Rational Choice *Rules for Choice Under Ignorance*

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The Decision Matrix

		ωι	ω_2	 ω_j	• • •	ω_n
	\mathcal{A}_{I}	0 _{1,1}	<i>0</i> _{1,2}	0 ₁ ,j		0 _{1,n}
	\mathcal{A}_{2}	0 _{2,I}	<i>0</i> _{2,2}	0 _{2,j}		0 _{2,n}
(\mathcal{W})	• • •					
Acts	ai	0 <i>i</i> ,1	0 i,2	0 _{i,j}		0 <i>i</i> , <i>n</i>
,	•••					
	<i>A</i> _m	0 _{<i>m</i>,1}	$O_{m,2}$	0 _{m,j}		$O_{m,n}$

States of Affairs (Ω)

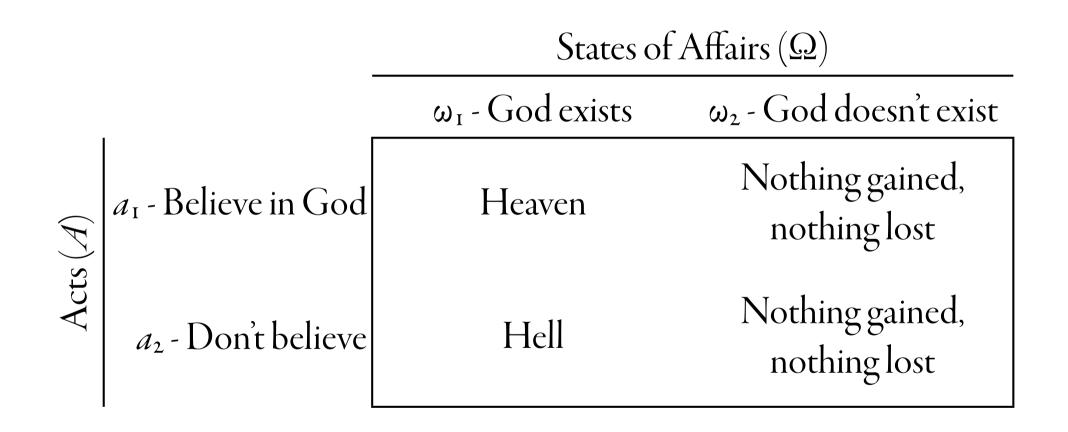
Choice Under Ignorance

In choice under ignorance, the following all hold:

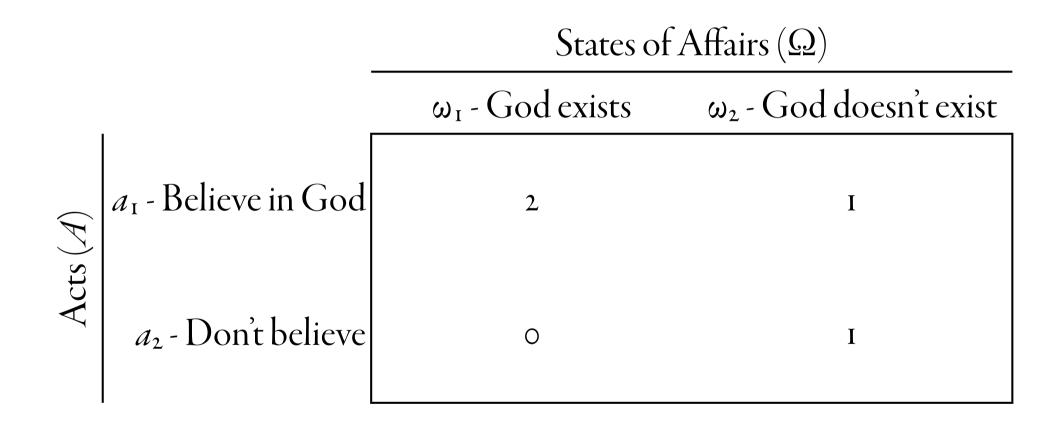
- 1. There are different outcomes for different states of affairs relevant to the decision,
- 2. For each combination of action and state of affairs, you *do* know the outcome, and

