Rational Choice

The Philosophy of Probability

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World Cup Bid

As the host of the World Cup in 2002, Qatar's national football team automatically participates. What is the probability that Qatar's national team will win its first match in that tournament?

The Nature of Probability

Suppose that you believe that the probability of Qatar's national team winning its first match is 0.45. What exactly does this number mean?

The Nature of Probability

There are two broad sets of positions concerning what the numbers of probability mean:

Objectivism: The numbers correspond to *facts* about the external world concerning whether an event will or will not occur.

Subjectivism: The numbers correspond to the degree to which a person *believes* an event will or will not occur.

Probabilistic Objectivism

There are three major competing interpretations of probability made by objectivists:

1. The classical interpretation,
2. The frequency interpretation, and
3. The propensity interpretation.

The Classical Interpretation

According to the **classical** interpretation, the probability of an event E expresses a ratio:

$$P(E) = \frac{\text{Number of possible outcomes where E may occur}}{\text{Total number of possible outcomes}}.$$

The crucial assumptions are that (1) the possible outcomes are mutually exclusive, and (2) each is *equally* probable of occurring.

The Classical Interpretation

For example let E = "a fair coin lands heads up on at least one of two coin flips". What is P(E)?

Notice that there is a total of four equally possible outcomes involved here: (1) HH, (2) HT, (3) TH, and (4) TT. Of these four outcomes, event E occurs in exactly three of them: (1) HH, (2) HT, and (3) TH. So putting this together, P(E) = ¾.

The Classical Interpretation

This definition of probability conforms to the Kolmogorov axioms of probability.

For instance, K3 holds that if events A and B cannot both be true, then P(A or B) = P(A) + P(B). Suppose that the total number of possible outcomes is k, where event A occurs in i of these outcomes while event B occurs in j of them. So P(A) = i/k, and P(B) = j/k. Since A and B cannot both occur in the same outcome, P(A or B) = $i+j/k$. Finally, notice that $i+j/k = i/k, + j/k$, and so K3 holds.

Problems

Circularity: Cannot define "probability" by referencing the idea of "equally probable".

Uniqueness: For a unique event, like Qatar first football match in 2022, the outcomes are "win" and "lose", which are hardly "equally probable". So what does it mean to say one outcome is more probable?

Infinity: If there are an infinite number of outcomes, then the probability for any single outcome is 0.

The Frequency Interpretation

According to the **frequency** interpretation, the probability of an event E expresses a ratio:

$$P(E) = \frac{\text{Number of trials where E has occurred}}{\text{Total number of trials}}.$$

The Frequency Interpretation

Once again, let E = "a fair coin lands heads up on at least one of two coin flips". What is P(E)?

According to the frequentist, I need to run many trials, where in each trial I flip the coin twice. I then count the total number of trials done and the number of trails where at least one heads occurs. So suppose I do the trial 1,000 times and E happens in 767 of these trials. In this case, P(E) = 767/ 1,000 = 0.767.

The Frequency Interpretation

This definition of probability also conforms to the Kolmogorov axioms of probability.

Again, K3 holds that if events A and B cannot both be true, then P(A or B) = P(A) + P(B). Suppose that the total number of trials is k, where event A occurs in i of these trials while event B occurs in j of them. So P(A) = i/k, and P(B) = j/k. Since A and B cannot both occur in the same trial, P(A or B) = $i+j/k$. Finally, notice that $i+j/k = i/k, + j/k$, and so K3 holds.

Problem

But what happens if I do the trial 1,000 more times, and E occurs in 736 of these trials?

Does $P(E) = 736/1{,}000 = 0.736$, or

Does $P(E) = (767 + 736)/(1{,}000 + 1{,}000) = 0.7515$?

Both are problematic for the frequency interpretation because it says probability is *objective*, and if the physical object (the coin) has not changed, then how can the probability change as more trials are done?

Problem

The problem with the frequency interpretation is that it is sensitive to the **reference class**, which is the set of trials that are used to determine the probability. It is extremely difficult to know which is the right one to use, and it gets even more complicated if only one trial is possible, as in the case of unique events like Qatar's opening match at the 2022 World Cup.

The Frequency Interpretation

Some frequentists, like John Venn (of the diagram) believe that the proper reference class is the result of conducting the trial an *infinite* number of times. We do not rely solely on the *observed* trials. The probability of E is its **limiting frequency**, the ratio of it occurring in that infinite number of trials.

Problems

No Limit: What if there is no convergence on a single number? The ratio could oscillate forever.

Ignorance: We may have no idea what the probabilities are for many events (especially unique ones, like Qatar's first match in 2022).

Identity: Can two trials really be exactly the "same" trial? Maybe *every* event is unique.

The Propensity Interpretation

According to the **propensity** interpretation, the probability of an event E represents the tendency of that event to occur given certain objective features of the external world.

For instance, a fair coin is around 50% likely to land heads up because its physical features, coupled with the laws of gravity and so forth, cause it to land heads up about once in every two flips. Similar arguments may be constructed even for unique events.

Problems

Circularity: What is meant by the "tendency" for an event to occur? This almost sounds circular.

Objectivity: In what sense do "tendencies" exist in the physical, objective world?

Ignorance: Similar to limiting frequencies, we may have no idea what the probabilities are for many events.

Problem

The most serious problem with the propensity view is its understanding of conditional probability. Recall that if we know $P(A \mid B)$, we can use Bayes' theorem to determine $P(B \mid A)$. For instance, let A = "the train arrives on time at its destination" and B = "the train departed its origin on time". Both $P(A \mid B)$ and $P(B \mid A)$ make sense as probabilities, but $P(B \mid A)$ does *not* make sense as a propensity because propensity assumes temporal and causal directions.

The Ecumenical Approach

According to the **ecumenical** approach, different events may be treated with different interpretations of probability. It is a pluralistic view.

So the probability that the coin lands heads up is interpreted *objectively*, while the probability that Qatar's national football team wins its first match at the 2022 World Cup is interpreted *subjectively*.

The Ecumenical Approach

The main problem with such an approach is deciding how to interpret the probability of "mixed" events.

For instance, what is the proper interpretation of the event that the coin lands heads up and Qatar wins its first match in the World Cup. That is, is the number representing P(H and Q) making an objective or subjective claim? Or, if both, how can a single number be both objective and subjective simultaneously?

Next Class...

We will look at the form of subjective probability endorsed by most Bayesians.