Rational Choice

Rules for Choice Under Ignorance

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The Decision Matrix

States of Affairs (Ω)

Acts (A)	ω_1	ω_2	$\ldots$	ω_j	$\ldots$	ω_n
a_1	$o_{1,1}$	$o_{1,2}$		$o_{1,j}$		$o_{1,n}$
a_2	$o_{2,1}$	$o_{2,2}$		$o_{2,j}$		$o_{2,n}$
$\ldots$						
a_i	$o_{i,1}$	$o_{i,2}$		$o_{i,j}$		$o_{i,n}$
$\ldots$						
a_m	$o_{m,1}$	$o_{m,2}$		$o_{m,j}$		$o_{m,n}$

Choice Under Ignorance

In choice under ignorance, the following all hold:

1. There are different outcomes for different states of affairs relevant to the decision,

2. For each combination of action and state of affairs, you *do* know the outcome, and

3. You *do not* know how likely (i.e., how probable) each state of affairs is.

Example: Pascal's Initial Choice

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts ($\mathcal{A}$)	a_1 - Believe in God	Heaven	Nothing gained, nothing lost
	a_2 - Don't believe	Hell	Nothing gained, nothing lost

Example

Acts (A)	States of Affairs (Ω): ω_1 - God exists	ω_2 - God doesn't exist
a_1 - Believe in God	2	1
a_2 - Don't believe	0	1

Dominance Rules

Weak dominance: $a_i \succcurlyeq a_j$ if and only if for every state of affairs $\omega_x \in \Omega$, $u(o_{i,x}) \geq u(o_{j,x})$.

Strict dominance: $a_i \succ a_j$ if and only if the following both hold:

1. For every state of affairs $\omega_x \in \Omega$, $u(o_{i,x}) \geq u(o_{j,x})$.
2. There is some state ω_y such that $u(o_{i,y}) > u(o_{j,y})$.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	2	1
	a_2 - Don't believe	0	1

How does each dominance rule rank these actions?

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	3	2
	a_2 - Don't believe	0	1

How does each dominance rule rank these actions?

Example

Acts (A)	States of Affairs (Ω): ω_1 - God exists	States of Affairs (Ω): ω_2 - God doesn't exist
a_1 - Believe in God	3	1
a_2 - Don't believe	0	2

What's the problem with the rules in this case?

Security Rules

Maximin: $a_i \succcurlyeq a_j$ if and only if $\text{Min}^n{}_{k=1}[u(o_{i,k})] \geq \text{Min}^n{}_{k=1}[u(o_{j,k})]$.

$\text{Min}^n{}_{k=1}[u(o_{i,k})]$ represents the lowest possible utility value that a_i might return.

Leximin: $a_i \succ a_j$ if and only if there is some positive integer q such that $q\text{-Min}^n{}_{k=1}[u(o_{i,k})] > q\text{-Min}^n{}_{k=1}[u(o_{j,k})]$ and for all integers $p < q$, $p\text{-Min}^n{}_{k=1}[u(o_{i,k})] = p\text{-Min}^n{}_{k=1}[u(o_{j,k})]$.

$q\text{-Min}^n{}_{k=1}[u(o_{i,k})]$ returns the q^{TH} lowest possible utility value that a_i might return.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	2	1
	a_2 - Don't believe	0	1

How does each security rule rank these actions?

Example

Acts (A)	States of Affairs (Ω): ω_1 - God exists	ω_2 - God doesn't exist
a_1 - Believe in God	3	1
a_2 - Don't believe	1	2

How does each security rule rank these actions?

Optimism Rule

Maximax: $a_i \succcurlyeq a_j$ if and only if $\mathrm{Max}^n{}_{k=1}[u(o_{i,k})] \geq \mathrm{Max}^n{}_{k=1}[u(o_{j,k})]$.

$\mathrm{Max}^n{}_{k=1}[u(o_{i,k})]$ represents the highest possible utility value that a_i might return.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	2	1
	a_2 - Don't believe	0	1

How does the optimism rule rank these actions?

Optimism-Pessimism Rule

Optimism-Pessimism: $a_i > a_j$ if and only if

$$\alpha \times \text{Max}^n{}_{k=1}[u(o_{i,k})] + (1 - \alpha) \times \text{Min}^n{}_{k=1}[u(o_{i,k})] >$$
$$\alpha \times \text{Max}^n{}_{k=1}[u(o_{j,k})] + (1 - \alpha) \times \text{Min}^n{}_{k=1}[u(o_{j,k})].$$

α is the optimism-pessimism index, where

$\alpha = 1$ means complete *optimism*
(just focus on best-case with maximax), and

$\alpha = 0$ means complete *pessimism*
(just focus on worst-case with maximin).

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts ($\mathcal{A}$)	a_1 - Believe in God	10	2
	a_2 - Don't believe	0	2

Suppose $\alpha = 0.5$. How does the optimism-pessimism rule rank these actions?

(Notice: We have introduced interval information. Why?)

Minimax Regret Rule

Minimax Regret: $a_i > a_j$ if and only if

$$\begin{aligned}&\mathrm{Max}\{u(o_{i,1}) - \mathrm{Max}^n{}_{k=1}[u(o_{k,1})],\\ &\qquad u(o_{i,2}) - \mathrm{Max}^n{}_{k=1}[u(o_{k,2})], \ldots\} >\\ &\mathrm{Max}\{u(o_{j,1}) - \mathrm{Max}^n{}_{k=1}[u(o_{k,1})],\\ &\qquad u(o_{j,2}) - \mathrm{Max}^n{}_{k=1}[u(o_{k,2})], \ldots\}.\end{aligned}$$

This is complicated. So let's break this down into steps.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	10	1
	a_2 - Don't believe	0	2

Start with the decision matrix, with all its utility info.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	**10**	1
	a_2 - Don't believe	0	**2**

We convert this *decision* matrix into a *regret* matrix by **first** highlighting the best outcome for each state of affairs. This effectively computes $\text{Max}^{n}_{k=1}[u(o_{k,x})]$ for each state of affairs ω_x.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	$10 - 10 = 0$	$1 - 2 = -1$
	a_2 - Don't believe	$0 - 10 = -10$	$2 - 2 = 0$

We convert this *decision* matrix into a *regret* matrix by **second** subtracting the value of this best outcome from all the outcomes for that state of affairs. This effectively computes $\text{Max}^{n}{}_{k=1}[u(o_{k,x})]$ for each cell in the decision matrix.

Example

		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	0	-1
	a_2 - Don't believe	-10	0

Now this is the regret matrix (*not* the original decision matrix). We then perform maximin on this matrix. For this example, which option is now better according to minimax regret?

The Problem of Plurality

A major concern with choice under ignorance is that there is little consensus on which rule is the correct one for rational choice. Each rule has its own benefits and burdens. How would you choose between them?

Next Class…

We will discuss one last rule for choice under ignorance, and then we shall discuss principles for assessing choice under ignorance, and then see which of these rules satisfies these principles.