Rational Choice

Prospect Theory

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Choice 1

You are given QR 2,000. You now must choose between the following two tickets:

- Ticket 5: A 50% chance of losing QR 1,000, or
- Ticket 6: A 100% chance of losing QR 500.

Choice 2

You are given QR 0. You now must choose between the following two tickets:

- Ticket 7: A 45% chance of getting QR 6,000, or
- Ticket 8: A 90% chance of getting QR 3,000.

Choice 3

You are given QR 1,000. You now must choose between the following two tickets:

- Ticket 9: A 50% chance of getting QR 1,000, or
- Ticket 10: A 100% chance of getting QR 500.

Choice 4

You are given QR 0. You now must choose between the following two tickets:

- Ticket 11: A 0.1% chance of getting QR 6,000, or
- Ticket 12: A 0.2% chance of getting QR 3,000.

The Certainty Effect

So far, we have seen how forms of actuarial and utility risk aversion lead to a phenomenon psychologists call the **certainty effect**. In other words, people tend to prefer *certain* gains over equally large *expected* gains. This is seen in experiments testing Allais' Paradox.

Allais' Paradox

	1 Blue	10 Red	89 Green
Ticket 1	QR 3M	QR 3M	QR 3M
Ticket 2	QR 0	QR 15M	QR 3M
Ticket 3	QR 3M	QR 3M	QR 0
Ticket 4	QR 0	QR 15M	QR 0

Small Probabilities

Another well-established effect discovered by psychologists is the **effect of small probabilities**. That is, people reason incorrectly about small probabilities. When probabilities are large, people tend to understand what they mean. However, as probabilities get smaller, people tend to ignore them, or treat them as identical, and so use just the utilities to make the decision.

Small Probabilities

		45 Blue	45 Red	10 Green
Choice 2	Ticket 7	QR 6,000	QR 0	QR 0
	Ticket 8	QR 3,000	QR 3,000	QR 0

		1 Blue	1 Red	998 Green
Choice 4	Ticket 11	QR 6,000	QR 0	QR 0
	Ticket 12	QR 3,000	QR 3,000	QR 0

Small Probabilities

This is a violation of the Von Neumann-Morgenstern independence axiom, though not in the same way as Allais' paradox. (Why not?) Notice the following:

$$T_{11} = (1/450)(T_7) \oplus (449/450)(\text{QR } 0 \text{ for sure}), \text{ and}$$

$$T_{12} = (1/450)(T_8) \oplus (449/450)(\text{QR } 0 \text{ for sure}).$$

So according to the independence axiom, $T_7 > T_8$ if and only if $T_{11} > T_{12}$.

Small Probabilities

With larger probabilities most people understand that 90% is twice as likely as 45%. However, when small probabilities are involved, people tend to focus on utilities instead. They often treat 0.1% equally as likely as 0.2%, and so use the utilities to make the decision.

Perceived Gains Versus Losses

Yet another well-established effect discovered by psychologists is the **reflection effect**. According to this, people are sensitive about whether the outcomes are present as gains or as losses, even though the actual end-state outcomes might be identical.

Perceived Gains Versus Losses

		50 Blue	50 Red
Choice 1	Ticket 5	QR 2,000 – QR 1,000	QR 2,000
	Ticket 6	QR 2,000 – QR 500	QR 2,000 – QR 500
Choice 3	Ticket 9	QR 1,000	QR 1,000 + QR 1,000
	Ticket 10	QR 1,000 + QR 500	QR 1,000 + QR 500

Perceived Gains Versus Losses

People are risk averse against *uncertain gains*, and so they tend to choose the sure gain of QR 500 of T_{10}. They are also risk averse against *certain losses*, and so they reject taking the sure loss of QR 500 of T_6. Of course, if you compare their respective final end-states of money, there is no difference between T_{10} and T_6.

Prospect Theory

Daniel Kahneman and Amos Tversky studied these three effects and derives what is known as **prospect theory**. According to this theory, the principle of expected utility must be augmented to take into account how people perceive probabilities and how they understand gains versus losses.