3. You *do not* know how likely (i.e., how probable) each state of affairs is.

Example: Pascal's Initial Choice

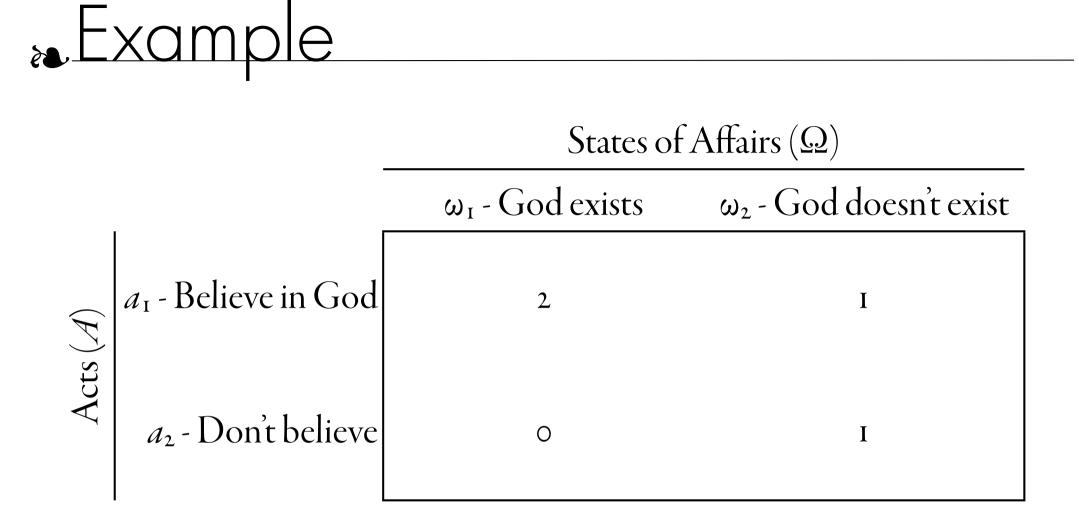


».Example

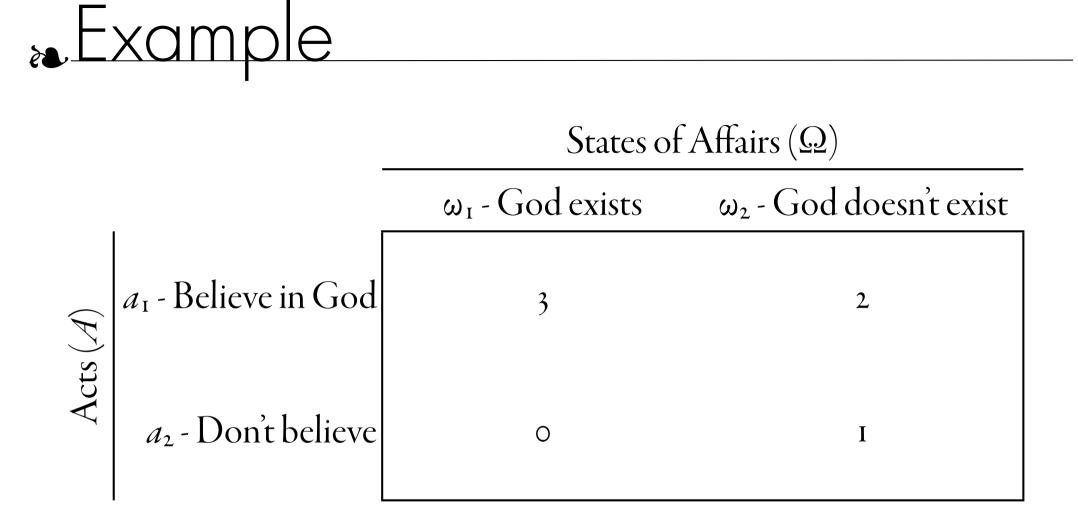


- Weak dominance: $a_i \ge a_j$ if and only if for every state of affairs $\omega_x \in \Omega$, $u(o_{i,x}) \ge u(o_{j,x})$.
- **Strict dominance:** $a_i > a_j$ if and only if the following both hold:

