Rational Choice

Evaluating Choice Under Ignorance

David Emmanuel Gray

Carnegie Mellon University in Qatar

The Decision Matrix

		States of Affairs (Ω)				
Acts (A)		ω_1	ω_2	$\ldots$	ω_j	$\ldots$	ω_n

Acts (A)	ω_1	ω_2	$\ldots$	ω_j	$\ldots$	ω_n
a_1	$o_{1,1}$	$o_{1,2}$		$o_{1,j}$		$o_{1,n}$
a_2	$o_{2,1}$	$o_{2,2}$		$o_{2,j}$		$o_{2,n}$
$\ldots$						
a_i	$o_{i,1}$	$o_{i,2}$		$o_{i,j}$		$o_{i,n}$
$\ldots$						
a_m	$o_{m,1}$	$o_{m,2}$		$o_{m,j}$		$o_{m,n}$

Choice Under Ignorance

In choice under ignorance, the following all hold:

1. There are different outcomes for different states of affairs relevant to the decision,

2. For each combination of action and state of affairs, you *do* know the outcome, and

3. You *do not* know how likely (i.e., how probable) each state of affairs is.

The Principle of Insufficient Reason: $a_i > a_j$ if and only if $\text{avg}(a_i) > \text{avg}(a_j)$.

$\text{avg}(a_x)$ represents the average utility value that a_x might return when giving each of the n states of affairs equal weight, i.e., $\text{avg}(a_x) = \sum^{n}_{k=1}[(1/n) \times u(o_{x,k})]$.

Example

States of Affairs (Ω)

Acts (A)	ω_1	ω_2	ω_3
a_1	3	0	9
a_2	9	3	12
a_3	0	3	6

Example

		States of Affairs (Ω)			
		ω_1	ω_2	ω_3	avg
Acts (A)	a_1	3	0	9	4
	a_2	9	3	12	
	a_3	0	3	6	

$$\text{avg}(a_1) = \left[\left(\tfrac{1}{3}\right) \times 3\right] + \left[\left(\tfrac{1}{3}\right) \times 0\right] + \left[\left(\tfrac{1}{3}\right) \times 9\right] = 4.$$

Example

States of Affairs (Ω)

Acts (A)	ω_1	ω_2	ω_3	avg
a_1	3	0	9	4
a_2	9	3	12	8
a_3	0	3	6	

$$\text{avg}(a_1) = \left[\left(\tfrac{1}{3}\right) \times 9\right] + \left[\left(\tfrac{1}{3}\right) \times 3\right] + \left[\left(\tfrac{1}{3}\right) \times 12\right] = 8.$$

Example

Acts (A)		States of Affairs (Ω)			
		ω_1	ω_2	ω_3	avg
	a_1	3	0	9	4
	a_2	9	3	12	8
	a_3	0	3	6	3

$$\text{avg}(a_1) = \left[\left(\tfrac{1}{3}\right) \times 0\right] + \left[\left(\tfrac{1}{3}\right) \times 3\right] + \left[\left(\tfrac{1}{3}\right) \times 6\right] = 3.$$

Example

States of Affairs (Ω)

Acts (A)

	ω_1	ω_2	ω_3	avg
a_1	3	0	9	4
a_2	9	3	12	8
a_3	0	3	6	3

Principle of insufficient reason says to choose a_2.

The Principle of Insufficient Reason

The idea is that if one has no reason to think that one state of affairs is more or less likely than another, then it is rational to assign these states equal probability.

The major concern with this approach is that this introduces probabilities—indeed, one very precise probability distribution—to a decision where there is explicitly no information concerning likelihoods.

The Problem of Plurality

We have now seen eight rules for choice under ignorance. It is, of course, possible to imagine others (e.g., a V-Admissibility rule). The problem is that these often issue conflicting prescriptions. So which is the "correct" rule for choice under ignorance?

Evaluating Choice Under Ignorance

Appealing to intuition about the "best" rule of choice under ignorance does not appear helpful. As a result most decision theorists devise conditions that a rule ought to satisfy.

Evaluating Choice Under Ignorance

Weak Pareto: If for every state of affairs $\omega_x \in \Omega$, $u(o_{i,x}) > u(o_{j,x})$, then $a_i \succ a_j$.

The textbook misleadingly calls this *strict dominance*. Avoid the confusion, and call it *weak Pareto* instead.

All the choice rules we have discussed satisfy this.

Ordering: The judgments $\succ$ over the actions generated by the decision rule is a preference relation.

Of the eight rules we have seen, the two dominance rules (weak and strict) fail to satisfy this condition.

Example

States of Affairs (Ω)

Acts ($\mathcal{A}$)	ω_1	ω_2	ω_3
a_1	1	0	3
a_2	3	1	4
a_3	0	1	2

Evaluating Choice Under Ignorance

Irrelevant Alternatives: Let $a_i, a_j \in A \subseteq B$. $a_i \succ a_j$ when the act set is A if and only if $a_i \succ a_j$ when the act set is B.

The idea is that the ordering of two options does not change when options are added (the "only if" part) or when other options are removed (the "if" part) from the set of available actions.

