#### **Rational Choice** *Evaluating (hoice Under Ignorance*

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#### The Decision Matrix

		ωι	$\omega_2$	 $\omega_j$	• • •	$\omega_n$
$(\mathcal{V})$	$\mathcal{A}_{\mathrm{I}}$	0 <sub>1,1</sub>	<i>0</i> <sub>1,2</sub>	0 <sub>1</sub> ,j		0 <sub>1,n</sub>
	$\mathcal{A}_{2}$	0 <sub>2,I</sub>	<i>0</i> <sub>2,2</sub>	0 <sub>2,j</sub>		0 <sub>2,n</sub>
	• • •					
Acts	ai	0 <i>i</i> ,1	0 i,2	0 <sub>i,j</sub>		0 <i>i</i> , <i>n</i>
,	•••					
	<i>A</i> <sub>m</sub>	0 <sub><i>m</i>, 1</sub>	$O_{m,2}$	0 <sub>m,j</sub>		$O_{m,n}$

States of Affairs  $(\Omega)$ 

## Choice Under Ignorance

In choice under ignorance, the following all hold:

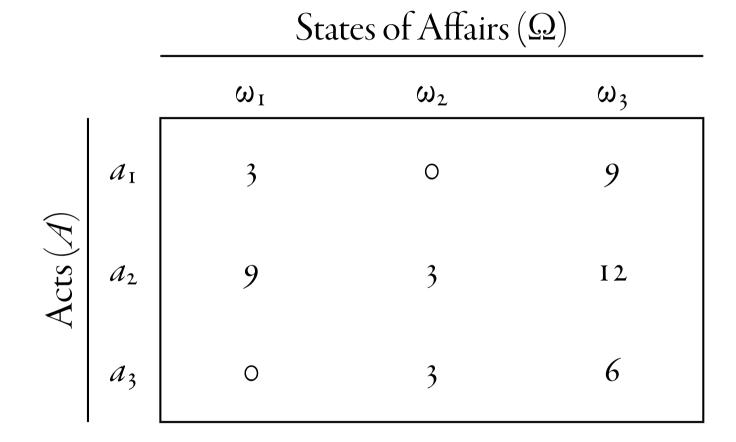
- 1. There are different outcomes for different states of affairs relevant to the decision,
- 2. For each combination of action and state of affairs, you *do* know the outcome, and

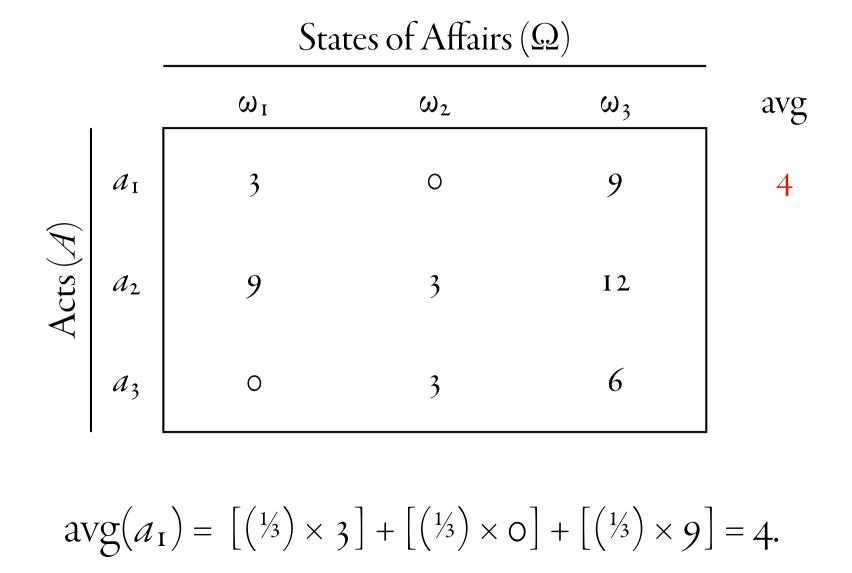
3. You *do not* know how likely (i.e., how probable) each state of affairs is.

