Rational Choice

Multi-Attribute Decision Making

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Choice Under Certainty

States of Affairs (Ω)

Acts ($\mathcal{A}$)

	ω_1
a_1	o_1
a_2	o_2
$\dots$	
a_i	o_i
$\dots$	
a_m	o_m

Notice that choosing an *action* in this situation is identical with choosing an *outcome*. That is, choosing act a_i is equivalent to choosing outcome o_i.

Multi-Attribute Decision Making

Utility Functions (U)

Outcomes (O)

$$
\begin{array}{c|ccccccc}
 & u_1 & u_2 & \ldots & u_j & \ldots & u_l \\
\hline
o_1 & u_1(o_1) & u_2(o_1) & & u_j(o_1) & & u_l(o_1) \\
o_2 & u_1(o_2) & u_2(o_2) & & u_j(o_2) & & u_l(o_2) \\
\ldots & & & & & & \\
o_i & u_1(o_i) & u_2(o_i) & & u_j(o_i) & & u_l(o_i) \\
\ldots & & & & & & \\
o_m & u_1(o_m) & u_2(o_m) & & u_j(o_m) & & u_l(o_l)
\end{array}
$$

The Decision Matrix

States of Affairs (Ω)

$$
\text{Acts } (\mathcal{A}) \quad
\begin{array}{c}
\\ a_1 \\ a_2 \\ \ldots \\ a_i \\ \ldots \\ a_m
\end{array}
\begin{array}{cccccc}
\omega_1 & \omega_2 & \ldots & \omega_j & \ldots & \omega_n \\
\hline
o_{1,1} & o_{1,2} & & o_{1,j} & & o_{1,n} \\
o_{2,1} & o_{2,2} & & o_{2,j} & & o_{2,n} \\
 & & & & & \\
o_{i,1} & o_{i,2} & & o_{i,j} & & o_{i,n} \\
 & & & & & \\
o_{m,1} & o_{m,2} & & o_{m,j} & & o_{m,n}
\end{array}
$$

Multi-Attribute Decision Making

In multi-attribute decision making, there is not a single utility function ranking outcomes, but a set of several utility functions doing so. The challenge is that these functions may have different ranking.

The Challenge of Rational Choice

In the case of multi-attribute decision making, the challenge concerns how to use these separate utility functions to derive a rational method for arriving at a single set of judgments concerning the outcomes.

There are a variety of rules that have been devised for making decisions like these.

Weighted Averaging

Weighted Average: An "all-things-considered" utility function u may be constructed from a set of utility functions $U = \{u_1, u_2, \ldots, u_l\}$ by assigning non-negative weights $w_1, w_2, \ldots, w_l$ summing to one (i.e., $w_j \geq 0$ and $\sum_{j=1}^{l}[w_j] = 1$), and then using these weights to take calculate a weighed average for each outcome:

$$u(x) = \sum_{j=1}^{l}[w_j \times u_j(x)].$$

Example

Options (O)	Utility Functions (U): SAT	GPA
Huey	2400	3.20
Dewey	2040	4.00
Louie	2130	3.40

Example

Options (O)	Utility Functions (U): SAT	Utility Functions (U): GPA
Huey	2400	3.20
Dewey	2040	4.00
Louie	2130	3.40
Weight	$w_1 = 0.5$	$w_2 = 0.5$

Example

Options (O)	SAT	GPA	u
	Utility Functions (U)		
Huey	2400	3.20	
Dewey	2040	4.00	
Louie	2130	3.40	
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

Example

	Utility Functions (U)		
Options (O)	SAT	GPA	u
Huey	2400	3.20	1201.6
Dewey	2040	4.00	
Louie	2130	3.40	
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

$$u(\text{Huey}) = (0.5 \times 2400) + (0.5 \times 3.20) = 1201.6.$$

Example

Options (O)	SAT	GPA	u
Huey	2400	3.20	1201.6
Dewey	2040	4.00	1022
Louie	2130	3.40	
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

Utility Functions (U): SAT, GPA

$$u(\text{Dewey}) = (0.5 \times 2040) + (0.5 \times 4.00) = 1022.$$

Example

Options (O)	SAT	GPA	u
Huey	2400	3.20	1201.6
Dewey	2040	4.00	1022
Louie	2130	3.40	1066.7
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

Utility Functions (U): SAT, GPA

$$u(\text{Louie}) = (0.5 \times 2130) + (0.5 \times 3.40) = 1066.7.$$

Example

	Utility Functions (U)		
Options (O)	SAT	GPA	u
Huey	2400	3.20	1201.6
Dewey	2040	4.00	1022
Louie	2130	3.40	1066.7
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

What's the Problem?

Normalization

In order to do comparisons across different utility functions, normalization is often done. One approach to normalization works as follows:

$$\text{normalized value} = \frac{\text{value} - \text{min possible}}{\text{max possible} - \text{min possible}}$$

Example

In the case of Huey, Dewey, and Louie: GPA ranges from 0.00 to 4.00 and SAT ranges from 600 to 2400.

Example

Options (O)	Utility Functions (U): SAT	GPA
Huey	2400	3.20
Dewey	2040	4.00
Louie	2130	3.40

Example

	Utility Functions (U)		
Options (O)	SAT	GPA	u
Huey	1.00	0.80	0.90
Dewey	0.80	1.00	0.90
Louie	0.85	0.85	0.85
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

Accept either Huey and Dewey.

Problem 1

The type of normalization makes a big difference for weighted averaging. Without normalization, only admit Huey. With the normalization proposed here, then either Huey and Dewey may be admitted. However, other forms of normalization might remove Dewey as an option.

