Rational Choice

Detecting Order and Randomness

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Mother's Lucky Coin

Suppose you want to see the new Hunger Games film, but your mom wants you to stay home. She says she will flip her lucky coin: tails you go to the movie, heads you stay home. Before you agree, you want her to prove that this coin is fair. So she flips it eleven times. Which sequence would make you more likely to think that this is a fair coin?

Sequence A: T–H–H–T–H–T–H–T–H–T–H, or

Sequence B: H–H–H–H–T–T–H–T–H–T–T.

Mother's Lucky Coin

Most people believe that sequence A is random like a fair coin because there is lots of variation, whereas sequence B has a few runs where the coin keeps coming up the same, suggesting some manipulation.

In fact, however, it is sequence B that suggests the coin is actually fair! It is sequence A, on the other hand, that suggests something fishy is going on with the coin or how your mother is flipping it.

Mother's Lucky Coin

To understand why sequence B is more consistent with a fair coin, first consider that for any two fair coin flips, you are 50% likely to see an alteration, that is an H–T or a T–H. You are also 50% likely to see a streak or either T–T or H–H. This means if you flip a fair coin x times, you should expect to see about $x/2$ alterations in the resulting sequence.

Now notice that sequence B has 5 alterations, which is close to $11/2$. Sequence A, however, has 9!

The Problem

Our everyday conception of randomness presumes too much variation. We think randomized sequences should look haphazard with many alterations. In fact, when confronted with an actual random sequence, people will perceive an underlying order because streaks are thought to be unusual for random events.

This problem manifests itself often in the real world.

Dear Abby

DEAR ABBY: My husband and I just had our eighth child. Another girl, and I am really one disappointed woman. I suppose I should thank God that she was healthy, but, Abby, this one was supposed to have been a boy. Even the doctor told me the law of averages were [sic] in our favor 100 to one.

Dear Abby

The reasoning the woman (and apparently her incompetent doctor) used says that the probability of having a boy or a girl is about 50-50. As a result, $P(8\text{Girl}) \approx (\tfrac{1}{2})^8 \approx 0.0039$. This is really unlikely.

Of course, this is irrelevant for anticipating the gender of the eighth child *after* the rest have already been born. The gender of one child is independent of the genders of his or her older sibling, and so by definition 6.2, $P(\text{Girl} \mid 7\text{Girl}) = P(\text{Girl}) \approx 0.50$.

The Gambler's Fallacy

This illustrates what is known as the **gambler's fallacy**. The idea is that after one event has occurred repeatedly in a random sequence then another outcome "is due" to happen. The assumption is that the chances of independent, random events "mature" if they have not occurred for a while.

Rule of Threes

Notice that Michael Jackson, Farrah Fawcett, and Ed McMahon died around the same time? This is just further proof that celebrities die in groups of three!

Rule of Threes

Let the probability of a celebrity dies on any given day be $0 < p < 1$. Suppose a celebrity dies today, is it more likely that a second celebrity dies (1) tomorrow or (2) n days after tomorrow?

The probability for (1) is p, whereas the probability for (2) is $(1-p)^{n-1}p$. It turns out, perhaps surprisingly, that $p > (1-p)^{n-1}p$. (Why?) So even if celebrity deaths are random, it is very likely that they occur in "bunches".

The Hot Hand in Basketball

Does a basketball player have a better chance of making a shot after having just made his last two or three shots than he does after having just missed his last two or three shots?

The Hot Hand in Basketball

More than 90% of a large sample of amateur and professional basketball players, basketball fans, and sports reporters answer “yes”. They claim that there is a hot hand or streak shooting in the sport.

The Hot Hand in Basketball

The idea behind this **hot hand** is that a player can build up momentum. Making several baskets in a row increases the chance of also hitting the next one; several misses in a row, however, decreases the chance of a hit on the next attempt. So, for instance, $P(\text{Hit} \mid 2\text{Hit}) > P(\text{Hit}) > P(\text{Hit} \mid 2\text{Miss})$.

Notice that this phenomenon is the exact opposite of the gambler's fallacy.

The Hot Hand in Basketball

Tom Gilovich, Robert Vallone, and Amos Tversky argue that there is no evidence of this phenomenon actually occurring. (Very depressing for basketball fans—including Tversky, who loved basketball and wanted to prove this phenomenon did exist!)

The Hot Hand in Basketball

There is no evidence of a hot hand when looking at the conditional probabilities for scoring by the Philadelphia '76ers during the 1980–1981 season:

	P(Hit | 3Miss)	P(Hit | 2Miss)	P(Hit)	P(Hit | 2Hit)	P(Hit | 3Hit)
Julius Erving	0.52	0.51	0.52	0.52	0.48
Andrew Toney	0.52	0.53	0.46	0.40	0.34

The Hot Hand in Basketball

Similar results with conditional probabilities were seen when looking at the free throws of the Boston Celtics during the 1980–1981 and 1981–1982 seasons, as well as within controlled experiment involving Cornell University's varsity basketball teams.

