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# Deception in Simple Voting Games

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The calculations of sophisticated voters who successively eliminate undesirable strategies are analyzed in three-person voting games in which one voter with complete information can, as a deceiver, induce the other two voters with incomplete information to vote in such a way as to ensure a better outcome than the deceiver could ensure in a game of complete information. Deception which is "tacit," wherein a deceiver votes consistently with his announced preference scale, is distinguished from deception which is "revealed," wherein a deceiver's action deviates from his announced preference scale. Among the conclusions drawn from the study is that revealed deception is generally a more potent tool than tacit deception in securing a more-preferred outcome, and deception opportunities are greater the more disagreement there is among the nondeceivers.

Voting not directly in accordance with one's preferences is usually referred to as *strategic* or *insincere* voting. The ostensible purpose of strategic voting is to bring about an outcome that one could not obtain by voting sincerely.

For example, you may agree to vote insincerely on an issue another voter cares very much about, but you do not, if he will in turn support you, by also voting insincerely, on an issue you care very much about but he does not. If you are both pivotal voters on the separate issues, such a vote trade will bring about the desired outcome on the issue about which each of you cares most.

This form of strategic voting, which is commonly referred to as logrolling, requires some degree of cooperation among two or more voters. Strategic voting, however, need not be based on cooperative agreements being reached or even require communication among voters. In fact, the concept of sophisticated voting, as developed by Farquharson (1969), Kramer (1972), Niemi *et al.* (1974), and others, is a noncooperative form of strategic voting that is motivated only by an awareness by all voters of each others' preference scales. Given this mutual awareness in a game of

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complete information, sophisticated voting means roughly that voters try to anticipate each other's voting strategies and, based on this anticipation, successively eliminate strategies that could lead to the adoption of their least-preferred outcomes.

One of the purposes of this paper is to show that such elimination attempts may fail, even for relatively "powerful" voters in a voting body. Indeed, as the example in the next section illustrates, a sophisticated voter may succeed only in ensuring his very worst outcome, which would seem to rob sophisticated voting of its claim to being "strategic," that is, of value in insuring against worst outcomes. Is there any way to rescue this rationale of sophisticated voting, or at least its calculations, for the hapless voter done in by the outcomes that sophisticated voting produces?

In this paper we shall demonstrate that it is possible to retain the calculations of sophisticated voting but relax the assumption that all voters have complete information about each other's preference scales. By so changing the information structure of a game, but not the sophisticated calculations of its players, we shall show that a voter who is hurt by everybody's knowledge of his (true) preference scale when information is complete may, through deception, have a recourse in a game of incomplete information: By announcing a preference scale different from his (true) preference scale, he may be able to induce an outcome better for himself than the sophisticated outcome that is induced when information is complete.

Specifically, if the preference scale of a voter is not known by the other voters, but their preference scales are known by him (and each other), the voter with complete information, call him the *deceiver*, may be able to misrepresent his preference scale in such a way that the sophisticated calculations his misrepresentation induces work to his advantage. As we shall show, a deceiver may realize this advantage either through "tacit" or "revealed" deception (to be defined later).

For a simple three-number voting body considering three alternatives under a plurality voting procedure, we shall offer a quantitative assessment of the advantages of tacit and revealed deception for three different power configurations, and all possible preference configurations, of the three voters. Among the conclusions that we draw from our analysis of games defined by the different power/preference configurations, and vulnerable to misrepresentation by either one or two deceivers, is that revealed deception is generally a more potent tool than tacit deception in securing a more-preferred outcome, and deception opportunities are greater the more disagreement there is among the nondeceivers.

## THE LIMITS OF SOPHISTICATED VOTING: AN EXAMPLE

Consider a voting body consisting of three *voters* $V = \{ v_1, v_2, v_3 \}$ who must choose among three alternatives $A = \{ a_1, a_2, a_3 \}$. Assume that the preference scales of the three voters, or their ordinal rankings of the alternatives, are as follows:

$$v_1: (a_1, a_2, a_3),$$
$$v_2: (a_2, a_3, a_1),$$
$$v_3: (a_3, a_1, a_2).$$

Thus, in this representation $v_1$ most prefers $a_1$, next most prefers $a_2$, and least prefers $a_3$.

Assume that the choice of an alternative is made according to a *plurality procedure*, whereby the alternative with the most votes wins. If each voter casts one vote, and votes for his most-preferred alternatives (i.e., sincerely), then the outcome will be a three-way tie among the three alternatives.