Problem 2

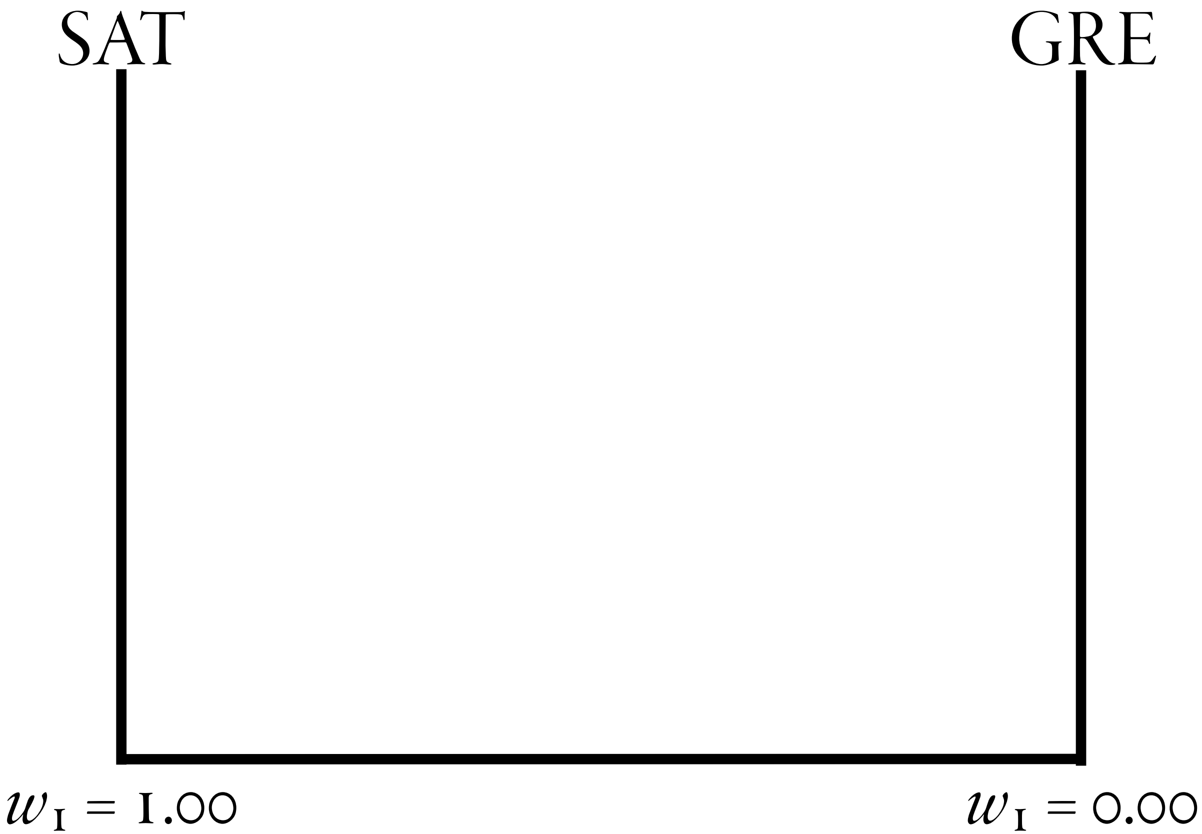
In addition, the precise choice of weights obviously makes a big difference for weighted averaging. In the example, putting more weight on SAT favors Huey, whereas putting more weight on GPA favors Dewey.

V-Admissibility

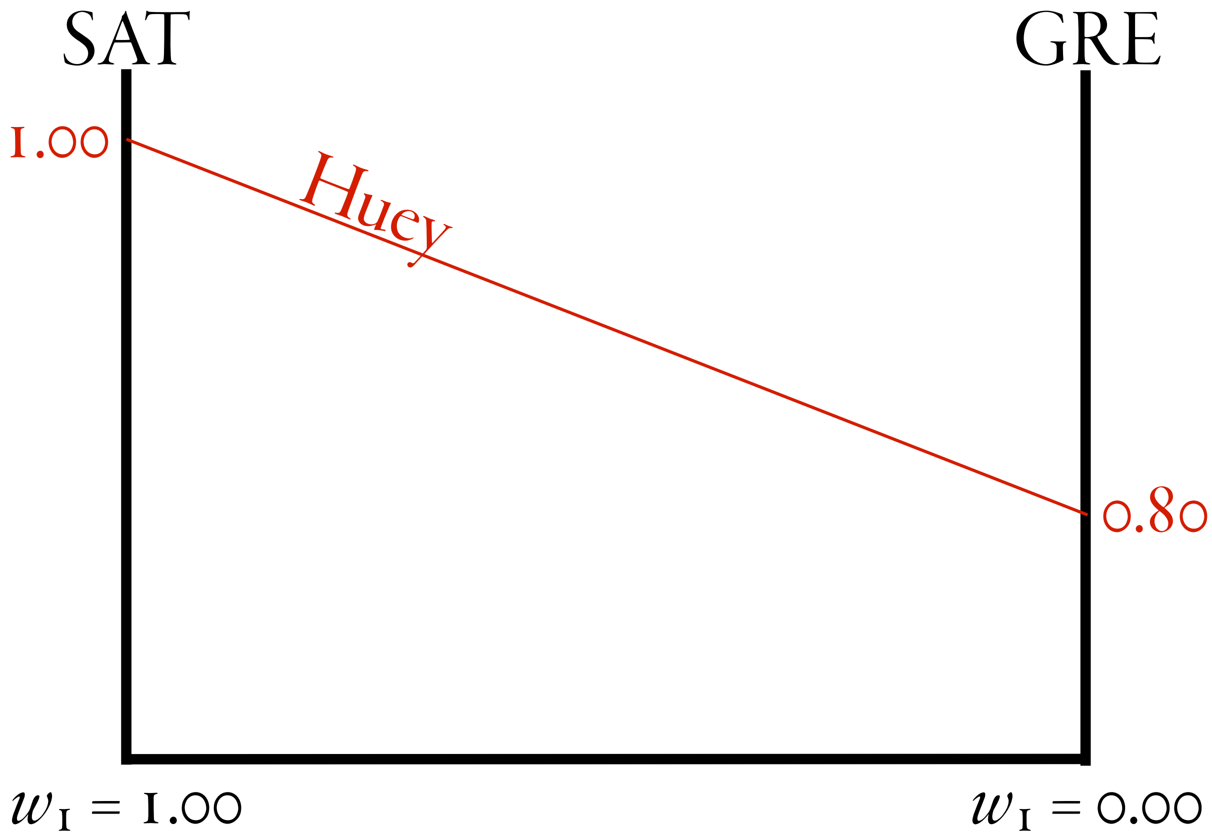
An outcome is **V-Admissible** if it is optimal according to at least one weighted average.