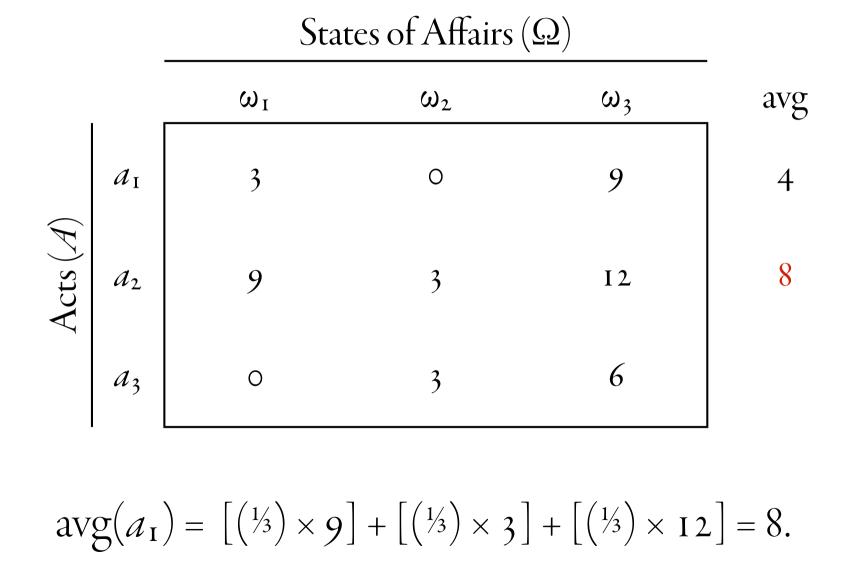
#### The Principle of Insufficient Reason

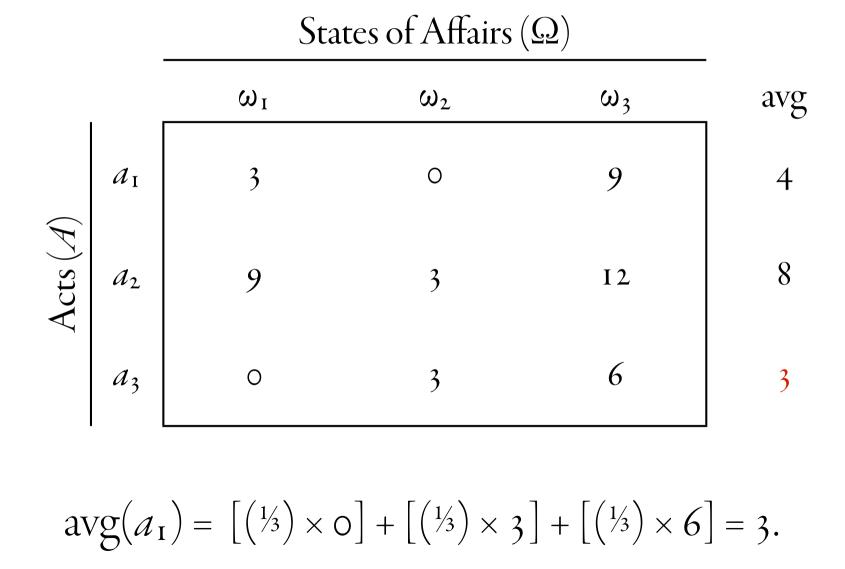
**The Principle of Insufficient Reason:**  $a_i > a_j$  if and only if  $avg(a_i) > avg(a_j)$ .

