# Rational Choice

*Introduction to Bayesianism*

**David Emmanuel Gray**

*Carnegie Mellon University in Qatar*

# A Routine Test

Suppose you undergo a routine test for HIV, and the test comes back positive! The most commonly used HIV test returns positive for a person with HIV 99.7% of the time; whereas it returns positive for a healthy person 1.5% of the time.

**Question:** Do you think you have HIV?

# Bayesian Decision Theory

Bayesianism is traditionally made up of two distinct, though interrelated, components:

1. The epistemic component, and
2. The deliberative component.

The first concerns what a rational agent should *believe*, while the second concerns what a rational agent should *do* given a set of beliefs.

# Bayesian Epistemology

According to the epistemic component of Bayesianism, a rational agent should have beliefs representable by a *subjective* probability distribution, and those beliefs should be updated (when new information is found) in accordance with Bayes' Theorem. Apart from these structural constraints, a rational agent is free to believe whatever she wants.

# Bayes' Theorem

**Theorem 6.5 (Bayes'):** $P(B \mid A) = \dfrac{P(B) \times P(A \mid B)}{P(A)}$, given that $P(A) \neq 0$.

**Theorem 6.6:** $P(B \mid A) = \dfrac{P(B) \times P(A \mid B)}{[P(B) \times P(A \mid B)] + [P(\underline{B}) \times P(A \mid \underline{B})]}$, given that $P(A) \neq 0$.

$P(B)$ is known as the **prior probability**. It specifies the likelihood of $B$ *before* the new information ($A$) was available. $P(B \mid A)$ is therefore known as the updated, **posterior probability**.

# Updating Via Bayes' Theorem

A random person like you is about 0.5% likely to be HIV positive:

$$P(Y = HIV) = 0.005.$$

According to the standardly used HIV test::

$$P(T = + \mid Y = HIV) = 0.997 \text{ (**Sensitivity**), and}$$

$$P(T = + \mid Y \neq HIV) = 0.015 \text{ (**False Positives**).}$$

# Updating Via Bayes' Theorem

The test is positive, so Bayes' Theorem is used to update the probability that you are HIV positive:

$$P(Y = HIV \mid T = +)$$

$$= \frac{P(Y = HIV) \times P(T = + \mid Y = HIV)}{[P(Y = HIV) \times P(T = + \mid Y = HIV)] + [P(Y \neq HIV) \times P(T = + \mid Y \neq HIV)]}$$

$$= \frac{0.005 \times 0.997}{[0.005 \times 0.997] + [(1 - 0.005) \times 0.015]}$$

$= 0.250$, or about 25%.

It is still unlikely that you are HIV positive.

# Updating Via Bayes' Theorem

Indeed, in all likelihood, you will be retested. What if that second test also comes back positive?

Since the test results are independent:

$$P(T1 = + \text{ and } T2 = + \mid Y = \text{HIV}) =$$
$$P(T1 = + \mid T = \text{HIV}) \times P(T2 = + \mid T = \text{HIV}) =$$
$$0.997 \times 0.997 =$$
$$0.994009, \text{ and}$$

$$P(T1 = + \text{ and } T2 = + \mid Y \neq \text{HIV}) =$$
$$0.015 \times 0.015 =$$
$$0.000225.$$

# Updating Via Bayes' Theorem

Once again, use Bayes' Theorem is used to update the probability that you are HIV positive:

$$P(Y = \text{HIV} \mid T1 = + \text{ and } T2 = +)$$

$$= \frac{P(Y = \text{HIV}) \times P(T1 = + \text{ and } T2 = + \mid Y = \text{HIV})}{[P(Y = \text{HIV}) \times P(T1 = + \text{ and } T2 = + \mid Y = \text{HIV})] + [P(Y \neq \text{HIV}) \times P(T1 = + \text{ and } T2 = + \mid Y \neq \text{HIV})]}$$

$$= \frac{0.005 \times 0.994009}{[0.005 \times 0.994009] + [(1 - 0.005) \times 0.000225]}$$

$= 0.9569$, or about 95.69%.

It is *now* quite likely that you are HIV positive :'(

# Bayesian Deliberation

1. A rational agent's beliefs can be represented by a subjective probability distribution, which is defined in terms of the agent's preference judgments.

2. A rational agent's desires can be represented by a utility function, which is defined in terms of the agent's preference judgments.

3. A rational agent's decisions are made as if they are done in accordance with the principle of utility.

# Bayesian Deliberation

Bayesianism says you can look at a person's decisions and (by revealed preference theory) determine her preference judgments. From these you can then infer *both* a utility function *and* a subjective probability distribution consistent with her decisions. Thus you can model her as if she was obeying the principle of expected utility. For this to work, though, her preference judgments must satisfy certain axioms (like completeness and transitivity).

# Arguments for Bayesianism

1. It provides an elegant and (relatively) simple unified theory concerning decision making.

2. It presumes nothing about the nature and metaphysics of probability.

3. It offers a high degree of mathematical precision.

# Argument Against Bayesianism

Bayesianism is often accused of "putting the cart before the horse". That is, Bayesianism requires that you *already* have a complete and transitive set of preference judgments over options. Once you have these judgments, then Bayesianism can model your desires (utilities) and beliefs (probabilities). So what Bayesians use as *input*, other decision theorists want as *output*.

# Argument Against Bayesianism

This means that Bayesianism may be useful as a *descriptive* tool for modeling decision makers, but it does not provide much in the way of *normative* guidance because it fails to say how a decision maker ought to choose. Instead, it assumes that the decision maker already has made her preference judgments, and so she already knows what to do. She does not need any further guidance.

# Bayesianism

In this last major unit of the course, we will spend some time fleshing out the details of Bayesianism. You can then come back to these arguments and see to what extent you find Bayesianism an attractive (or unattractive) theory of decision making.

# Next Class…

We will look at some non-Bayesian approaches to the philosophy of probability. This will provide some useful points of contrast.