To prevent such an indeterminate situation from occurring, assume that one voter (say, $v_1$) can, as chairman, cast a tie-breaking vote in the event of a three-way tie. Then if each voter votes sincerely, the tie-breaking vote of $v_1$ will be decisive and $a_1$ will be the alternative that wins. Sincere voting under the plurality procedure, therefore, results in the selection of the chairman's most-preferred alternative, $a_1$.

What will the outcome be in this example if voting is sophisticated? Without offering a formal definition of sophisticated voting here, we can easily illustrate its calculations in our example.

First, we ask whether any voting has a *straightforward strategy*, i.e., a voting strategy that is unconditionally best whatever voting strategies the other voters adopt. The answer is that the chairman, $v_1$, has such a strategy: "Vote for $a_1$." If the other two voters both vote for $a_2$ or both vote for $a_3$, the chairman cannot prevent the adoption of the alternative they both choose by voting for an alternative different from $a_1$. On the other hand, if the other two voters vote for different alternatives, and $v_1$ votes for $a_1$, $a_1$ will always win, which is the chairman's first choice. Since the chairman can do no better, and may do worse, if he votes for an alternative other than $a_1$, his strategy "vote for $a_1$" is straightforward (or *dominant*, in the language of game theory).

Following a simplification in the Farquharson (1969) reduction method suggested by Brams (1975, pp. 67–78), we assume a voter immediately adopts a straightforward strategy if he has one. Given the straightforward strategy choice "vote for $a_1$" by $v_1$, we then ask what, if any, strategies $v_2$ and $v_3$ will definitely *not* adopt because they are dominated by some other strategy.

As can be seen from the $3 \times 3$ outcome matrix shown in Fig. 1, the choice by $v_2$ of "vote for $a_1$" always leads to the adoption of his worst outcome, $a_1$, whereas his other two strategies do not, so he can eliminate "vote for $a_1$" from further consideration. Similarly, $v_3$ can eliminate "vote for $a_1$" and "vote for $a_2$" from further consideration, because both of these strategies are dominated by "vote for $a_3$," which leads to out-

| | | $v_3$ Vote for $a_1$ | $v_3$ Vote for $a_2$ | $v_3$ Vote for $a_3$ | |
|---|---|---|---|---|---|
| $v_2$ | Vote for $a_1$ | $a_1$ | $a_1$ | $a_1$ | Dominated |
| | Vote for $a_2$ | $a_1$ | $a_2$ | $a_1$ | |
| | Vote for $a_3$ | $a_1$ | $a_1$ | $a_3$ | |
| | | Dominated | Dominated | | |

FIG. 1. Outcome matrix for $v_2$ and $v_3$, given that $v_1$ adopts his straightforward strategy "vote for $a_1$."

comes as good as, and in at least one contingency better than, his other two strategies.

So far we have shown that, given that $v_1$ chooses his straightforward (dominant) strategy, "vote for $a_1$," then $v_3$ also has a dominant strategy, "vote for $a_3$." Assuming that $v_1$ and $v_3$ choose these strategies as their "most desirable," then $v_2$ can further eliminate "vote for $a_2$" as "undesirable" since it results in the choice of $a_1$, which is inferior to the alternative he can ensure ($a_3$) if he chooses his strategy, "vote for $a_3$." In this manner, sophisticated voters can successively narrow down their strategy choices to a "most-desirable" set, which in our example results in the choice of $a_3$.

The fact that the sophisticated outcome, $a_3$, is the chairman's worst choice seems, to say the least, bizarre. The chairman with the tie-breaking vote, who would appear to be the most powerful member of the voting body, suffers the most when voting is sophisticated.[1] In effect, the perception that the chairman is the most powerful member does him in when voters are sophisticated, i.e., induces them to "gang up" on him.

One should note that this is not because the alternative that the two regular members support ($a_3$) is in any sense "socially preferred." In fact, the configuration of preferences of the three voters is such as to create a paradox of voting and make no alternative socially preferred, i.e., majorities are cyclical.

