of strong concavity [26] and composition rules that preserve concavity [27]: i) component-wise maximum of two affine functions is a convex function, ii) the P_{th} power of a convex and positive function is a convex function if $P > 1$, and iii) the opposite of a convex function is a concave function. □

Based on Lemma 4 and the Theorem 1, a VI solution for $\mathrm{VI}(\widehat{\varPhi},\ \widehat{\boldsymbol{U}})$ can be calculated through a best-response based algorithm similar to Algorithm 1. Denote the VI solution obtained at iteration k with $\boldsymbol{\alpha}_k = (\boldsymbol{\alpha}_s^k)$, where $\boldsymbol{\alpha}_s^k = (\alpha_{s,r}^k),\ s \in \mathcal{S},\ r \in \mathcal{R}$. Then, we can update ρ_k and $\boldsymbol{u}_k$ as follows

$$\rho_{k+1} = \rho_k + \varDelta\rho, \quad (23)$$

$$u_{k+1}^r = \max\left(0,\ u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}}\alpha_{s,r}^k - 1\right)\right), \quad (24)$$

where $\varDelta\rho$ is any fixed positive constant.

The proposed algorithm is summarized in Algorithm 2, and for the algorithm we have that Lemma 5 holds true.

Lemma 5. *The proposed penalized algorithm always converges to a VI solution for* $\mathrm{QVI}(\varPhi, \boldsymbol{F})$*, which is also a NE solution for* $\mathrm{GNE}(\varPhi, \boldsymbol{U})$.

Algorithm 2: Penalization-Based Algorithm for DRS

Step 1: Initialize $\rho_0 = 0,\ \boldsymbol{u}_0 = 0$, and set $k = 0$.
Step 2: Calculate VI solution $\boldsymbol{\alpha}_k$ by solving VI problem VI($\widehat{\Phi},\ \widehat{\boldsymbol{U}}$), in which each communication session solves

$$\begin{aligned} \underset{\boldsymbol{\alpha}_s}{\text{maximize}} \quad & \widehat{U}_s(\boldsymbol{\alpha}_s,\ \boldsymbol{\alpha}_{-s}) \text{ in (22)} \\ \text{subject to} \quad & \boldsymbol{\alpha}_s \in \Phi_s. \end{aligned} \tag{25}$$

Step 4: Update ρ_k and $\boldsymbol{u}_k$ according to (23) and (24), respectively. Set $k \leftarrow k+1$.
Step 5: If condition (10) is satisfied for each relay node $r \in \mathcal{R}$, stop. Otherwise, go to Step 2.

Proof. To prove the lemma, it is enough to prove i) the proposed DRS algorithm in Algorithm 2 converges, and ii) every accumulation point corresponds to a VI solution of QVI($\Phi, \boldsymbol{F}$) formulated in Section 4.3.2. By redefining the domain set Φ in QVI($\Phi, \boldsymbol{F}$) using a set of vectors of functions, it can be proven that the constraining functions satisfy the implication condition in the Theorem 3 in [30]. Then, according to the Theorem 3 in [30], we have that $\max\left(0, u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}} \alpha_r^s - 1\right)\right)$ in (22) is bounded and hence that, the penalization item in (22) tends to zero as ρ_k tends to infinity, implying that the proposed iterative algorithm in Algorithm 2 converges. Moreover, according to Theorem 3 in [30], each accumulation point corresponds to a QVI solution of QVI($\Phi, \boldsymbol{F}$). $\square$

4.4 Joint Spectrum Management and Relay Selection

We have so far solved two individual problems: i) power allocation with given fixed relay selection, and ii) relay selection with given spectrum profile. We can solve the overall problem formulated in Section 3 by solving the two individual problems iteratively.