Example

Example



Example



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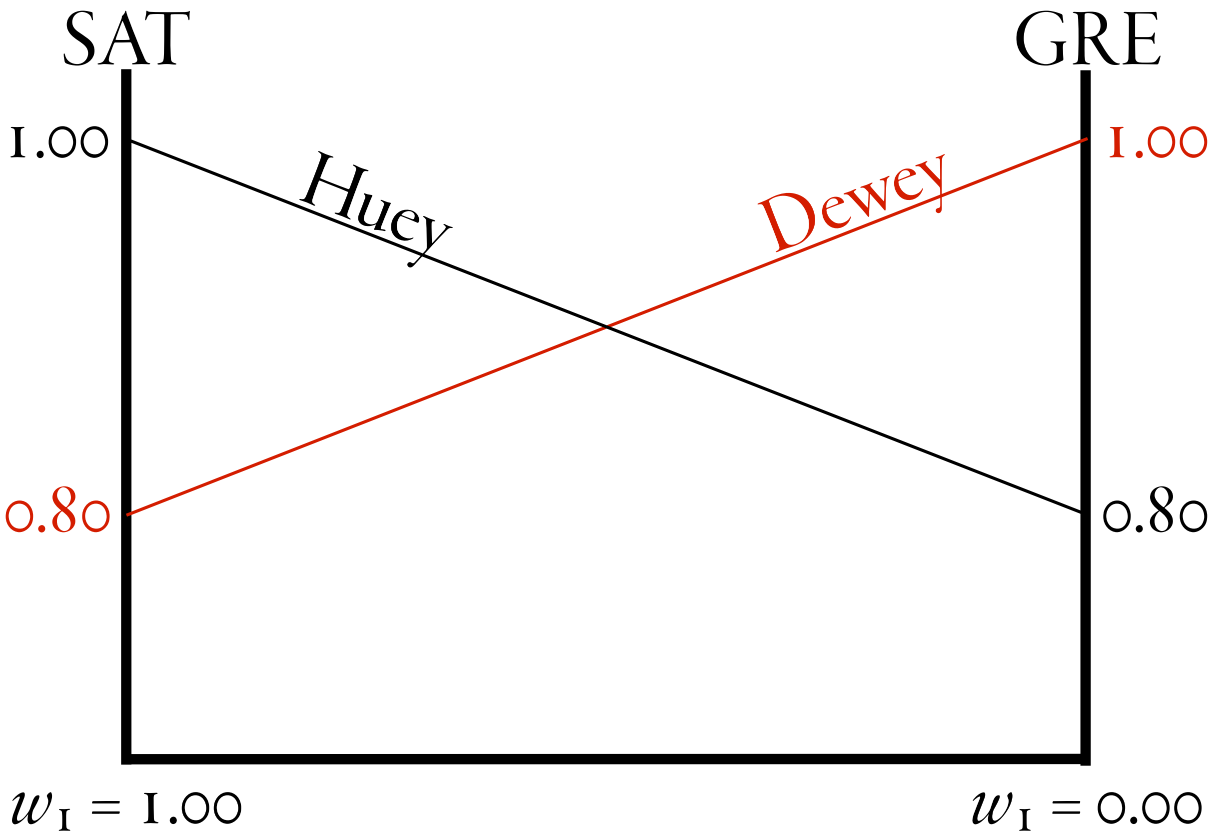