## TACIT AND REVEALED DECEPTION

What, if any, recourse does the chairman ($v_1$), as the most disadvantaged voter, have once sincerity (innocence?) in voting is lost? A chairman is often in the unique position, after the other voters have already

[1] Elsewhere this and related phenomena associated with sophisticated voting under the plurality procedure have been called the *paradox of the chairman's position (vote)* (Brams, 1976, Chap. 6; Farquharson, 1969, p. 50).

committed themselves, of being the last voter to have to make a strategy choice. Yet this position does not furnish a ready solution to his problem if voting is truly sophisticated, for sophisticated voting implies that voters act upon both their own preferences and a knowledge of the preferences of the other voters. If complete information about all voters' preference scales is available before voting, as it must be for voting to be sophisticated, then the order of voting is immaterial: All voters can predict sophisticated choices beforehand and act accordingly.

Let us assume for purposes of the subsequent analysis that the chairman, by virtue of his unique position, can obtain information about the preference scales of the other two voters but they cannot obtain information about his preference scale. Assume further that each of the two regular members is informed of the other regular member's preference scale. Now, if voting is to be truly sophisticated, the chairman must inform the regular members of his preference scale; but, given these information assumptions, he is not compelled to tell the truth. The question is: Can a chairman, by announcing a preference scale different from his true preference scale, induce a better (manipulated) sophisticated outcome for himself?

Given that voting is sophisticated, but (a priori) information on preferences is no longer complete, the chairman, because of his tie-breaking vote, will still have a straightforward strategy: vote for his most-preferred outcome. Thus, the other voters need only know his (announced) first choice, and not his complete preference scale, to determine what his sophisticated (and sincere) strategy choice will be.

Define a *deceptive strategy* on the part of the chairman to be any announced most-preferred outcome that differs from his *honest* most-preferred outcome. We call the use of a deceptive strategy by the chairman *tacit deception*, since the other members, not knowing his honest preference scale, would not be able to determine whether or not his announcement was an honest representation of his most-preferred outcome. Tacit deception will be profitable for the chairman if it induces a more-preferred social choice for him than an honest representation of his preferences, given sophisticated voting by the other voters.

To illustrate this concept, assume that the chairman announces $a_2$, rather than $a_1$, to be his most-preferred alternative. Then the other two voters, $v_2$ and $v_3$, taking "vote for $a_2$" to be the chairman's ($v_1$'s) straightforward strategy, will now view the outcome matrix to be that shown in Fig. 2. Since "vote for $a_2$" always leads to $v_2$'s most-preferred alternative ($a_2$), and neither of his other (dominated) strategies can ensure the choice of this alternative, this strategy is a dominant choice for $v_2$.

On the other hand, $v_3$ has only one dominated strategy ("vote for $a_2$"). Since his other two (undominated) strategies lead to the same outcome ($a_2$) when $v_2$ adopts his dominant strategy, $v_3$ cannot further reduce his

| | | $v_3$: Vote for $a_1$ | $v_3$: Vote for $a_2$ | $v_3$: Vote for $a_3$ | |
|---|---|---|---|---|---|
| $v_2$ | Vote for $a_1$ | ~~$a_1$~~ | ~~$a_2$~~ | ~~$a_2$~~ | Dominated |
| | Vote for $a_2$ | $a_2$ | ~~$a_2$~~ | $a_2$ | |
| | Vote for $a_3$ | ~~$a_2$~~ | ~~$a_2$~~ | ~~$a_3$~~ | Dominated |
| | | | Dominated | | |

FIG. 2. Outcome matrix for $v_2$ and $v_3$, given that $v_1$ announces his first choice to be $a_2$.

strategies to a single "most-desirable" choice. Consequently, the outcome $a_2$ associated with $v_3$'s two undominated strategies is the (manipulated) sophisticated outcome induced by $v_1$'s misrepresentation of $a_2$ as his most-preferred alternative.

Thus, as chairman, $v_1$ can prevent the adoption of his worst alternative ($a_3$) by announcing his most-preferred alternative to be $a_2$. Should he announce his most-preferred alternative to be $a_3$, one can show in a series of reductions similar to that which we have just described that this misrepresentation would lead to the (manipulated) sophisticated outcome $a_3$, which clearly would not be profitable for $v_1$, that is, not change the (unmanipulated) sophisticated outcome in his favor.

Can the chairman do better than ensure his next most-preferred alternative $a_2$ by misrepresenting his preferences, given that he has complete information but the other voters do not? If he is to vote consistently with his announced first choice, and hence not reveal that his announcement is a misrepresentation of his (true) preference scale, the answer is "no."