Prospect Theory

Recall that the expected utility of an action is formalized as follows:

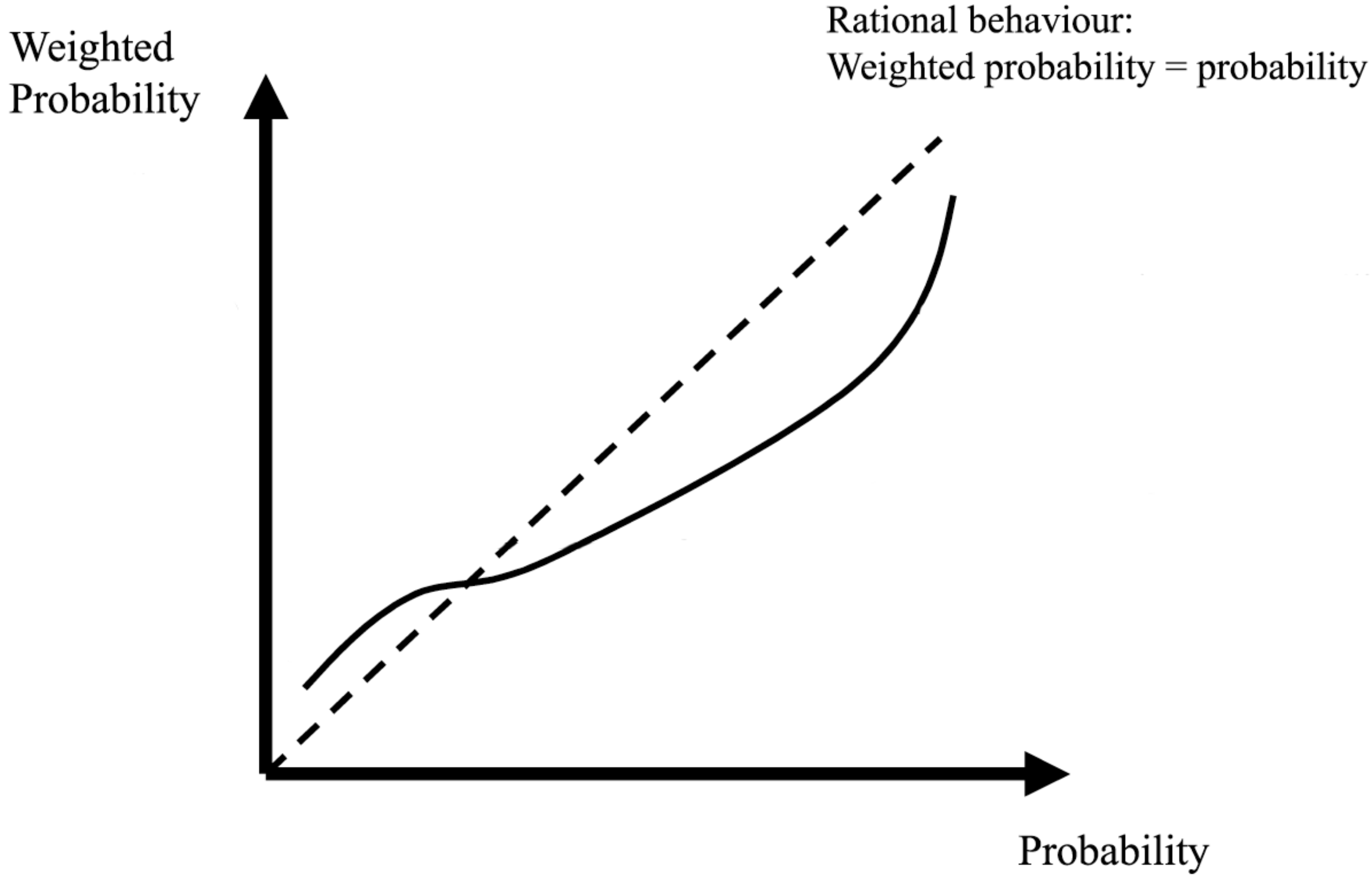
$$v(a_i) = \sum^{n}_{j=1}[p_j \times u(o_{i,j})].$$