The vectors of power allocation $\boldsymbol{P}$ and $\boldsymbol{Q}$ are initialized based on any feasible power allocation, e.g., equal power allocation over all channels for each source and relay node. Then, based on $\boldsymbol{P}$ and $\boldsymbol{Q}$, a VI solution of relay selection $\boldsymbol{\alpha}$ can be obtained through Algorithm 2. Note that in Section 4.3, we assume that each $\alpha_r^s,\ r \in \mathcal{R},\ s \in \mathcal{S}$ is real. Hence, the VI solution $\boldsymbol{\alpha}$ might not be feasible for the original problem formulated in Section 3, where each α_r^s takes only integer values of 0 or 1. To get a feasible $\boldsymbol{\alpha}$, we perform a *round*$(\cdot)$ operation to each α_r^s as follows

$$\widehat{\alpha}_r^s = \ round(\alpha_r^s),\ r \in \mathcal{R},\ s \in \mathcal{S}. \tag{26}$$

If rounding gives an unfeasible solution, e.g., $\widehat{\alpha}_r^s = \widehat{\alpha}_r^w = 1,\ s, w \in \mathcal{S},\ s \neq w$, the relay is assigned to session s if $\alpha_r^s \geq \alpha_r^w$, and assigned to session w otherwise. Denote the resulting vector of feasible relay selection as $\widehat{\boldsymbol{\alpha}}$. Then, with given $\widehat{\boldsymbol{\alpha}}$, we solve the problem of power allocation using Algorithm 1. The above iteration continues until the objective function in (7), i.e., sum utility of all communication sessions, does not change any more or the maximum number of iterations is reached. The overall iterative algorithm is summarized in Algorithm 3.

4.5 Implementation Issues and Future Work

The proposed algorithm can be directly applied to a scenario with multiple co-existing pre-established source-destination pairs. In addition, it can be used to optimally control resource allocation for an independent set of transmissions with primary interference constraints (i.e., no transmitters and receivers in common) periodically scheduled by a separate scheduling algorithm, where idle nodes can be used as potential relays.

Algorithm 3: Joint Spectrum Management and Relay Selection

Step 1: Initialize $\boldsymbol{P}$ and $\boldsymbol{Q}$ based on equal power allocation.
Step 2: Given $\boldsymbol{P}$ and $\boldsymbol{Q}$, calculate QVI solution $\boldsymbol{\alpha}$ using Algorithm 2.
Step 3: Calculate feasible relay selection vector $\widehat{\boldsymbol{\alpha}}$ using (26).
Step 4: Calculate VI solution $\boldsymbol{x} = (\boldsymbol{P},\ \boldsymbol{Q})$ using Algorithm 1.
Step 5: If utility does not change any more for all communication sessions or, maximum number of iterations is reached, stop. Otherwise, go to Step 2.

There are several potential alternative implementations for the proposed algorithm. To allow each source and potential relay to select the best subchannels, the power level of noise plus interference on each subchannel need to be measured at the destination and relay and then fed back to the source. Alternatively, they can be estimated at the source based on control information overheard on a control channel. This can be carried out through a cooperative MAC protocol. A good example of such protocols is the CoCogMAC proposed in [16], which uses a three-way handshake to exchange Request-to-Send (RTS), Clear-to- Send (CTS) and Relay-Ready-to-Relay (RTR) frames among the source, destination and the selected relay to inform the source (and neighboring devices) of transmitted power chosen on each channel. For relay selection, each destination node needs to estimate the quality of all subchannels from itself to the corresponding source, while each relay needs to measure the channel quality from itself to each source and to each destination. This can be also carried out through a protocol similar to CoCogMAC. Additionally, to implement the pricing strategy in relay selection, each potential relay needs to periodically broadcast a "price" frame to claim its price on a common control channel that can be implemented out of band, in a time-sharing or in a code-division fashion.

It is worth pointing out that in the proposed distributed algorithm it is assumed that the channel condition and the topology vary slowly with respect to the convergence time scale of the algorithm. Therefore, the proposed algorithm can be guaranteed to converge before the obtained CSI information becomes less exact or outdated. Algorithms that can track fast variations of the wireless channel in real time, as well as robustness of the algorithm to incomplete or outdated information, will be addressed in our future work.

5. PERFORMANCE EVALUATION

Simulation Scenario. We consider a communication area of size 1500×1500 m^2. Source, destination and relay nodes are randomly placed in the area. The average channel gain between two nodes, say $m,\ n$, is determined by the distance between m and n, i.e.,

$$G_{m,n} = |d(m,n)|^{-4}, \tag{27}$$

where 4 represents the path loss factor, and $d(m,n)$ represents the distance between m and n. The maximum transmission power for each source and relay node is set to 0.5 W. The average noise power is set to 10^{-10} W. The bandwidth

Table 1: Simulation Parameters.