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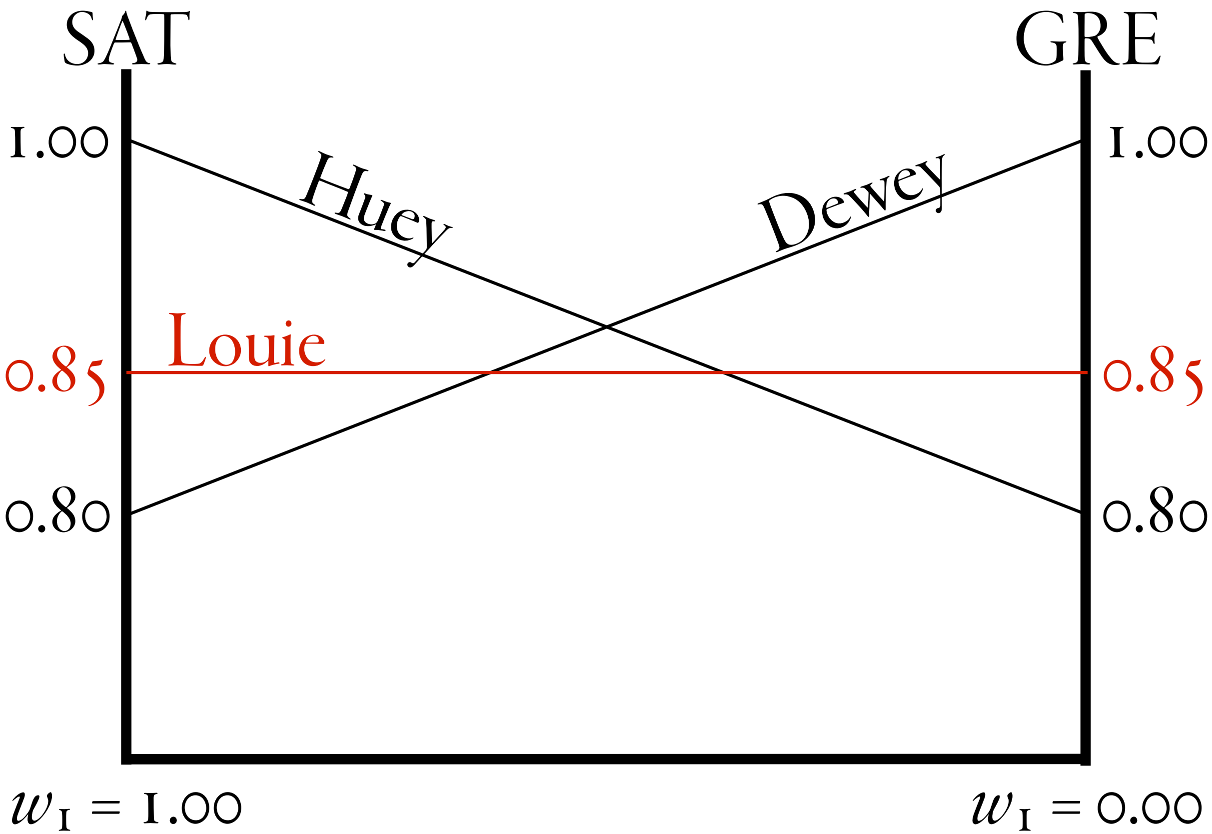

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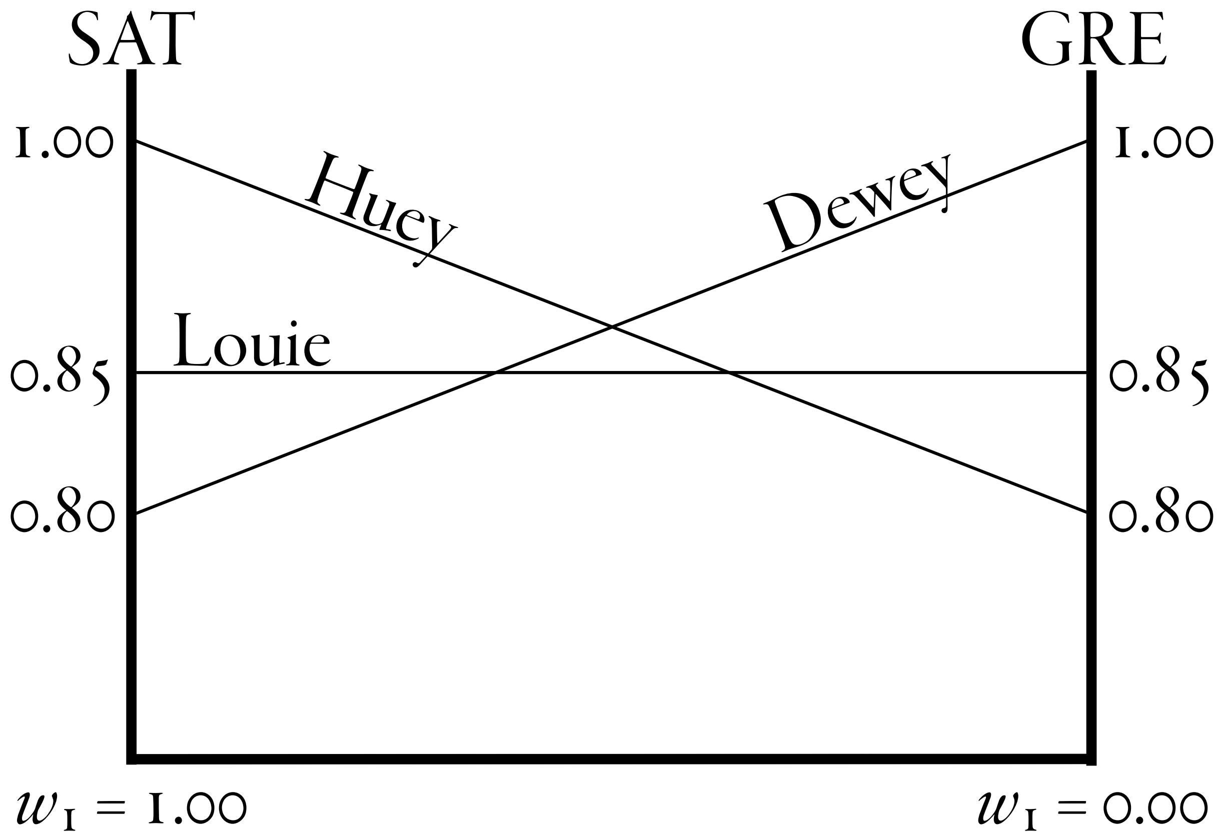