The situation is quite different, however, if the chairman feels no compunction to hide his misrepresentation by voting consistently with his announcement when the vote is actually taken. Deception by the chairman is possible at this stage because the two outcomes associated with undominated strategies of $v_2$ and $v_3$ in Fig.2 (both $a_2$) are not in equilibrium *with respect to $v_1$'s (true) preference scale*. That is, unbeknownst to $v_2$ and $v_3$, not only would $v_1$ prefer $a_1$ to $a_2$, but he can also bring about this outcome (his most preferred) by voting for $a_1$ instead of $a_2$, whichever undominated strategy $v_3$ selects ($v_2$ has a single dominant strategy).

Thus, $v_1$ can induce his best alternative by faking his (true) preference scale—announcing his first choice to be $a_2$—and then, contrary to his announcement, voting honestly, for $a_1$, in the end. We call this kind of deception, which involves not only announcing a deceptive strategy but taking *deceptive action* as well (i.e., voting differently than is implied by one's announced preference scale), *revealed deception*. It is deception that is revealed in the voting process and is clearly profitable for the chairman in the situation we have described.

To be sure, if the game that the voters play does not end with voting on the three alternatives, but is likely to have repercussions on voting on other issues that are anticipated to come before the voting body as well, then the cost of revealing oneself may, over the long run, make revealed deception unprofitable. For since revealed deception becomes apparent after the vote, unless it is secret, it probably cannot be used very frequently without undermining the credibility of one's future announcements. Thus, it may be better for the chairman in our example to accept the second choice he can ensure through tacit deception rather than sacrifice the credibility, and hence inducement value, of his future announcements to obtain his first choice on the immediate issue.

We shall have more to say about this kind of short-run versus long-run trade-off later. First, however, to gain a more precise notion of how frequently, and under what circumstances, opportunities for tacit and revealed deception occur, we shall systematically analyze deception possibilities in some simple three-person voting games. This analysis is based on assumptions described in the next section.

## ASSUMPTIONS OF THE ANALYSIS

In the subsequent analysis, we make the following assumptions:

1. There are three voters $V = \{v_1,v_2,v_3\}$ voting on three alternatives $A = \{a_1,a_2,a_3\}$ under the plurality procedure.
2. One voter (the deceiver) has complete information about the preference scales of the other two voters, but the other two voters, who have complete information about each other's preference scales, have no (a priori) information about the preference scale of the deceiver.
3. The deceiver's announcement of his preference scale is believed by the nondeceivers, i.e., they accept his announcement as a (true) representation of his preferences, and the deceiver knows this.
4. On the basis of the (a priori or announced) information available to each voter, voting by all voters is sophisticated (with some modifications, to be indicated shortly).
5. The goal of the deceiver is to announce a preference scale (true or misrepresented) that induces his best possible outcome.

We begin our analysis by identifying, but not listing here, preference configurations of the three voters in which the deceiver can improve upon the sophisticated outcome (given complete information) by misrepresenting his preferences in a game of incomplete information. We carry out this analysis for three distinct "power" configurations:

1. All voters are equal, i.e., each casts one vote.
2. One voter has an extra tie-breaking vote (as was the case for the chairman in our previous example).

3. One voter has a veto, i.e., he can block the choice of either or both of the other two voters.

In the case of the first and third power configurations, we assume that in the event that there is a three-way tie, or a veto is cast, the outcome reverts to the status quo, which is assumed to be one of the three alternatives before the voting body. This assumption precludes an outcome in the outcome matrix from being indeterminate, though it does not preclude indeterminacy in the reduction process, whereby more than one sophisticated outcome may be associated with the ultimately irreducible set of "most-desirable" (undominated) strategies of the three voters.

Because there is a good deal of indeterminacy associated with sophisticated voting under the plurality procedure, especially when one voter has a veto, we have made one emendation in Farquharson's (1969) reducttion method, besides the modification mentioned earlier that assumes voters immediately adopt straightforward strategies if they have them, that decreases somewhat the number of indeterminate outcomes. This emendation seems reasonable in light of the following example, which provides a further illustration of the calculations of sophisticated voting.

Suppose, in the case of the chairman with a tie-breaking vote, that the preference scales of the three voters are as follows, where $v_1$ is the chairman:

$$v_1:\ (a_1,a_2,a_3),$$
$$v_2:\ (a_3,a_2,a_1),$$
$$v_3:\ (a_3,a_2,a_1).$$

Note that the preference scales of $v_2$ and $v_3$ are identical and the reverse of the chairman's scale.