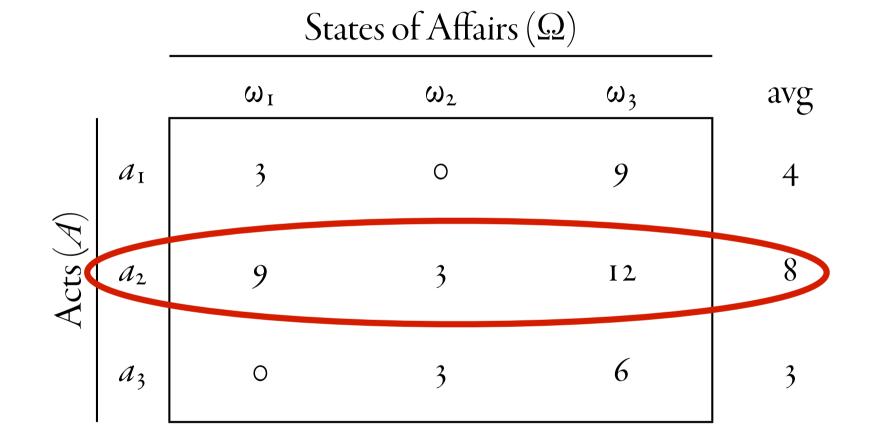
avg( $a_x$ ) represents the average utility value that  $a_x$  might return when giving each of the *n* states of affairs equal weight, i.e., avg( $a_x$ ) =  $\sum_{k=1}^{n} [(1/n) \times u(o_{x,k})].$ 











#### Principle of insufficient reason says to choose $a_2$ .

### The Principle of Insufficient Reason

The idea is that if one has no reason to think that one state of affairs is more or less likely than another, then it is rational to assign these states equal probability.

The major concern with this approach is that this introduces probabilities—indeed, one very precise probability distribution—to a decision where there is explicitly no information concerning likelihoods.

### The Problem of Plurality

We have now seen eight rules for choice under ignorance. It is, of course, possible to imagine others (e.g., a V-Admissibility rule). The problem is that these often issue conflicting prescriptions. So which is the "correct" rule for choice under ignorance?

### ».Evaluating Choice Under Ignorance

Appealing to intuition about the "best" rule of choice under ignorance does not appear helpful. As a result most decision theorists devise conditions that a rule ought to satisfy.

### ».Evaluating Choice Under Ignorance

- Weak Pareto: If for every state of affairs  $\omega_x \in \Omega$ ,  $u(o_{i,x}) > u(o_{j,x})$ , then  $a_i > a_j$ .
- The textbook misleadingly calls this *strict dominance*. Avoid the confusion, and call it *weak Pareto* instead.
- All the choice rules we have discussed satisfy this.

### Evaluating Choice Under Ignorance

**Ordering:** The judgments > over the actions generated by the decision rule is a preference relation.

Of the eight rules we have seen, the two dominance rules (weak and strict) fail to satisfy this condition.

#### 

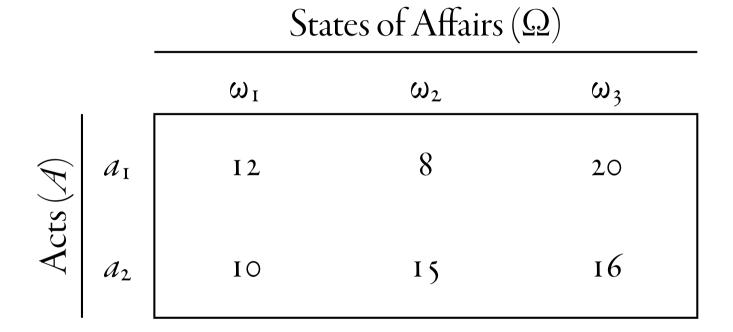
#### States of Affairs $(\Omega)$

### ».Evaluating Choice Under Ignorance

**Irrelevant Alternatives:** Let  $a_i, a_j \in A \subseteq B$ .  $a_i > a_j$  when the act set is A if and only if  $a_i > a_j$  when the act set is B.

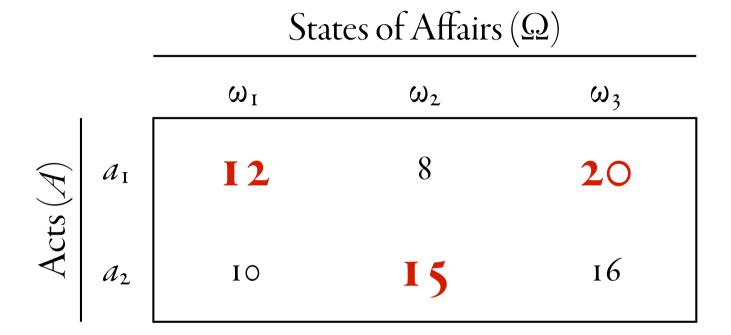
The idea is that the ordering of two options does not change when options are added (the "only if" part) or when other options are removed (the "if" part) from the set of available actions.

