Rational Choice

Risk Aversion

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Allais' Paradox

		Blue	Red	Green
Choice 1	Ticket 1	QR 3M	QR 3M	QR 3M
	Ticket 2	QR 0	QR 15M	QR 3M
Choice 2	Ticket 3	QR 3M	QR 3M	QR 0
	Ticket 4	QR 0	QR 15M	QR 0

Risk Aversion

Many people may fall prey to Allais' paradox because they prefer a ticket giving them a certain outcome over a ticket that involves the risk of getting nothing. That is, they are adverse against the risks involved.

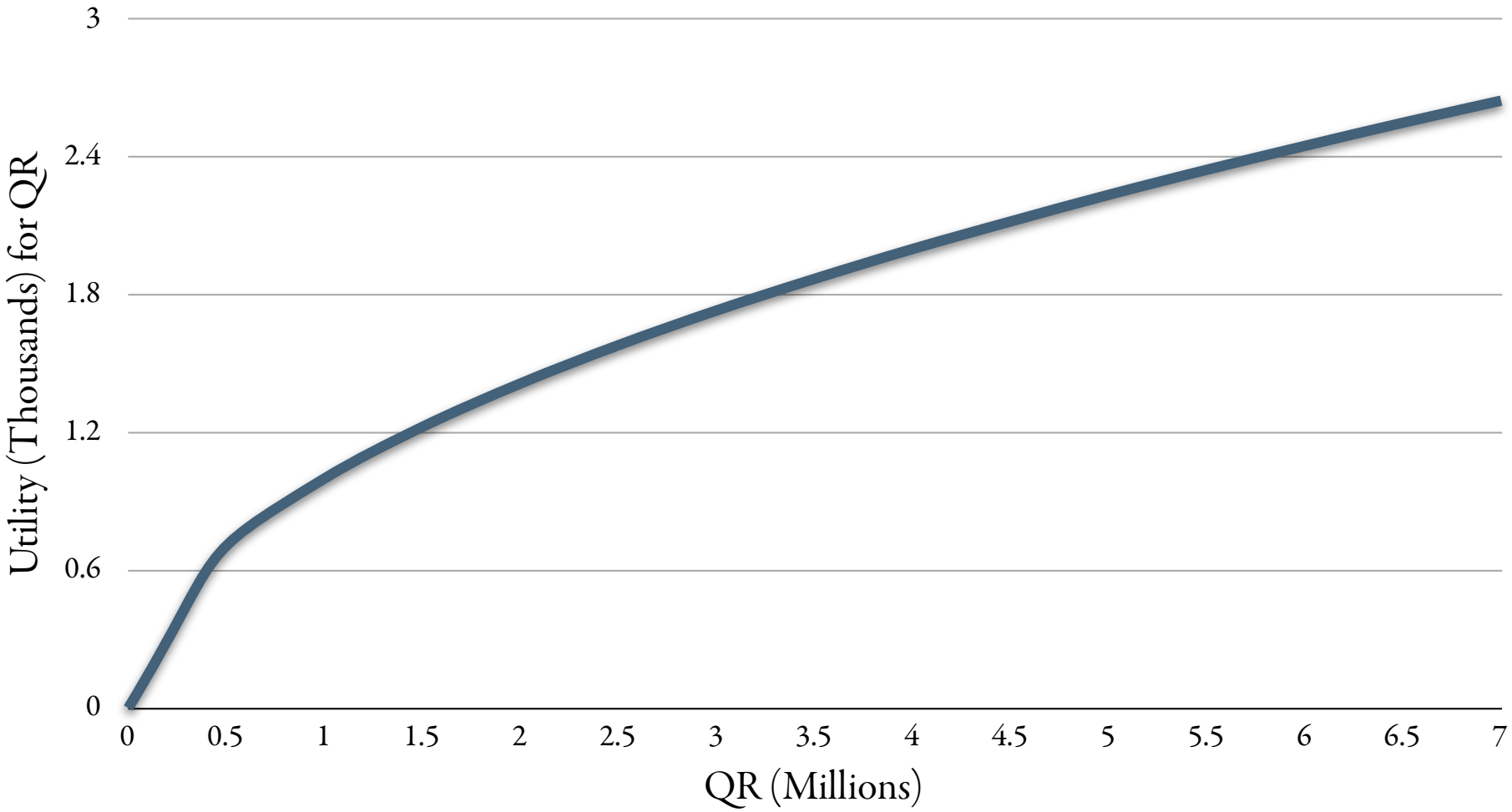
In general, many arguments for rejecting the principle of expected utility (and the independence axiom in particular) maintain that it is rational to be risk averse in situations like this.