Communication Area (m^2)	**AWGN Power** (W)	B (KHz)	ϵ (%)
1500×1500	10^{-10}	64	95
Network Topology Index	**Number of Sessions**: S	**Number of Relays**: R	**Number of Channels**: F
1	2	10	4
2	3	5	5
3	5	5	5
4	10	5	5
5	10	5	2

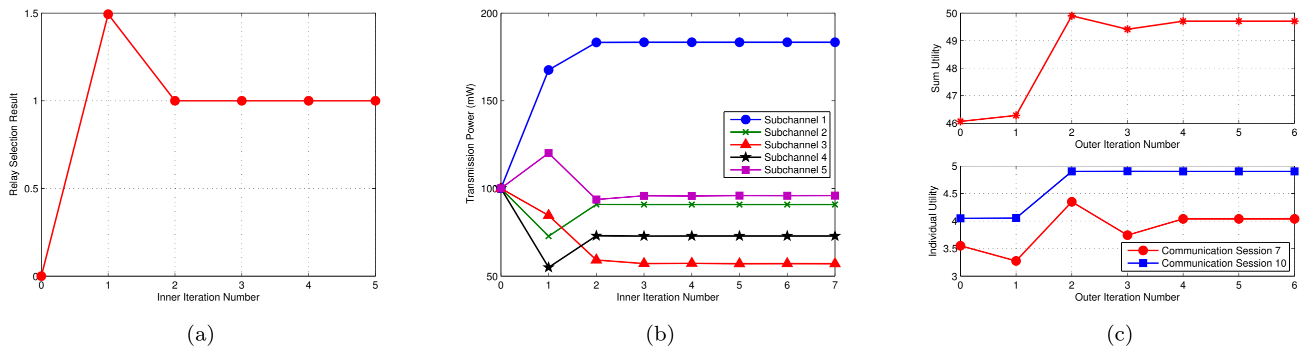


Figure 1: Convergence performance of the proposed distributed solution algorithm. Network topology index is 4. (a): Example results of relay selection. Values greater than 1 means that the relay is selected by multiple communication sessions. (b): Example results of power allocation over channels. (c): Sum utility (top) and examples of individual utility (bottom).

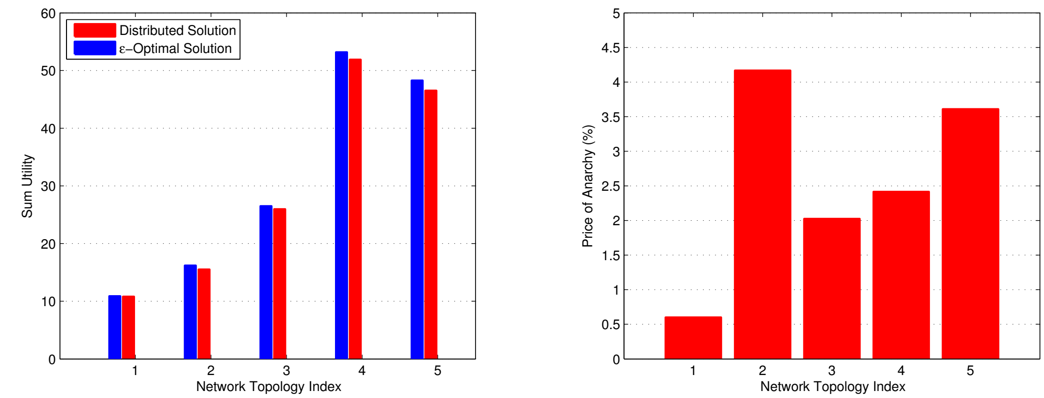
of each channel is set to 64 kHz. To approximate the cooperative link capacity to a continuously differentiable function, the approximation parameters P in (19) is set to $P = 5$. Different network topology parameters are employed in our experiments to model sufficient, moderate and scarce radio resources (relay nodes and spectrum) compared to the number of communication sessions. These parameters are summarized in Table 1, where S, R, F represent the number of total communication sessions, relay nodes, and available channels, respectively.