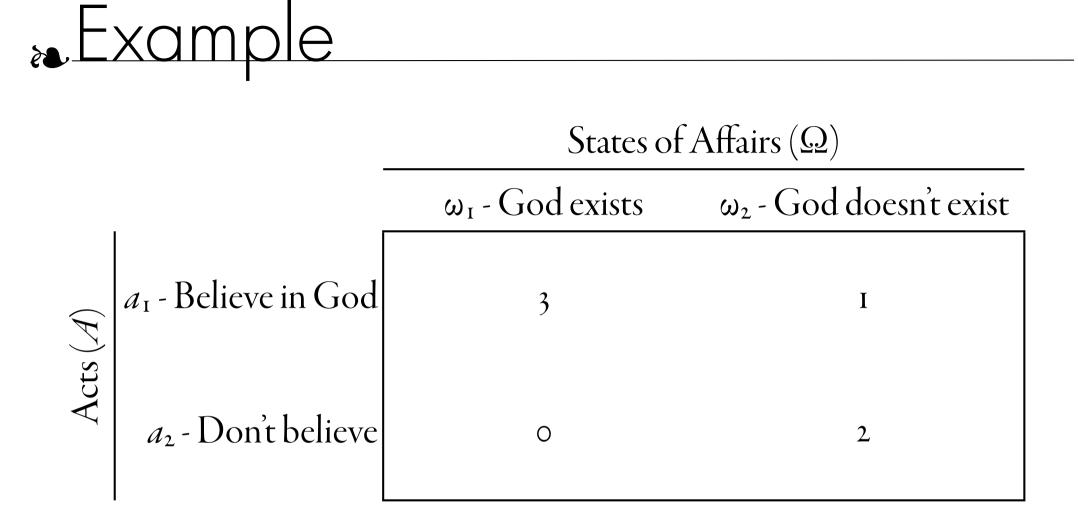
1. For every state of affairs $\omega_x \in \Omega$, $u(o_{i,x}) \ge u(o_{j,x})$. 2. There is some state ω_y such that $u(o_{i,y}) > u(o_{j,y})$.



How does each dominance rule rank these actions?



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What's the problem with the rules in this case?

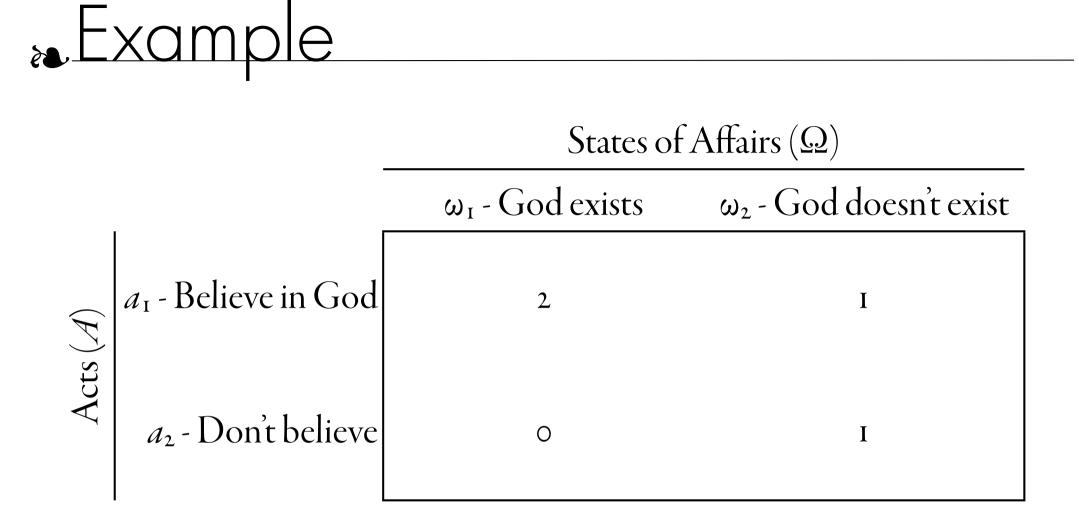


Maximin: $a_i \ge a_j$ if and only if $\operatorname{Min}_{k=1}^n [u(o_{i,k})] \ge \operatorname{Min}_{k=1}^n [u(o_{j,k})]$.