Given that the chairman adopts his straightforward strategy "vote for $a_1$," then the dominated strategies of $v_2$ and $v_3$ will be as shown in Fig. 3. If each voter eliminated his dominated strategy "vote for $a_1$," however,

| | | $v_3$: Vote for $a_1$ | Vote for $a_2$ | Vote for $a_3$ | |
|---|---|---|---|---|---|
| | Vote for $a_1$ | ~~$a_1$~~ | ~~$a_1$~~ | ~~$a_1$~~ | Dominated |
| $v_2$ | Vote for $a_2$ | ~~$a_1$~~ | $a_2$ | $a_1$ | |
| | Vote for $a_3$ | ~~$a_1$~~ | $a_1$ | $a_3$ | |
| | | Dominated | | | |

FIG. 3. Outcome matrix for $v_2$ and $v_3$, given that $v_1$ adopts his straightforward strategy "vote for $a_1$."

the $2 \times 2$ reduced outcome matrix cannot be further reduced since the two remaining strategies of each voter are undominated.

Sophisticated voting in this case, therefore, admits all three possible alternatives, $a_1$, $a_2$, and $a_3$, depending on the sophisticated strategy choices of $v_2$ and $v_3$. This kind of indeterminacy seems implausible, though, since $v_2$ and $v_3$, by both choosing their strategy "vote for $a_3$," can ensure the choice of $a_3$, their best outcome. Since this outcome is the "prominent" outcome in the reduced matrix (see Schelling, 1960), it seems reasonable to assume that it will be chosen, even in the absence of communication among the voters. Accordingly, we have "adjusted" the sophisticated outcome in this case and similar cases to reflect the choice of $a_3$ rather than the nonchoice of an indeterminate outcome.

In the subsequent analysis, we assume that if there is a "prominent" sophisticated outcome in the reduced outcome matrix, with or without deception, it is always chosen. We may formalize this assumption as follows:

> *Prominence Assumption*. When the reduction process of sophisticated voting admits a set containing more than one alternative (i.e., there is no single sophisticated outcome), and one and only one of these alternatives is preferred by a decisive coalition of voters (i.e., is prominent), that outcome is *the* sophisticated outcome.

By a *decisive coalition*, we mean a coalition of voters whose choice of the prominent alternative cannot be blocked.

The Prominence Assumption by no means renders determinate outcomes in all voting situations that yield more than one sophisticated outcome. Consider, as an example, the following configuration of preference:

$$v_1\text{: } (a_1,a_2,a_3),$$
$$v_2\text{: } (a_2,a_3,a_1),$$
$$v_3\text{: } (a_3,a_2,a_1).$$

If we assume that $v_1$ again has the tie-breaking vote, the successive elimination of dominated strategies proceeds exactly as in our previous example, except for the fact that now there is no "prominent" outcome in the reduced outcome matrix (Fig. 3).

This is not to say that $v_2$ and $v_3$ would not prefer either $a_2$ or $a_3$ to $a_1$. Without communication, however, they have no way of coordinating their choices of strategies to avoid the selection of $a_1$. Even if they could communicate, however, since $v_2$ prefers $a_2$ to $a_3$ and $v_3$ prefers $a_3$ to $a_2$, communication alone would not necessarily resolve the evident conflict between the two voters over their first choices. Hence, one can offer no assurance that both voters will reach an agreement on which alternative to support in lieu of $a_1$. Because this and related voting situations afford no "prominent" alternative in the reduced outcome matrix, we consider the sophisticated outcome to be indeterminate.

There are some classes of games whose outcomes, while susceptible to deception by at least one voter, would appear to be quite stable. For example, if the status quo corresponds to the first choice of a voter with a veto, he is, in effect, a dictator: He can always obtain his best alternative by voting for the status quo (i.e., by exercising his veto). Not only does he have no reason to misrepresent his preferences, but the (perhaps deceptive) strategy announcements of the other voters cannot induce him to do otherwise since their strategy choices cannot block his best outcome.

The largest class of 2 × 2 ordinal games are "deception stable" in the above sense (Brams, 1977). Many three-person voting games are also deception stable, but our focus in the remainder of this paper will be on those games that are "*deception vulnerable*," that is, in which a voter can misrepresent his preference scale so as to induce a better outcome[2] by either tacit or revealed deception.