To evaluate the performance of the proposed distributed algorithms, we have developed a centralized algorithm based on the *branch-and-bound* framework and a combination of *reformulation linearization technique (RLT)* and convexification of non-convex problems. The centralized algorithm can be used to obtain a globally optimal solution with ϵ-optimality guarantee, with ϵ being the optimality precision. In simulations, ϵ is set to 95% representing that the achieved sum utility is equal or greater than 95% of the optimum.

To show convergence of the proposed distributed algorithms, results are obtained using one instance of each network topology and channel realization. For performance comparison among different algorithms, results are obtained by averaging over 30 simulations with different network topologies and channel realizations.

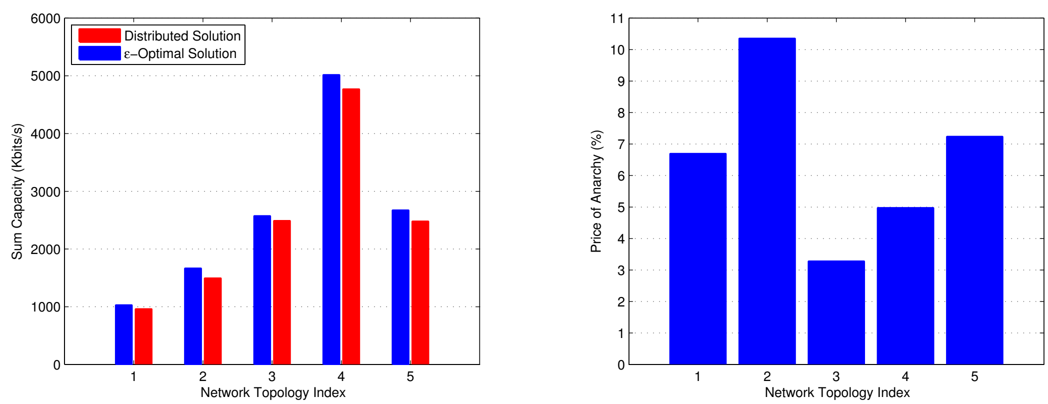
Convergence of Distributed Algorithm. The convergence performance of the proposed distributed solution algorithm is illustrated in Fig. 1. Network topology 4 is used in the simulation. In such a network topology, the number of relay nodes and also channels is less than, but comparable to, the number of concurrent communication sessions. Convergence of the distributed relay selection (DRS) algorithm at a relay node is shown in Fig. 1(a) with uniform power allocation over different channels. All relay selection variables are initialized to zero, i.e., no transmitter selects the relay node for cooperative relaying. After one iteration, at least two transmitters select the relay node. Notice that a transmitter might choose to use a relay node for only part time of a transmission, but use other relay nodes or direct link for the rest part. Consequently, the relay selection constraint (10) becomes violated for this relay node and he increases his price. As a result, the relay selection converges with all constraints in (10) satisfied. We can see that, the proposed penalization-based algorithm converges very fast. Distributed spectrum management (DSM) by power allocation is shown in Fig. 1(b). In power allocation, results of the relay selection obtained in Fig. 1(a) is employed. Here, power allocation for a source node over multiple channels is shown as an example. We can see that the proposed best-response algorithm converges within three iterations. Results of joint DRS and DSM are given in Fig. 1(c). The top figure shows the sum utility of all communication sessions, while the bottom figure shows individual utilities for two communication sessions. Individual utilities become stable after four iterations of DRS or DSM. To summarize, the proposed algorithms of DRS, DSM, and joint DRS and DSM, have a good convergence performance.

Performance of Distributed Algorithm and Price of Anarchy. Performance of the distributed solution algorithm is evaluated by comparing to the ϵ-optimal solution



(a) (b)

Figure 2: (a) Average performance for the distributed solution algorithm compared to the centralized algorithm in terms of sum utility. (b) Price of anarchy.