Of the eight rules we have seen, minimax regret fails to satisfy this condition.



#### Start with the decision matrix.

#### » Example



#### Identify the maximum value for each state (column).

### Example

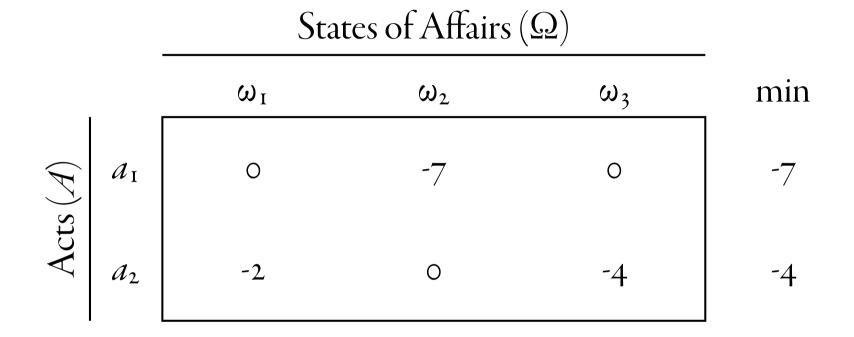


# Create the regret matrix by subtracting each column's maximum value from everything in that column.

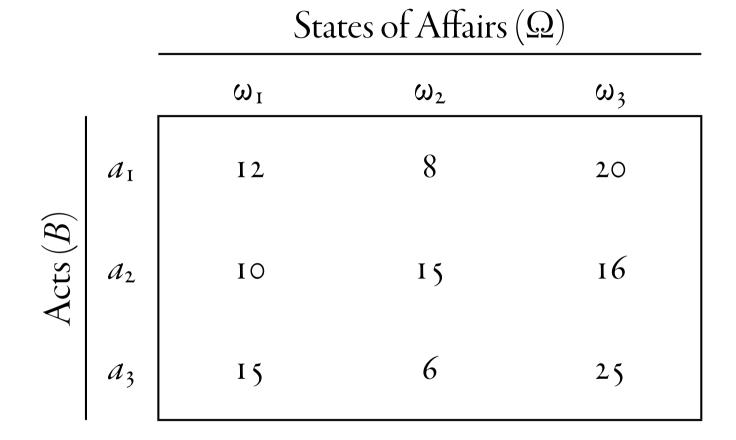
#### Example



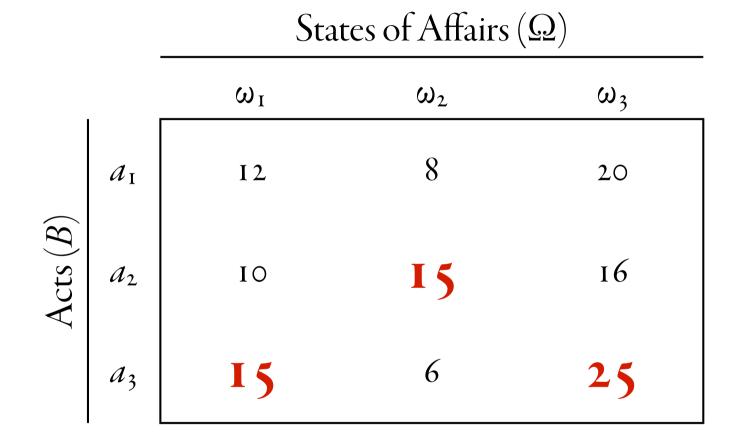
# Calculate the minimums for each row (i.e., find the maximum regret for each action).



In this case:  $a_2 > a_1$ .