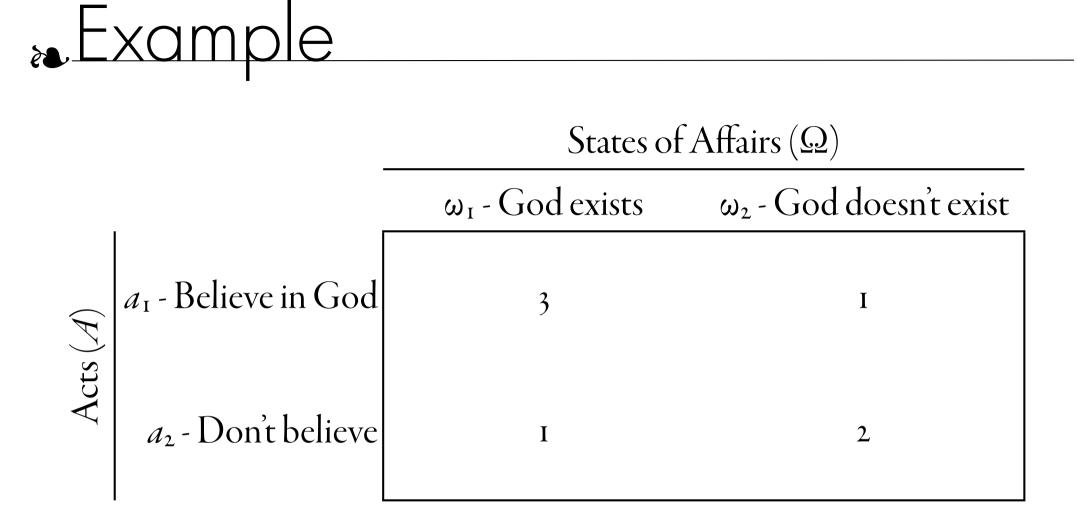
 $\operatorname{Min}_{k=1}^{n}[u(o_{i,k})]$ represents the lowest possible utility value that a_i might return.

Leximin: $a_i > a_j$ if and only if there is some positive integer q such that q-Min^{$n_{k=1}$} $[u(o_{i,k})] > q$ -Min^{$n_{k=1}$} $[u(o_{j,k})]$ and for all integers p < q, p-Min^{$n_{k=1}$} $[u(o_{i,k})] = p$ -Min^{$n_{k=1}$} $[u(o_{j,k})]$.

q-Min^{$n_{k=1}$}[$u(o_{i,k})$] returns the q^{TH} lowest possible utility value that a_i might return.



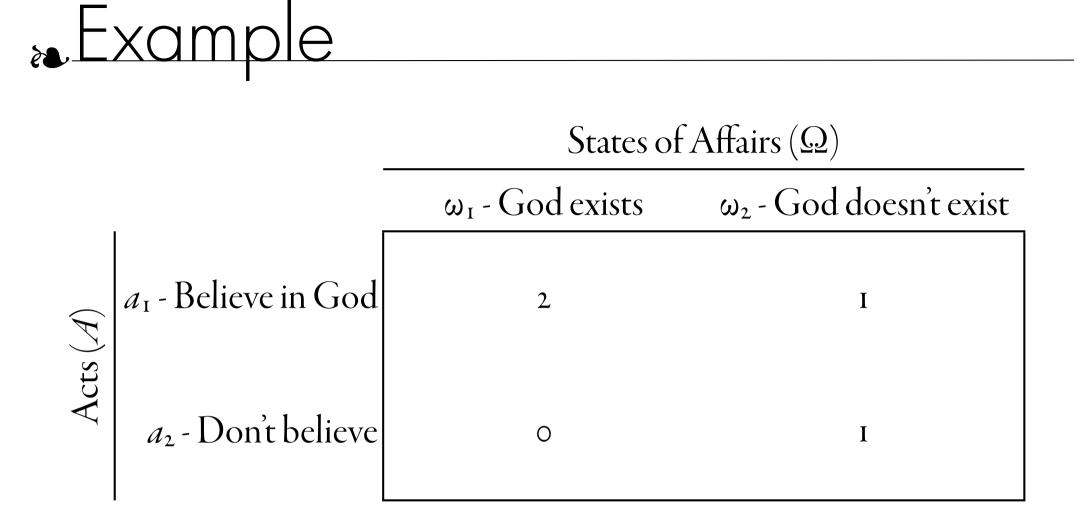
How does each security rule rank these actions?