(a) (b)

Figure 3: (a) Average performance for the distributed solution algorithm compared to the centralized algorithm in terms of sum capacity. (b) Price of anarchy.

achieved by the centralized algorithm. Five network topologies are used, and results are obtained by averaging over 30 independent simulations for each network topology. The results of this comparison is shown in Fig. 3(a) in terms of sum utility, while in Fig. 3(b) the corresponding price of anarchy is shown. In all cases, the distributed algorithm can achieve sum utility and sum capacity very close to the ϵ-optimal solution. The distributed algorithm can achieve 95.8%-99.4% of the ϵ-optimal solution in terms of sum utility, with an average of 97.4%. The maximum price of anarchy is just less than 4.5%. The two algorithms are also compared in terms of sum capacity as shown in Fig. 3(a) and Fig. 3(b) for the corresponding price of anarchy. The distributed algorithm also has good performance in terms of sum capacity. The distributed algorithm can achieve 89.7%-96.7% of the ϵ-optimal solution, with an average of 93.5%. The corresponding price of anarchy varies from 4% to 10%.

6. CONCLUSIONS

In this paper, we have studied distributed spectrum management and relay selection in cognitive and cooperative wireless networks. We first formulated the problem of joint spectrum management and relay selection, and then, decomposed it into two individual problems: i) spectrum management by power allocation with given fixed relay selection, and ii) relay selection with given fixed spectrum profile. A distributed solution algorithm is proposed for each subproblem and analyzed based on the variational inequality (VI) theory. We prove that the proposed algorithms converge to a VI solution, which is also a NE solution. Performance of the distributed algorithm is evaluated by comparing to the centralized solution. Simulation results indicate that the distributed algorithm has performance that is very close to the optimal solution. Convergence of the distributed algorithm is also verified using simulation results. The distributed algorithm can be used to schedule an independent set of transmissions, each of which is scheduled by a separate algorithm.

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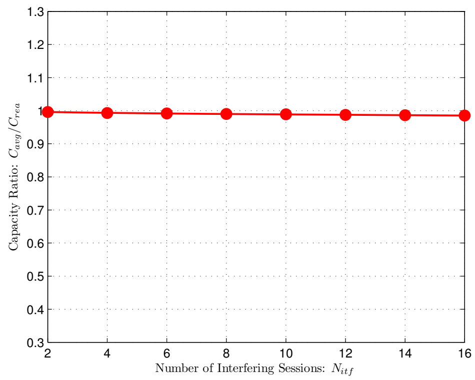

Figure 4: Comparison between the average-based interference model and exact interference in synchronization-based cooperative network.

APPENDIX

A. VALIDATION OF THE AVERAGE-BASED INTERFERENCE MODEL

We validate the average interference model by comparing it to the exact interference in practical cooperative wireless networks. Let us consider a cooperative network with global time-synchronization, in which all source nodes transmit in the first time-slot while all relay nodes transmit in the second. Then, for a wireless link l that uses direct transmission only, the interference measured at its destination node comes from all source nodes of a number N_{itf} of cooperative sessions in the first time-slot, while it comes from all relay nodes in the second. Then, the average capacity of the wireless link C_{rea} can be calculated as

$$C_{rea} = \frac{1}{2}(C_{slot1} + C_{slot2}), \tag{28}$$

where C_{slot1} and C_{slot2} represent the capacity in the first and second time-slot, respectively.

Use C_{avg} to represent capacity of the wireless link l calculated using the average-based interference model and then we compare it to C_{rea}. A communication area of 1000×1000 m^2 is considered and the number of interfering cooperative sessions varies from 2 to 16 with a step of 2. Other simulation parameters are set as in Section 5. Results of the comparison in terms of $\frac{C_{avg}}{C_{rea}}$ are shown in Fig. 4. Every point was plotted by averaging over 10^3 simulations. The value of C_{avg} is slightly lower but very close to that of C_{rea}, e.g., more than 99% and 98% of C_{rea} can be achieved when $N_{itf} = 2$ and $N_{itf} = 16$, respectively. We observe that the value of $\frac{C_{avg}}{C_{rea}}$ decreases very slightly as the number of interfering sessions increases, implying that the accumulation of performance degradation caused by the average-based interference model is negligible. Similar results can be also observed when wireless link l also uses cooperative relaying. Based on the above discussion, we can conclude that *the average-based interference model performs very well in tracking the cumulative effect of interference from different sources in practical cooperative wireless networks.*