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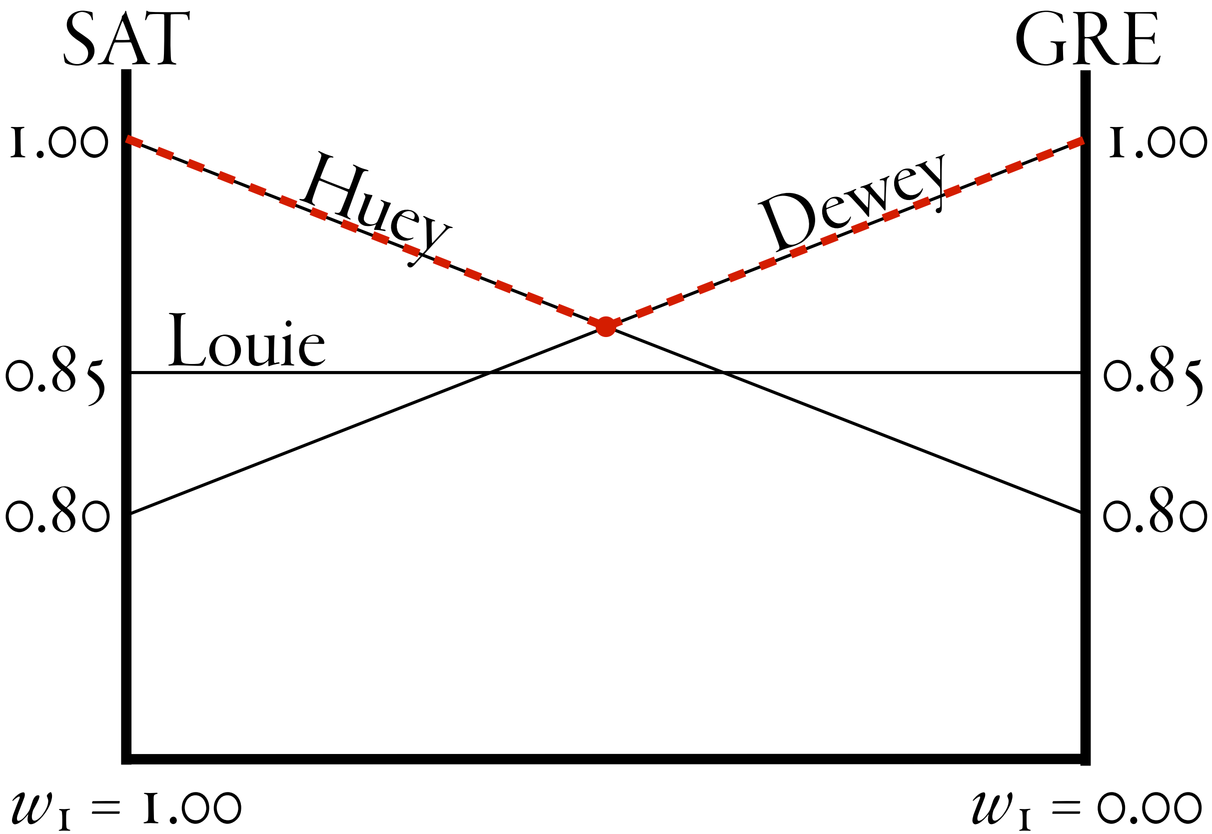