Of the eight rules we have seen, minimax regret fails to satisfy this condition.

Example

Acts (A)	States of Affairs (Ω): ω_1	ω_2	ω_3
a_1	12	8	20
a_2	10	15	16

Start with the decision matrix.

Example

Acts (A)	States of Affairs (Ω): ω_1	ω_2	ω_3
a_1	**12**	8	**20**
a_2	10	**15**	16

Identify the maximum value for each state (column).

Example

		ω_1	ω_2	ω_3
Acts (A)	a_1	0	-7	0
	a_2	-2	0	-4

States of Affairs (Ω)

Create the regret matrix by subtracting each column's maximum value from everything in that column.

Example

States of Affairs (Ω)

Acts (A)	ω_1	ω_2	ω_3	min
a_1	0	-7	0	-7
a_2	-2	0	-4	-4

Calculate the minimums for each row (i.e., find the maximum regret for each action).

Example

		States of Affairs (Ω)			
		ω_1	ω_2	ω_3	min
Acts (A)	a_1	0	-7	0	-7
	a_2	-2	0	-4	-4

In this case: $a_2 \succ a_1$.

Example

States of Affairs (Ω)

Acts (B)	ω_1	ω_2	ω_3
a_1	12	8	20
a_2	10	15	16
a_3	15	6	25

Now add a new action, a_3.

Example

States of Affairs (Ω)

Acts (B)	ω_1	ω_2	ω_3
a_1	12	8	20
a_2	10	**15**	16
a_3	**15**	6	**25**

Identify the maximum value for each state (column).

Example

		States of Affairs (Ω)		
		ω_1	ω_2	ω_3
Acts (B)	a_1	-3	-7	-5
	a_2	-5	0	-9
	a_3	0	-9	0

Create the regret matrix by subtracting each column's maximum value from everything in that column.

Example

		States of Affairs (Ω)			
		ω_1	ω_2	ω_3	min
Acts (B)	a_1	-3	-7	-5	-7
	a_2	-5	0	-9	-9
	a_3	0	-9	0	-9

Calculate the minimums for each row (i.e., find the maximum regret for each action).

Example

States of Affairs (Ω)

Acts (B)	ω_1	ω_2	ω_3	min
a_1	-3	-7	-5	-7
a_2	-5	0	-9	-9
a_3	0	-9	0	-9

Now $a_1 > a_2$! Adding an option made a difference.

Evaluating Choice Under Ignorance

Column Linearity: The judgments > over the actions do not change when a uniform constant value is added to (or subtracted from) the utility values for all the outcomes in one state of affairs.

Of the eight rules we have seen, maximin, leximin, maximax, and optimism-pessimism fail to satisfy this.

Example

States of Affairs (Ω)

Acts (A)	ω_1	ω_2	ω_3	min
a_1	12	8	20	8
a_2	10	15	16	10
a_3	15	6	25	6

According to maximin: $a_2 \succ a_1 \succ a_3$.

Example

		States of Affairs (Ω)		
		ω_1	ω_2	ω_3
Acts ($\mathcal{A}$)	a_1	12	13	20
	a_2	10	20	16
	a_3	15	11	25

Now add 5 to the utilities for state ω_2.

Example

		States of Affairs (Ω)			
		ω_1	ω_2	ω_3	min
Acts (A)	a_1	12	13	20	12
	a_2	10	20	16	10
	a_3	15	11	25	11

Now according to maximin: $a_1 > a_3 > a_2$! (E.g., went from $a_2 > a_1$ to $a_1 > a_2$.)

Evaluating Choice Under Ignorance

Column Duplication: The judgments > over the actions do not change if an identical state (a column) is added to the decision problem.

Of the eight rules we have seen, the principle of insufficient reason fails to satisfy this condition.

Example

		States of Affairs (Ω)		
		ω_1	ω_2	avg
Acts (A)	a_1	6	24	15
	a_2	24	0	12
	a_3	12	6	9

According to the principle of insufficient reason:

$a_1 \succ a_2 \succ a_3$.

Example

States of Affairs (Ω)

Acts (A)

	ω_1	ω_2	ω_3
a_1	6	24	6
a_2	24	0	24
a_3	12	6	12

Now duplicate state ω_1 and label it as ω_3.

Example

States of Affairs (Ω)

Acts (A)	ω_1	ω_2	ω_3	avg
a_1	6	24	6	12
a_2	24	0	24	16
a_3	12	6	12	10

Now according to the principle of insufficient reason:

$a_2 > a_1 > a_3$! (E.g., went from $a_1 > a_2$ to $a_2 > a_1$.)

The Problem of Plurality

The concern has emerged that no single rule for rational choice under ignorances seems to satisfy all the desirable properties for such a decision rule.

Next Class…

We will look at a debate in political philosophy concerning the correct rule concerning a choice under ignorance.

Exam #1 is one week from today. It will be in **lecture hall 2152** and begin promptly at 1:00PM. Show up and be seated by that time.

You are allowed to use one A4-sized page of notes. Everything else (including cell phone) must put in the aisle or back of the room. Plan accordingly.

I will provide you with two pencils, one pen, a simple calculator, and plenty of scratch paper.