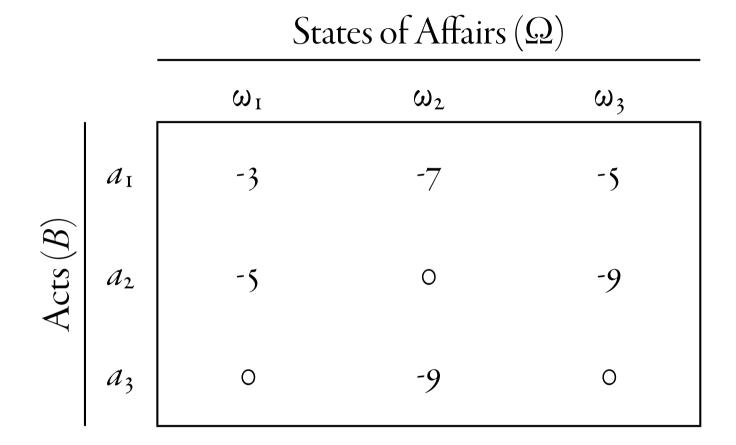


Now add a new action,  $a_3$ .



#### Identify the maximum value for each state (column).

### » Example



Create the regret matrix by subtracting each column's maximum value from everything in that column.



Calculate the minimums for each row (i.e., find the maximum regret for each action).

#### Example



#### Now $a_1 > a_2$ ! Adding an option made a difference.

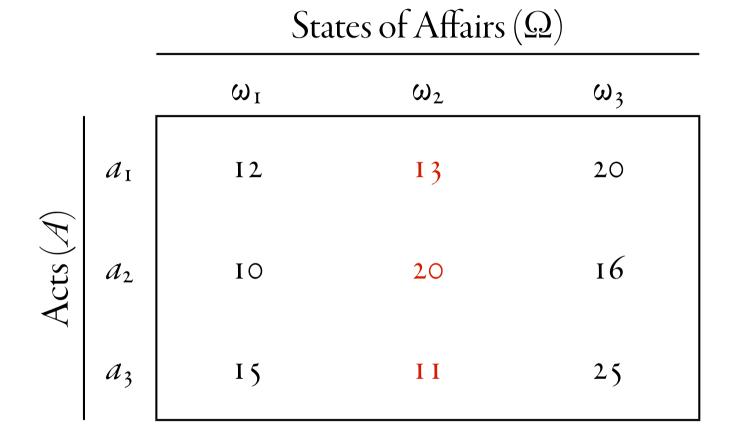
### ».Evaluating Choice Under Ignorance

**Column Linearity:** The judgments > over the actions do not change when a uniform constant value is added to (or subtracted from) the utility values for all the outcomes in one state of affairs.

Of the eight rules we have seen, maximin, leximin, maximax, and optimism-pessimism fail to satisfy this.



#### According to maximin: $a_2 > a_1 > a_3$ .



#### Now add 5 to the utilities for state $\omega_2$ .

### Example

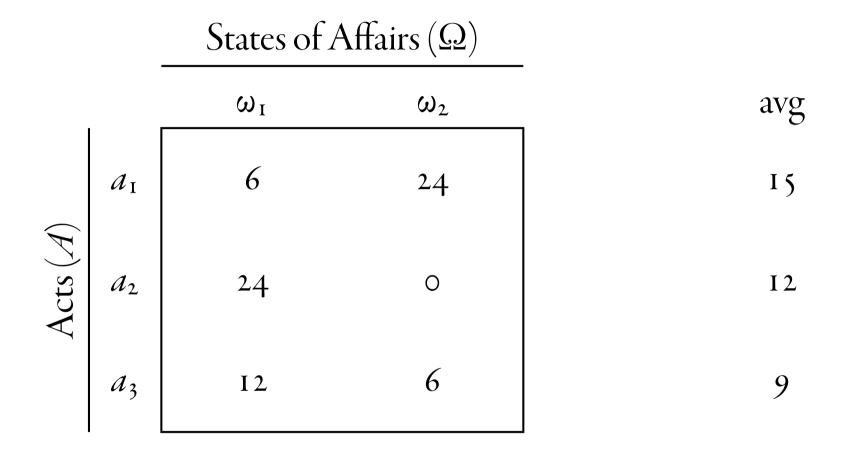
		Sta	_		
	I	ωı	$\omega_2$	ω	min
$\operatorname{Acts}(\mathcal{A})$	${\cal A}_{\rm I}$	I 2.	I 3	20	I 2
	$\mathcal{A}_2$	IO	20	16	IO
7	<i>a</i> <sub>3</sub>	15	ΙI	25	II

Now according to maximin: 
$$a_1 > a_3 > a_2!$$
 (E.g., went  
from  $a_2 > a_1$  to  $a_1 > a_2$ .)