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Optimism Rule

Maximax: $a_i \ge a_j$ if and only if $\operatorname{Max}_{k=1}^n [u(o_{i,k})] \ge \operatorname{Max}_{k=1}^n [u(o_{j,k})]$. $\operatorname{Max}_{k=1}^n [u(o_{i,k})]$ represents the highest possible utility value that a_i might return.



How does the optimism rule rank these actions?

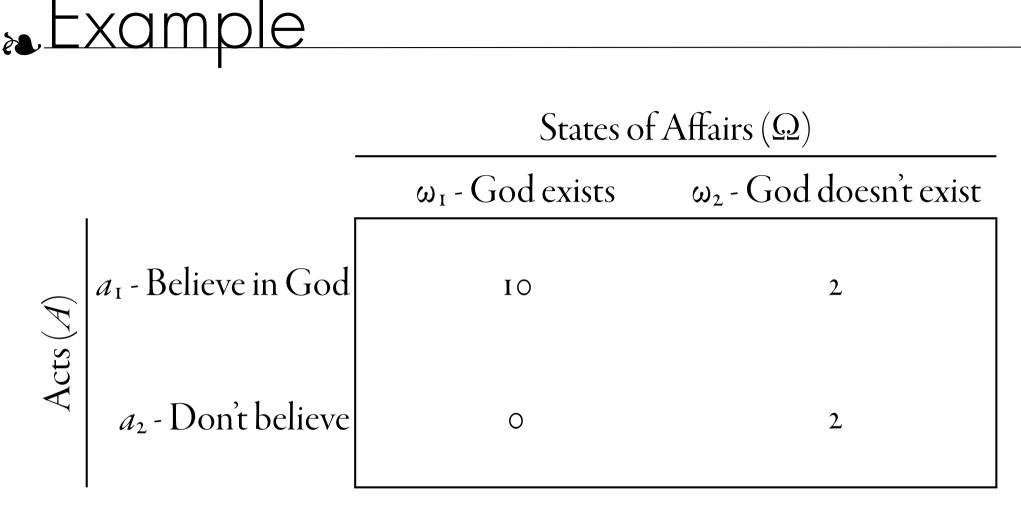
Optimism-Pessimism Rule

Optimism-Pessimism: $a_i > a_j$ if and only if $\alpha \times \operatorname{Max}_{k=1}^n [u(o_{i,k})] + (1 - \alpha) \times \operatorname{Min}_{k=1}^n [u(o_{i,k})] >$ $\alpha \times \operatorname{Max}_{k=1}^n [u(o_{j,k})] + (1 - \alpha) \times \operatorname{Min}_{k=1}^n [u(o_{j,k})].$

 α is the optimism-pessimism index, where

 $\alpha = 1$ means complete *optimism* (just focus on best-case with maximax), and

 α = 0 means complete *pessimism* (just focus on worst-case with maximin).