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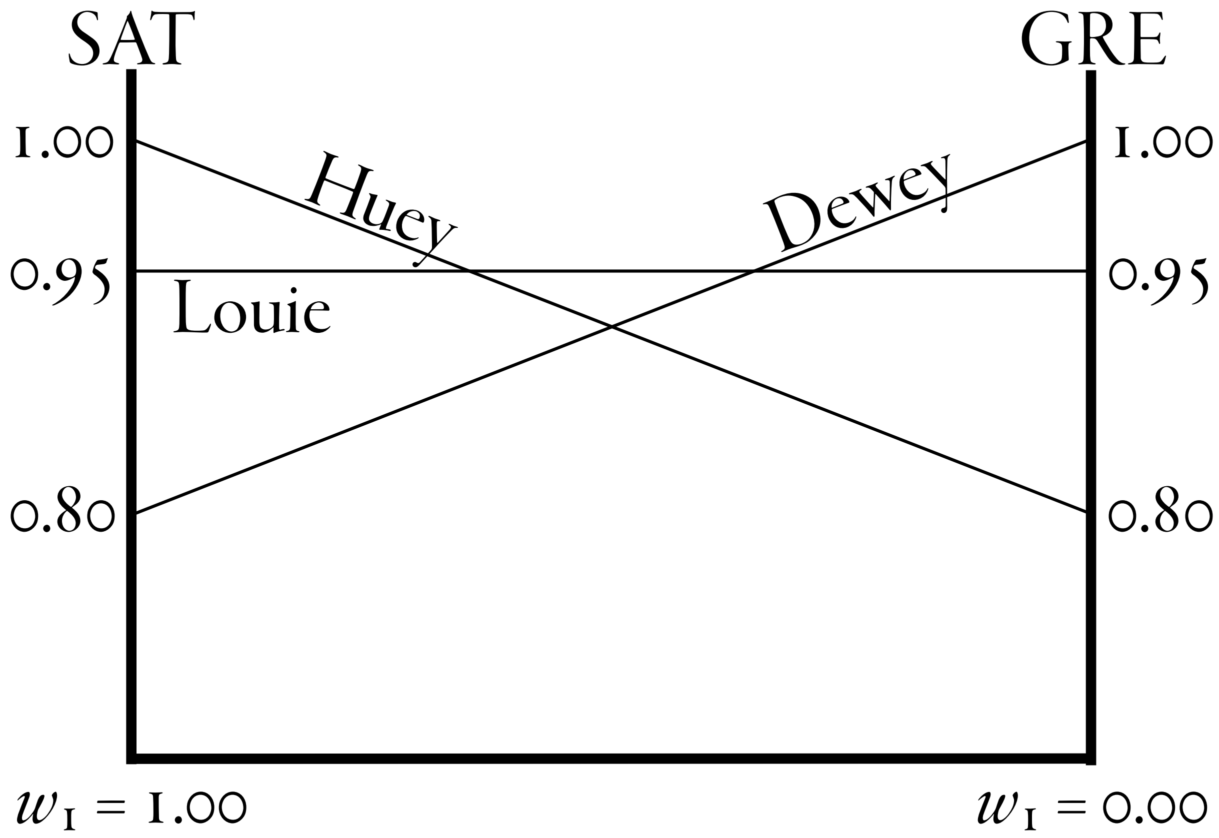
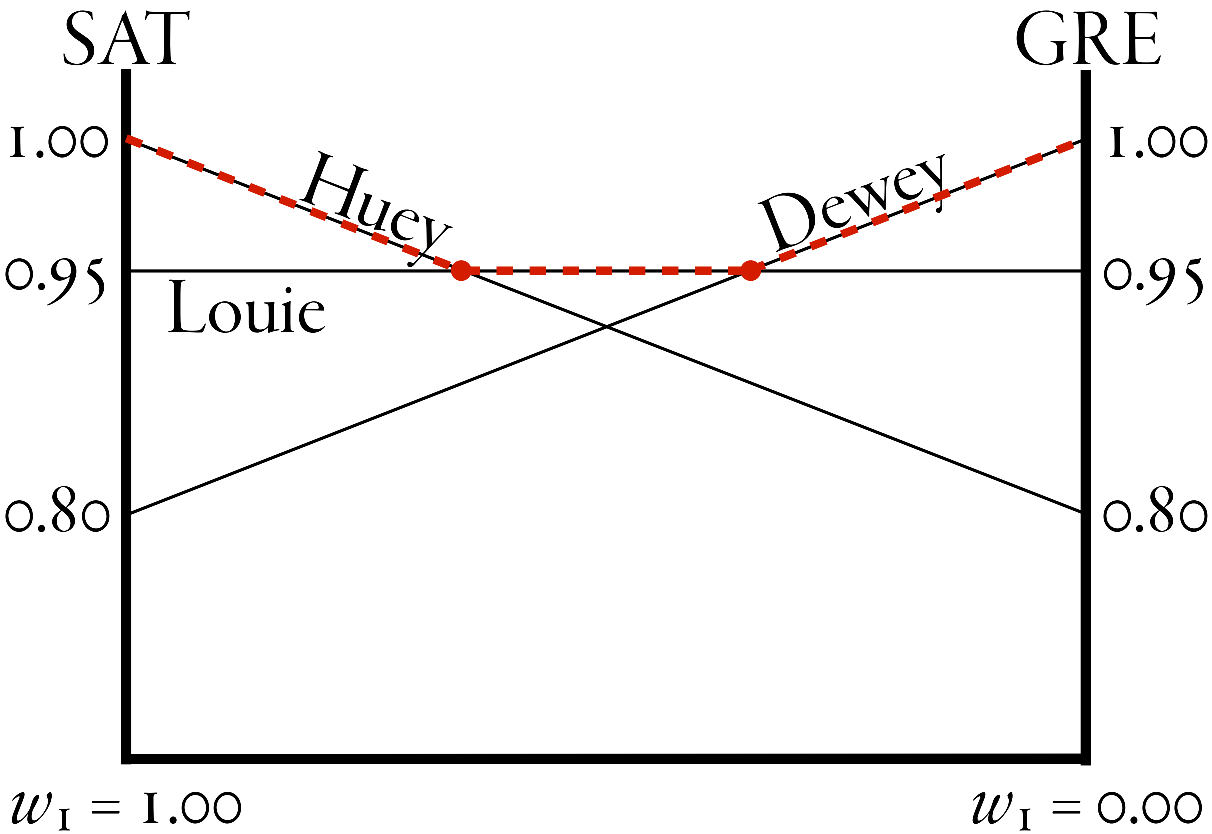