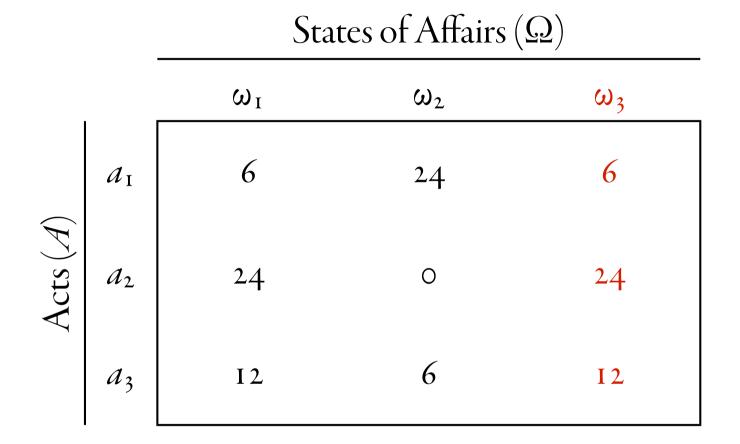
### ».Evaluating Choice Under Ignorance

**Column Duplication:** The judgments > over the actions do not change if an identical state (a column) is added to the decision problem.

Of the eight rules we have seen, the principle of insufficient reason fails to satisfy this condition.

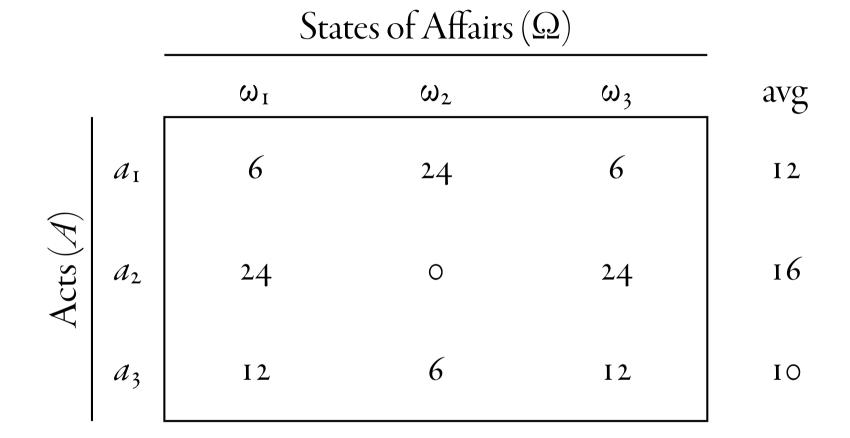


According to the principle of insufficient reason:  $a_1 > a_2 > a_3$ .



#### Now duplicate state $\omega_1$ and label it as $\omega_3$ .

#### Example



Now according to the principle of insufficient reason:  $a_2 > a_1 > a_3!$  (E.g., went from  $a_1 > a_2$  to  $a_2 > a_1$ .)

### The Problem of Plurality

The concern has emerged that no single rule for rational choice under ignorances seems to satisfy all the desirable properties for such a decision rule. We will look at a debate in political philosophy concerning the correct rule concerning a choice under ignorance.

Exam #1 is one week from today. It will be in lecture hall 2152 and begin promptly at 1:00PM. Show up and be seated by that time.

You are allowed to use one A4-sized page of notes. Everything else (including cell phone) must put in the aisle or back of the room. Plan accordingly.

I will provide you with two pencils, one pen, a simple calculator, and plenty of scratch paper.