Risk Aversion

Actuarial Risk Aversion: Preference for a smaller prize for certain over an actuarially equivalent lottery over larger and smaller prizes.

Do you prefer QR 3,000,000 for sure, or a 50-50 chance of winning QR 6,000,000 or nothing? In experimental settings, most people take the the certain outcome. This is known as the **certainty effect**.

Risk Aversion



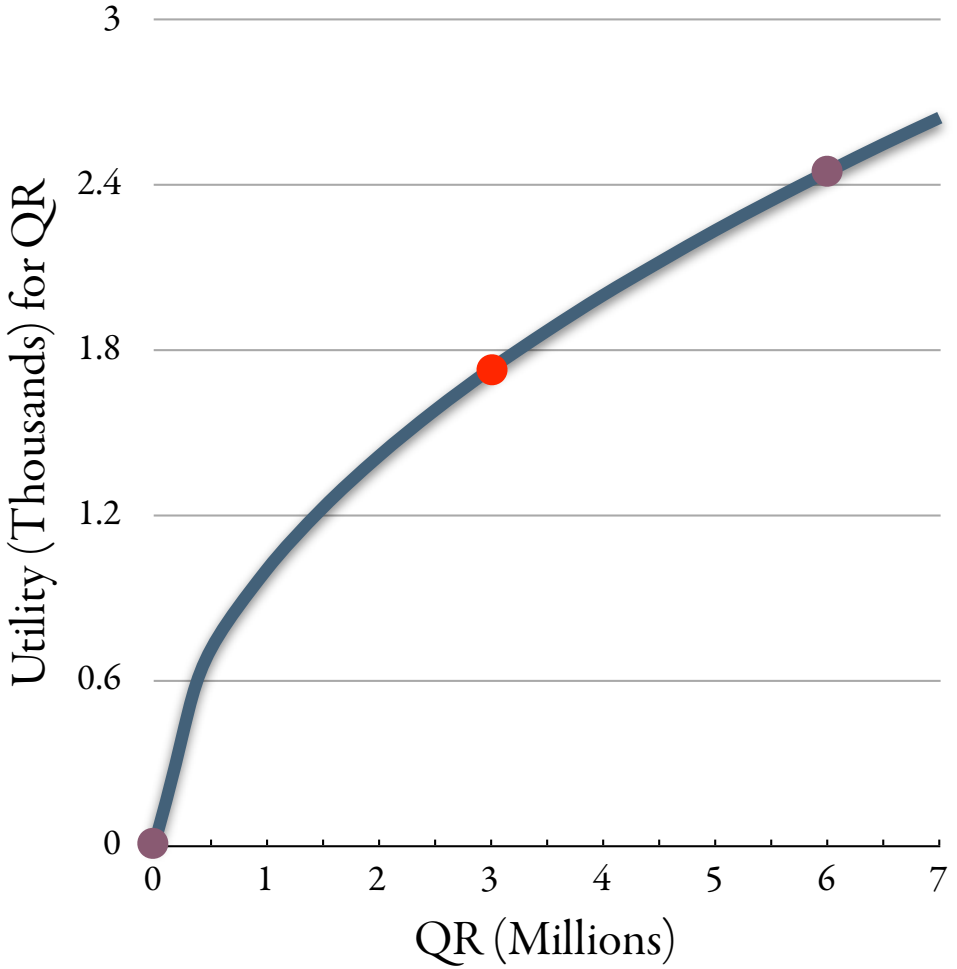
Risk Aversion

QR 3M for certain is far better than a 50-50 chance at QR 6M or nothing. In this case:

$u(\text{QR } 0) = 0$, and
$u(\text{QR } 6\text{M}) \approx 2.449\text{K}$.
So $0.5 \times u(\text{QR } 0\text{M}) + 0.5 \times u(\text{QR } 6\text{M})$
$\approx 1.225\text{K}$, but

$u(\text{QR } 3\text{M}) \approx 1.732\text{K}$.