## DECEPTION-VULNERABLE GAMES: GENERAL RESULTS

Given the assumptions described in the previous section, the following results about tacit and revealed deception were derived from our computer analysis of deception-vulnerable games:

1. Games that are vulnerable to tacit deception are also vulnerable to revealed deception, but not vice versa.
2. A deceiver can ensure at least as good an outcome, and sometimes better, by revealing his deception rather than keeping it tacit.
3. In all except one game, a deceiver's optimal tacit-deception announcements are a subset of his optimal revealed-deception announcements.

One implication of result No. 3 is that a deceiver does not generally have to decide beforehand whether he will reveal himself when he misrepresents his preference scale to the other voters. He can reserve a decision until he must make a strategy choice.[3]

To draw comparisons among the different power configurations described previously, consider the 6 × 6 × 6 = 216 games defined by the six possible preference scales of the three voters. In what proportion of the

[2] Since an indeterminate outcome allows the possibility that a deceiver's worst outcome will be chosen, we consider it better than the (certain) choice of his worst outcome but worse than the (certain) choice of his next-best outcome.

[3] Details on which games are deception vulnerable, and optimal tacit and revealed deception announcements (i.e., those which induce the best possible outcome for a deceiver) are available from the authors on request. Also, information on deception-vulnerable games in which there are two deceivers and one nondeceiver is available on request.

TABLE 1
Number (Proportion) of Deception-Vulnerable Games

| Deception | Regular voter in power configuration in which: All voters have one vote | Chairman has tie-breaking vote | Voter has veto | Chairman has tie-breaking vote | Voter has veto |
|---|---|---|---|---|---|
| Tacit | 16 (.07) | 12 (.06) | 18 (.08) | 24 (.11) | 8 (.04) |
| Revealed | 20 (.09) | 12 (.06) | 24 (.11) | 36 (.17) | 8 (.04) |

total of 216 games will the outcome be vulnerable to tacit or revealed deception by each voter?

These figures are given in Table 1 and indicate that deception opportunities vary greatly, depending on the type of voter and the power configuration. Regular voters have the fewest opportunities as deceivers when there is a tie-breaking chariman (6% for both tacit and revealed) and the most when there is one voter with a veto (8% for tacit, 11% for revealed). A tie-breaking chairman has both more tacit-deception opportunities (11 versus 4%) and more revealed-deception opportunities (17 versus 4%) than a voter with a veto.

The rewards of tacit and revealed deception are summarized in Table 2, in which we have given the number (proportion) of sophisticated, tacit-deception, and revealed-deception outcomes, for each type of voter and power configuration, that rank first, second, indeterminate, and last on each voter's preference scale. Table 2 shows that *all* voters benefit from tacit deception in at least some games, with the "number of favorable changes over sophisticated voting"[4] varying from 4 (voter with veto) to 11% (chairman with tie-breaking vote).

On the other hand, revealed deception does not always offer to a deceiver any benefit over tacit deception (and over sophisticated voting when tacit deception does not improve the sophisticated outcome). In particular, in the power configuration in which one voter (chairman) has a tie-breaking vote, a regular voter cannot improve on the tacit-deception outcome by revealing his deception, and neither can the voter with the veto in the power configuration in which one voter has a veto. In all three power configurations, however, there is one voter for whom revealed deception does provide an advantage, which, in quantitative terms,

[4] This value refers to the number of games in which the tacit deceiver can improve his outcome by raising it at least one rank (e.g., from last choice to indeterminate); an analogous concept, "number of favorable changes over tacit deception," is used in Table 2 to characterize the number of games in which the deceiver who reveals himself can improve on the tacit-deception outcome.

TABLE 2
Rankings of Game Outcomes of Deceiver for Sophisticated Voting, Tacit Deception, and Revealed Deception