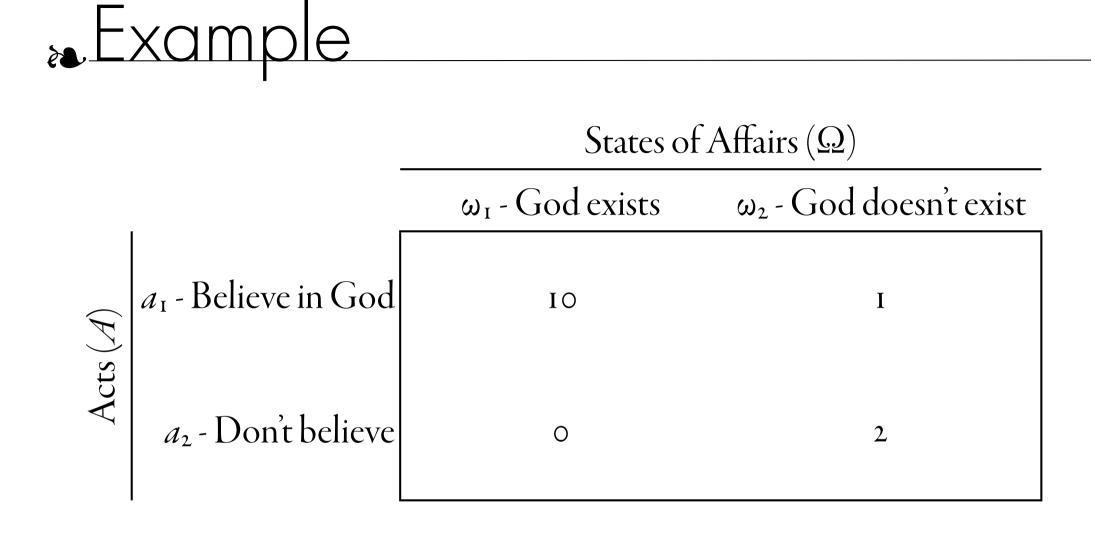
Suppose $\alpha = 0.5$. How does the optimism-pessimism rule rank these actions?

(Notice: We have introduced interval information. Why?)

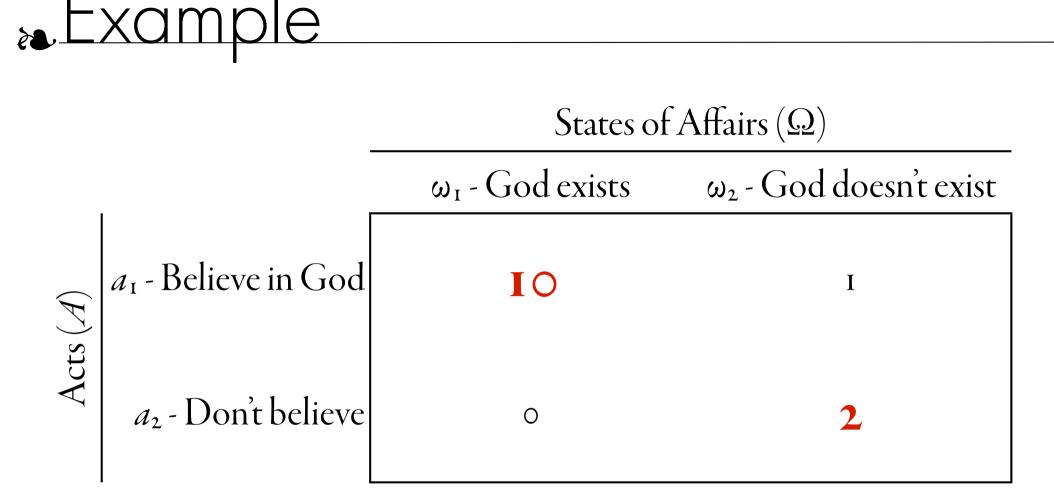
Minimax Regret Rule

 $\begin{aligned} \text{Minimax Regret: } a_i > a_j \text{ if and only if} \\ & \text{Max} \{ u(o_{i,1}) - \text{Max}^n_{k=1} [u(o_{k,1})], \\ & u(o_{i,2}) - \text{Max}^n_{k=1} [u(o_{k,2})], \dots \} > \\ & \text{Max} \{ u(o_{j,1}) - \text{Max}^n_{k=1} [u(o_{k,1})], \\ & u(o_{j,2}) - \text{Max}^n_{k=1} [u(o_{k,2})], \dots \}. \end{aligned}$