The Hot Hand in Basketball

There is also no evidence for the hot hand when looking at the total number of hit and miss runs. A hot hand should lead to fewer (but longer) runs.

	Number of Runs	Expected Number of Runs
Julius Erving	431	442.4
Andrew Toney	245	225.1

The Hot Hand in Basketball

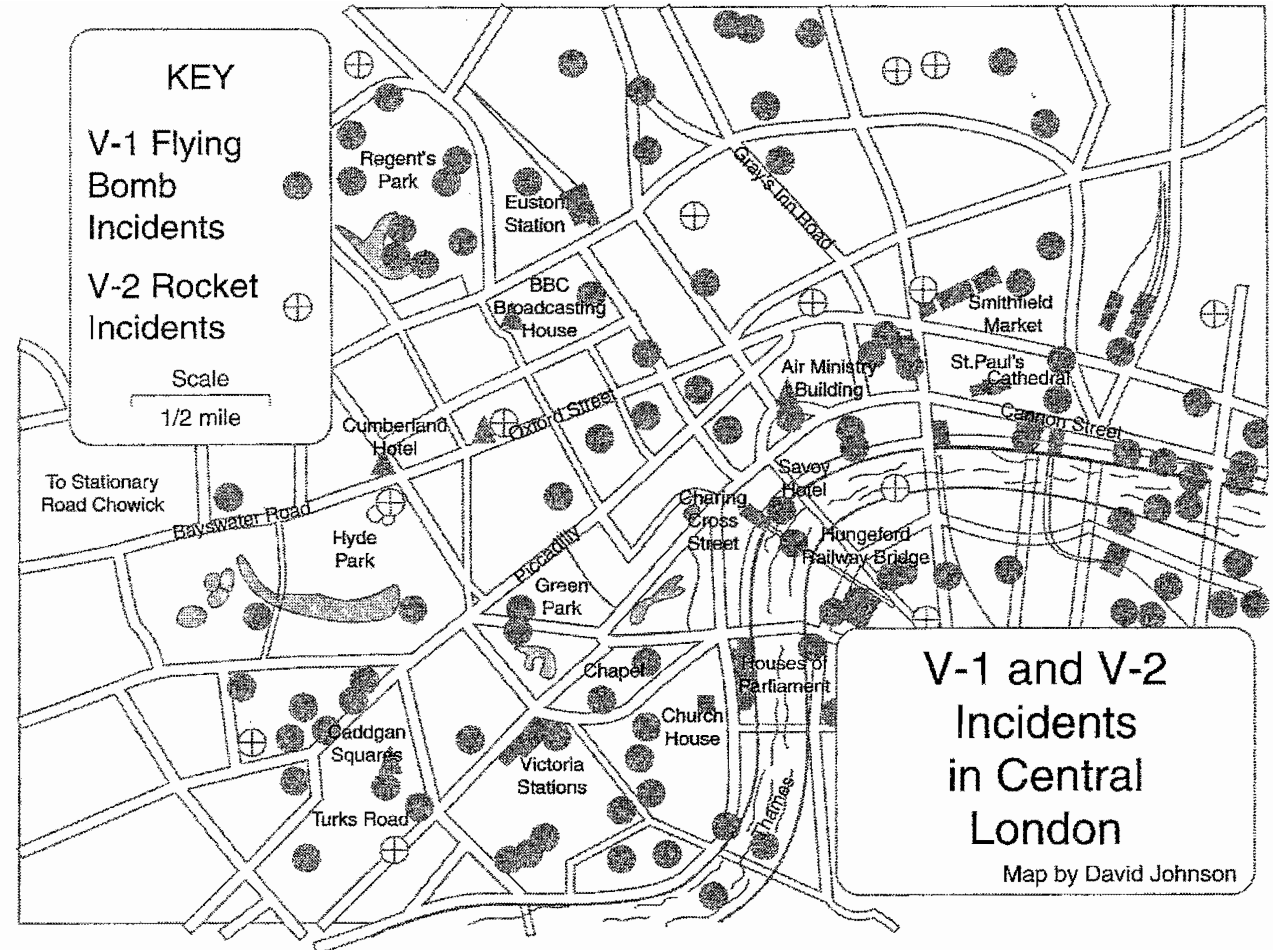
See the article for additional statistical analyses that suggest there is no hot hand in basketball, despite the fact that it is easy to imagine a causal process that might generate the patterns of a hot hand.

As you might imagine these studies caused some controversy among basketball fans and many have tried to find statistics disproving them. None have succeeded to discredit Gilovich et al.'s analysis.