| Voting | Regular voter in power configuration in which | | | | |
|---|---|---|---|---|---|
| | All voters have one vote | Chairman has tie-breaking vote | Voter has veto | Chairman has tie-breaking vote | Voter has veto |
| Sophisticated | | | | | |
| First | 132 (.61) | 132 (.61) | 104 (.48) | 132 (.61) | 160 (.74) |
| Second | 44 (.20) | 48 (.22) | 56 (.26) | 36 (.17) | 40 (.19) |
| Indeterminate | 12 (.06) | 12 (.06) | 8 (.04) | 12 (.06) | 8 (.04) |
| Last | 28 (.13) | 24 (.11) | 48 (.22) | 36 (.17) | 8 (.04) |
| Tacit Deception | | | | | |
| First | 140 (.65) | 144 (.67) | 112 (.52) | 132 (.61) | 168 (.78) |
| Second | 52 (.24) | 48 (.22) | 64 (.30) | 60 (.28) | 40 (.19) |
| Indeterminate | 0 (.00) | 0 (.00) | 0 (.00) | 0 (.00) | 0 (.00) |
| Last | 24 (.11) | 24 (.11) | 40 (.19) | 24 (.11) | 8 (.04) |
| Number favorable changes over sophisticates voting | 16 (.07) | 12 (.06) | 18 (.08) | 24 (.11) | 8 (.04) |
| Revealed deception | | | | | |
| First | 152 (.70) | 144 (.67) | 128 (.59) | 168 (.78) | 168 (.78) |
| Second | 40 (.19) | 48 (.22) | 48 (.22) | 24 (.11) | 40 (.19) |
| Indeterminate | 0 (.00) | 0 (.00) | 0 (.00) | 0 (.00) | 0 (.00) |
| Last | 24 (.12) | 24 (.11) | 40 (.19) | 24 (.11) | 8 (.04) |
| Number favorable changes over tacit deception | 12 (.06) | 0 (.00) | 16 (.07) | 42 (.19) | 0 (.00) |

ranges from 6 to 19% favorable outcome changes over tacit deception (see Table 2).

In sum, all three power configurations include games which are vulnerable to tacit deception at a lower level, and revealed deception at a higher level, by at least one voter. That is, at least one voter in each configuration can achieve some benefit from tacit deception, but even more from revealed deception, which attests to the value of distinguishing these concepts: They lead to genuinely different levels of satisfaction with the outcome.

As might be expected, if we measure "level of satisfaction" in terms of avoidance of indeterminate and last choices, the voter with the veto does best; only 16 (7%) of his sophisticated outcomes fall into these least-desirable categories. But he, like all other voters, cannot entirely avoid these outcomes by tacit or revealed deception (see Table 2).

A chairman with a tie-breaking vote matches the number of first-choice outcomes of the voter with the veto (78%) if he reveals his deception, but he does less well if he votes sophisticatedly or tacitly deceives. Among regular voters, those in a power configuration in which one voter has a veto fare worst, while those in the other two power configurations have quite similar distributions of first through last-choice outcomes, with and without deception.

What preference configurations of the two nondeceivers are most vulnerable to deception by a single deceiver? In Table 3 we have distinguished, for each type of voter in the three different power configurations, the preference configurations of the two nondeceivers in which:

1. they agree on the ranking of all alternatives (36 games);
2. they agree only on their first choices (36 games);
3. they agree only on their second choices (36 games);
4. they agree only on their last choices (36 games);
5. they disagree on all choices (72 games).

TABLE 3
Number of Deception-Vulnerable Games for Different Levels of Agreement of Nondeceivers

| Agreement on | Regular voter in power configuration in which: All voters have one vote | Chairman has tie-breaking vote | Voter has veto | Chairman has tie-breaking vote | Voter has veto |
|---|---|---|---|---|---|
| All choices | | | | | |
| Tacit changes | 0 | 0 | 0 | 0 | 0 |
| Revealed changes | 0 | 0 | 0 | 0 | 0 |
| First choices | | | | | |
| Tacit changes | 0 | 0 | 0 | 0 | 0 |
| Revealed changes | 0 | 0 | 0 | 0 | 0 |
| Second choices | | | | | |
| Tacit changes | 4 | 6 | 2 | 0 | 4 |
| Revealed changes | 0 | 0 | 0 | 0 | 0 |
| Last choices | | | | | |
| Tacit changes | 4 | 0 | 10 | 12 | 0 |
| Revealed changes | 4 | 0 | 8 | 12 | 0 |
| No choices | | | | | |
| Tacit changes | 8 | 6 | 6 | 12 | 4 |
| Revealed changes | 8 | 0 | 8 | 24 | 0 |

Not surprisingly, when the two nondeceivers have identical preference scales or have the first choice, the deceiver cannot induce a better outcome through either tacit or revealed deception. Only when there is greater conflict among the two nondeceivers (agreement only on their second or last choices or on no choices) can the deceiver induce favorable outcome changes over sophisticated voting through either tacit or revealed deception. Curiously, no games in which the nondeceivers have the same second choices can be improved on through revealed deception, but, as with tacit deception, the games most vulnerable to revealed deception are those in which the nondeceivers can at best agree on what they do not prefer (i.e., they agree only on their last choices or no alternatives at all).