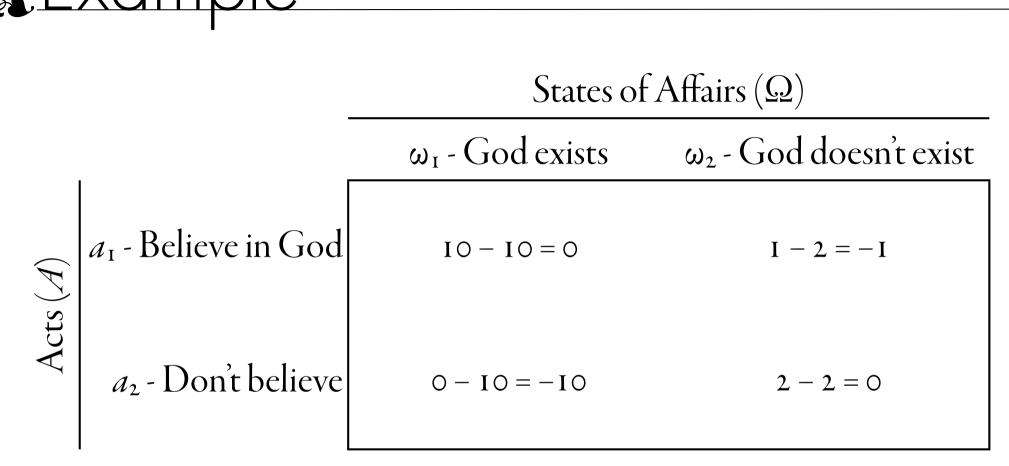
This is complicated. So let's break this down into steps.



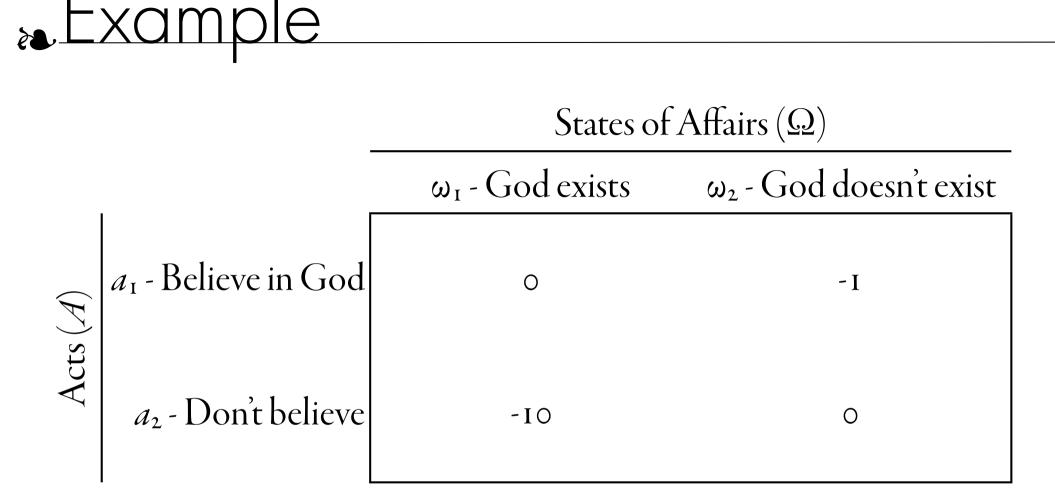
Start with the decision matrix, with all its utility info.



We convert this *decision* matrix into a *regret* matrix by **first** highlighting the best outcome for each state of affairs. This effectively computes $\operatorname{Max}_{k=1}^{n}[u(o_{k,x})]$ for each state of affairs ω_{x} .



We convert this *decision* matrix into a *regret* matrix by **second** subtracting the value of this best outcome from all the outcomes for that state of affairs. This effectively computes $Max_{k=1}^{n}[u(o_{k,x})]$ for each cell in the decision matrix.



Now this is the regret matrix (*not* the original decision matrix). We then perform maximin on this matrix. For this example, which option is now better according to minimax regret?

The Problem of Plurality

A major concern with choice under ignorance is that there is little consensus on which rule is the correct one for rational choice. Each rule has its own benefits and burdens. How would you choose between them?



We will discuss one last rule for choice under ignorance, and then we shall discuss principles for assessing choice under ignorance, and then see which of these rules satisfies these principles.