Prospect theory says there is a weighting function w_p for the probabilities and a weighting function w_u for utilities. Thus the principle of expected prospects is:

$$v(a_i) = \sum^{n}_{j=1}[w_p(p_j) \times w_u(u(o_{i,j}))].$$

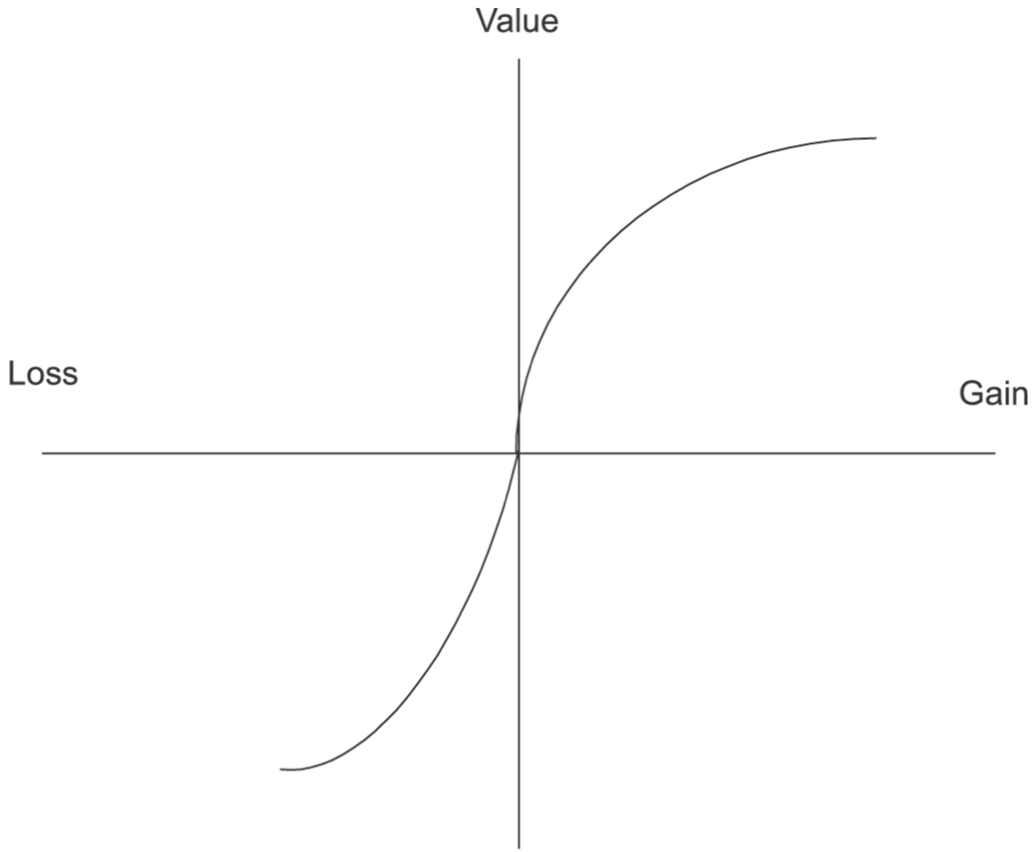
Prospect Theory

Kahneman and Tversky propose that the weighing function w_p for probabilities looks like this:



Prospect Theory

Kahneman and Tversky propose that the weighing function w_u for utilities looks like this:



There is some debate concerning the precise shape of these weighting functions. Even so, prospect theory turns out to have considerable predictive power over how people make decisions.

Next Class

We will look at another robust psychological phenomena: preference reversal. This is so strong, that even when faced with it, subjects fail to recognize that they have done something odd.

I have provided you with a transcript of an interview with a test subject illustrating this.