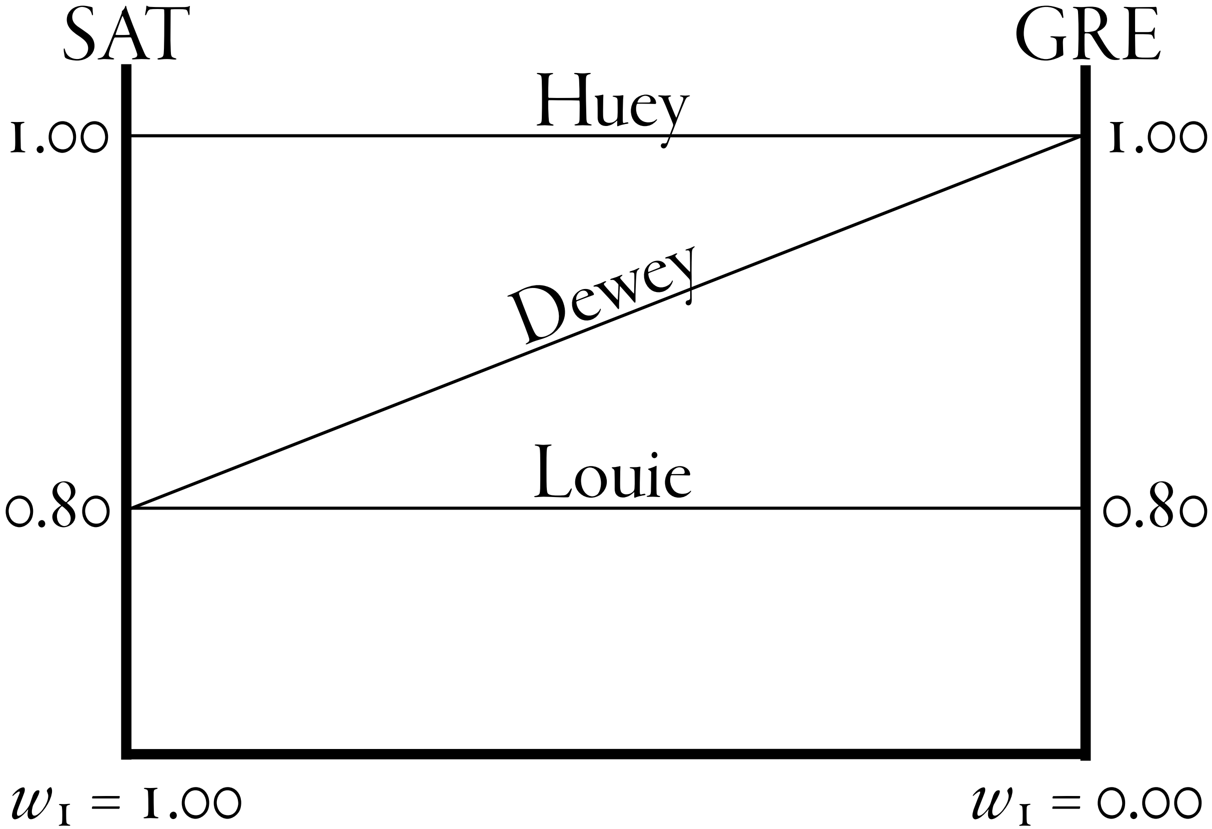
Example



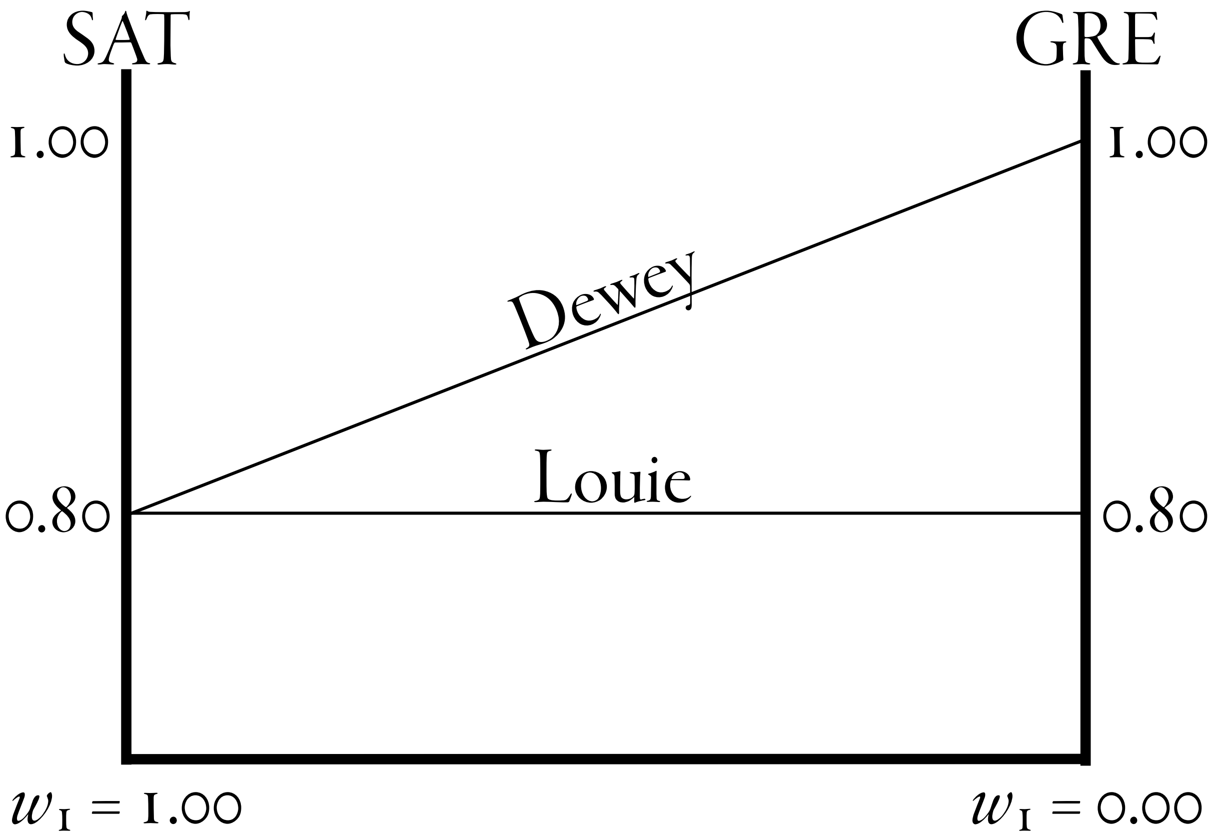
Problem

One concern with V-Admissibility is that is violates Sen's property beta (β).