B. PROOF FOR LEMMA OF STRONG CONCAVITY

PROOF. For our case, the approximation function $\widehat{C}_{cop}^{s,r,f}$ in (19) is a monotonically increasing function of the transmission power. The domain set $\mathcal{X}$ in the VI problem VI$(\mathcal{X}, \boldsymbol{F})$ is bounded. Therefore, the derivative of $\widehat{C}_{cop}^{s,r,f}$ with respect to the transmission power for source or relay node on each channel is positive and cannot be arbitrarily small. Hence, to prove that $\widehat{C}_{cop}^{s,r,f}$ is strong concave, we only need to show that it is a concave function.

For simplicity, we use $f(x) = \log(1+ax)$, $g(x,y) = \log(1+bx+cy)$, and $h(x,y) = \min(f(x), g(x,y))$, $(x,y) \in \Lambda$, to represent $C_{s2r}^{s,r,f}$, $C_{sr2d}^{s,r,f}$ and $C_{cop}^{s,r,f}$ in (19), respectively, with Λ being convex and closed domain set. Then, the approximation function $\hat{h}(x,y)$ for $h(x,y)$ can be expressed as

$$\hat{h}(x,y) = \left\{ \left[\left(\frac{1}{f(x)}\right)^P + \left(\frac{1}{g(x,y)}\right)^P \right]^{\frac{1}{P}} \right\}^{-1} \tag{29}$$

Take any point (x_0, y_0) in Λ, and also a direction $(\Delta x, \Delta y)$. Then, $\hat{h}(x,y)$ can be rewritten as a function of a scaler t, as follows

$$\hat{h}(t) = \left\{ \left[\left(\frac{1}{f(x_0 + t\Delta x)}\right)^P + \left(\frac{1}{g(x_0 + t\Delta x, y_0 + t\Delta y)}\right)^P \right]^{\frac{1}{P}} \right\}^{-1}. \tag{30}$$

Since function is concave if it is concave when restricted to any line in the domain [27], we only need to show that $\hat{h}(t)$ is a concave function of t.

The first derivative of $\hat{h}(t)$ can be calculated as

$$\hat{h}' = \frac{\frac{f'}{f(x_0+t\Delta x)^{P+1}} + \frac{g'}{g(x+t\Delta x, y+t\Delta y)^{P+1}}}{\left[\left(\frac{1}{f(x_0+t\Delta x)}\right)^P + \left(\frac{1}{g(x+t\Delta x, y+t\Delta y)}\right)^P\right]^{\frac{1}{P}+1}}. \tag{31}$$

Denote numerator and denominator of the right hand-side of (31) as u and v, respectively. Then, the second of derivative of $\hat{h}(t)$ can be calculated as

$$\hat{h}'' = \frac{u'v - uv'}{v^2}, \tag{32}$$

where $u'v - uv'$ can be further expressed in the form of $A \cdot B$ with A bing a positive item, and B can be expressed as follows

$$\begin{aligned} B \;=\; & \left(-\frac{(f')^2}{f^{P+1}g^P} - \frac{(f')^2}{f^{2P+1}g^P} - \frac{(g')^2}{g^{P+1}f^P} - \frac{(g')^2}{g^{2P+1}f^P} \right) \\ + \; & \left(-\frac{(f')^2}{f^{2P+1}} - \frac{(g')^2}{g^{2P+1}} + 2\frac{f'}{f^{P+1}}\frac{g'}{g^{P+1}} \right). \end{aligned} \tag{33}$$

The condition that SINR is greater than $e-1$ in Lemma 2 implies that $f > 1$ and $g > 1$ hold. Then, we have

$$B < -\left(\frac{f'}{f^{P+1}} - \frac{g'}{g^{P+1}} \right)^2 \leq 0. \tag{34}$$

Based on (31), (32) and (34), we can conclude that $\hat{h}' > 0$ and $\hat{h}'' < 0$, and hence, the approximation function $\hat{h}$ is concave. □