## SUMMARY AND CONCLUSION

We introduced the analysis in this paper with an example that vividly illustrated the unfortunate position a chairman with a tie-breaking vote might find himself in if voting were sophisticated in a game of complete information. This example also illustrated the advantage the chairman might derive if information in the game were incomplete and he could, as a single deceiver, misrepresent his preferences.

From this example, we then went on to analyze systematically three-person voting games of incomplete information in which one voter can, as a deceiver, induce for himself better outcomes than he could obtain if information were complete. We distinguished between games vulnerable to tacit deception, wherein a deceiver votes consistently with his announced preference scale, and those vulnerable to revealed deception, wherein a deceiver's action deviates from his announced preference scale.

Among our findings were that (1) those games that are vulnerable to tacit deception are also vulnerable to revealed deception, but not vice versa; (2) a deceiver can ensure at least as good an outcome, and sometimes better, by revealing his deception; and (3) in all except one game, a deceiver's optimal tacit-deception announcements are a subset of his optimal revealed-deception announcements. Since the latter findings says that a deceiver does not generally have to decide whether he will reveal himself when he makes his deceptive announcement, he can reserve a decision about whether to risk sacrificing his future credibility to obtain a more-preferred immediate outcome until he must make a strategy choice. It would seem that the cost of revealing oneself in all except one-shot games would be high if it makes all one's future announcements unbelievable, or even creates the risk of a future "credibility gap."

Voters in the three different power configurations we have studied—all voters have one vote, one voter (chairman) has an extra tie-breaking vote, and one voter has a veto—render between 4 and 17% of all games (based on all possible preference configurations of the three voters) vulnerable to either tacit or revealed deception. Although tacit deception is a weapon by which every voter can improve upon the sophisticated outcome (effective in between 4 and 11% of all games), revealed deception is not effective for all voters, in particular, regular voters when there is a tie-breaking chairman and a voter with a veto, who want to improve upon the tacit-deception outcome. It is an effective weapon, however, in the hands of at least one voter in every power configuration.

Thus, no power configuration we have studied is invulnerable to tacit deception at one level of satisfiction, and revealed deception at another level, for some preferences of the three voters. Indeed, we would speculate that there exist no nondictatorial voting rules or voter weights that can eliminate all incentives for deception, just as "strategy proofness" without deception is generally an impossible ideal if there are at least three alternatives (Gibbard, 1973; Satterthwaite, 1975; Pattanaik, 1976; Gärdenfors, 1976). In other words, it would appear always to be true, whatever the rules or weights, that some voter can benefit by misrepresenting his preferences and making the game being played appear to the nondeceivers to be one different than it is.[5] In this sense, deception involves mapping real into apparent games, but we have been unable to characterize the nature of this mapping in a simple way.

Our comparison of the different power configurations revealed that a chairman with a tie-breaking vote is the voter with the most tacit and revealed-deception opportunities. However, a voter with a veto can avoid more last-choice and indeterminate outcomes, and generally ensure more first-choice outcomes, with or without deception, than any other voter in the three power configurations. Our comparison of different preference configurations showed that there tend to be greater deception opportunities the more the nondeceivers disagree.

It is difficult to say to what extent our results can be extended to $n$-person situations generally. Since many interesting political situations involve very few significant actors, however, our models, as well as similar models for two-person games (Brams, 1977), would seem to provide useful guidelines for empirical research on political deception.

In this context, it is worth pointing out that our trichotomy of power configurations can often be identified in real-life nonvoting, as well as voting, situations.[6] Our second and third configurations, for example, represent situations, respectively, in which one actor's preferences prevail in the event of a deadlock and one actor can block the combined

[5] Recent work by Blin and Satterthwaite (1977) indicates that no nondictatorial decision rules are invulnerable to tacit deception, but they provide no systematic analysis of cases.

[6] Other theorists have drawn finer distinctions about the distribution of actor weights, or strengths, in triads (Caplow, 1968), but it is not clear that these distinctions are meaningful in other than experimental situations.

action of others. In fact, applications of the three-person models to deception in Vietnam (Zagare, 1977) and the two-person models to the Cuban missile crisis (Brams, 1977) convince us that they offer plausible explanations of why actors, on occasion, consciously misrepresent their preferences.

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