Hence, you should pick QR 3M for certain.



Risk Aversion

This shows, however, that actuarial risk aversion is not a problem for the principle of expected utility. All that is needed is a conversion function from outcomes to utility.

Furthermore, this does not explain the Allais paradox because it remains a paradox no matter the person's utility for money might be. (The math from the previous lecture demonstrates this.)

Risk Aversion

Utility Risk Aversion: Preference for a smaller amount of utility for certain over an actuarially equivalent lottery over larger and smaller amounts of utility.

Do you prefer 3,000,000 utility for sure, or a 50-50 chance of getting 6,000,000 utility or 0 utility?

Risk Aversion

If you are averse against utility risks, then you will put more weight on worse outcomes and less weight on better outcomes.

A decision rule like this, though, can often be converted into a utility scale that conforms to the principle of expected utility, just as was done with money for actuarial risks on the previous slides.

Risk Aversion

Even so, recall the **leximin rule** for decisions under ignorance. This was an extremely risk averse rule and it can also be used in decisions under risk if you are willing to ignore the probabilities. In that case, the decisions made in Allais' paradox might be explained. Furthermore, leximin is incompatible with the principle of expected utility, so leximin offers a truly genuine alternative to that principle.

Allais' Paradox

		Blue	Red	Green
Choice 1	Ticket 1	QR 3M	QR 3M	QR 3M
	Ticket 2	QR 0	QR 15M	QR 3M
Choice 2	Ticket 3	QR 3M	QR 3M	QR 0
	Ticket 4	QR 0	QR 15M	QR 0

Risk Aversion

This approach to aversion against utility risks does not explain Ellsberg's paradox, however. Recall, the reasons why people choose in Ellsberg's choices are different from those reasons given for the choices made in the Allais paradox.

Ellsberg's Paradox

		Blue	Red	Green
Choice 1	Ticket 1	QR 300	QR 0	QR 0
	Ticket 2	QR 0	QR 300	QR 0
Choice 2	Ticket 3	QR 300	QR 0	QR 300
	Ticket 4	QR 0	QR 300	QR 300

Risk Aversion

Epistemic Risk Aversion: Preference for lotteries where probabilities for the outcomes are known with certainty. When the probabilities are not certain, expect the worst.

Do you prefer a 35% chance of winning QR 300 or a chance of winning QR 300 that is between 0% to 65%?

Risk Aversion

The maximin criterion for expected utilities (**MMEU**)**:** Choose the alternative with the largest minimal expected utility.

This decision making rule is a form of maximin that *does* take probabilities into consideration, though now it considers ranges of probabilities when these probabilities are not known for certain, as in Ellsberg's paradox.

Ellsberg's Paradox

		Blue	Red	Green
Choice 1	Ticket 1	QR 300	QR 0	QR 0
	Ticket 2	QR 0	QR 300	QR 0
Choice 2	Ticket 3	QR 300	QR 0	QR 300
	Ticket 4	QR 0	QR 300	QR 300

Risk Aversion

In Ellsberg's paradox, the *minimal* expected utilities for the tickets are as follows:

Ticket 1: $0.35 \times u(\text{QR } 300) + 0.65 \times u(\text{QR } 0) = 0.35 \times u(\text{QR } 300)$.

Ticket 2: $0.35 \times u(\text{QR } 0) + 0.00 \times u(\text{QR } 300) + 0.65 \times \text{u}(\text{QR } 0) = 0$.

Ticket 3: $0.35 \times u(\text{QR } 300) + 0.65 \times u(\text{QR } 0) + 0.00 \times u(\text{QR } 300) = 0.35 \times \text{u}(\text{QR } 300)$.

Ticket 4: $0.35 \times u(\text{QR } 0) + 0.65 \times u(\text{QR } 300) = 0.65 \times \text{u}(\text{QR } 300)$.

In this case, the worst case for Ticket 1 is better than that for Ticket 2. Similarly, the worst case for Ticket 4 is better than that for Ticket 3.

Next Class

We will explore prospect theory, which is a modified version of expected utility theory created in response to how people tend to actually make decisions.