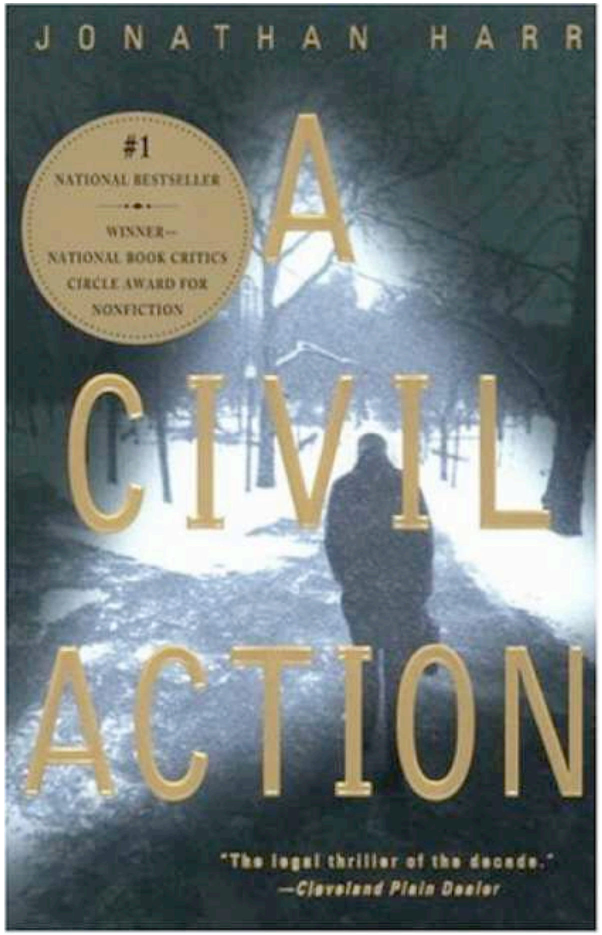
The Hot Hand in General

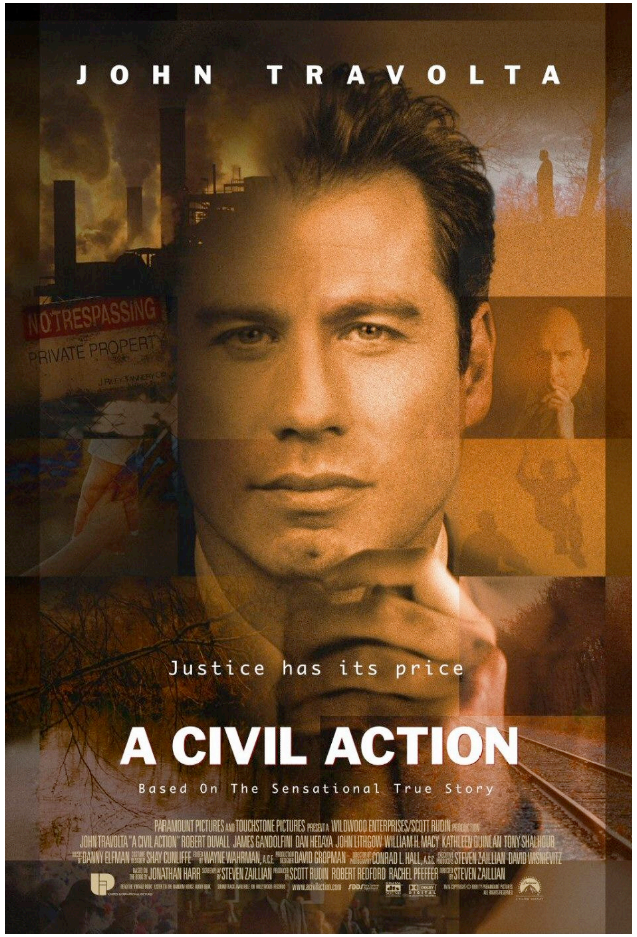
This article started a whole field of research in sport's statistics. It turns out that chaotic, in-your-face, player-on-player reactive sports do not exhibit streak performances (basketball, hockey, football, soccer, baseball, . . .). However, nonreactive, uniform-playing-field sports, subtle sequential dependences manifest themselves in performance, causing streaks and hot/cold hands to become statistically apparent (bowling, archery, billiards, golf, . . .).

Seeing Order Where There Isn't



Seeing Order Where There Isn't





Why Does This Happen?

Why do we expect too much alteration (and so too many short runs) from random processes? Conversely, why do we assume sequences with less alteration (and so fewer, but longer, runs) must come from non-random processes?

Next Class…

We move into the epilogue of the course, which has us reflect on the meaning of rational choice, and what status the axioms and rules of rational choice theory ought to have for our everyday decision making.