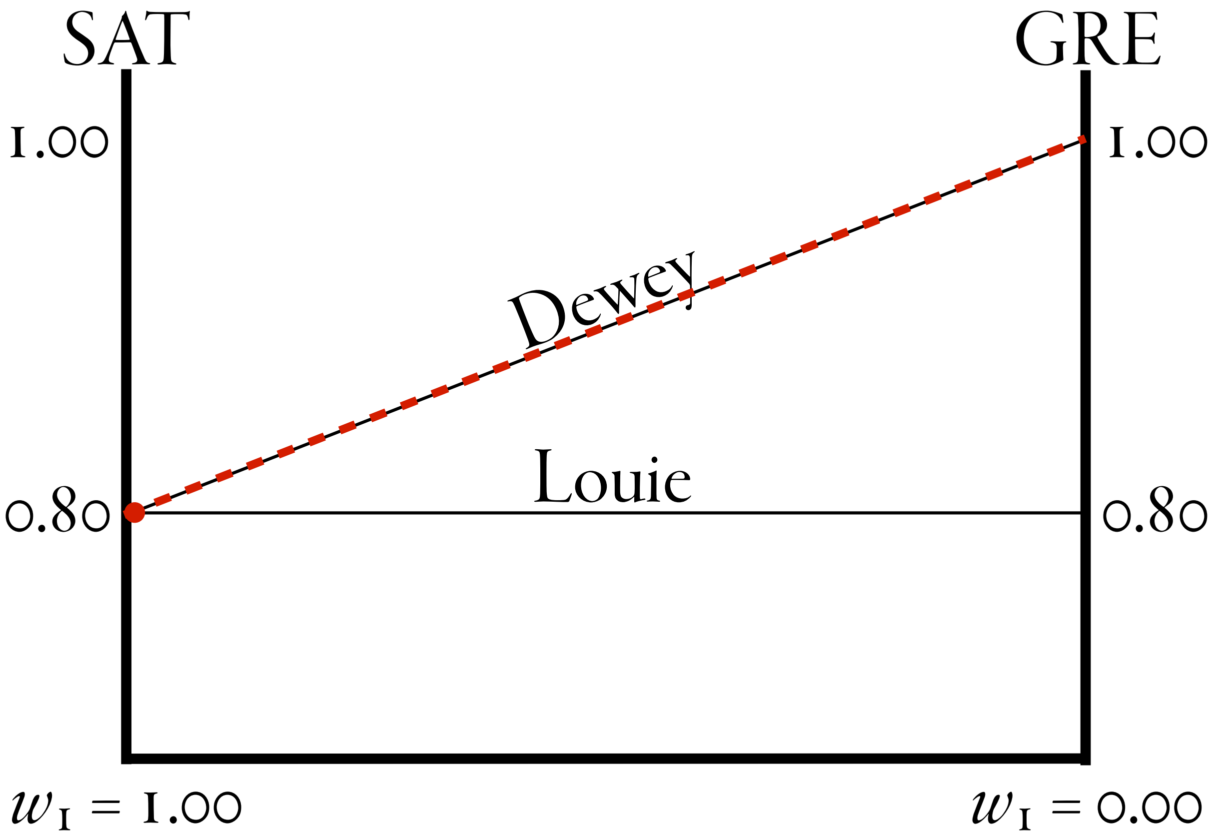
Example



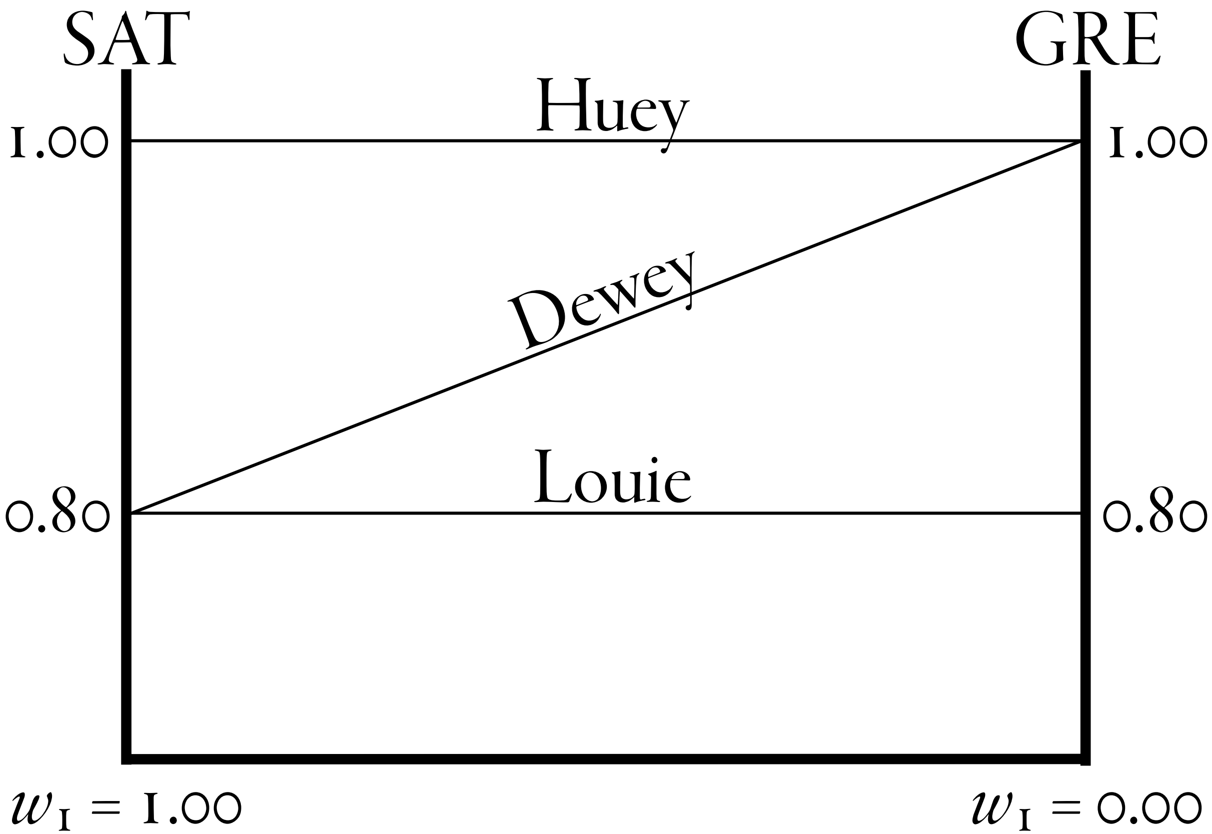
Example



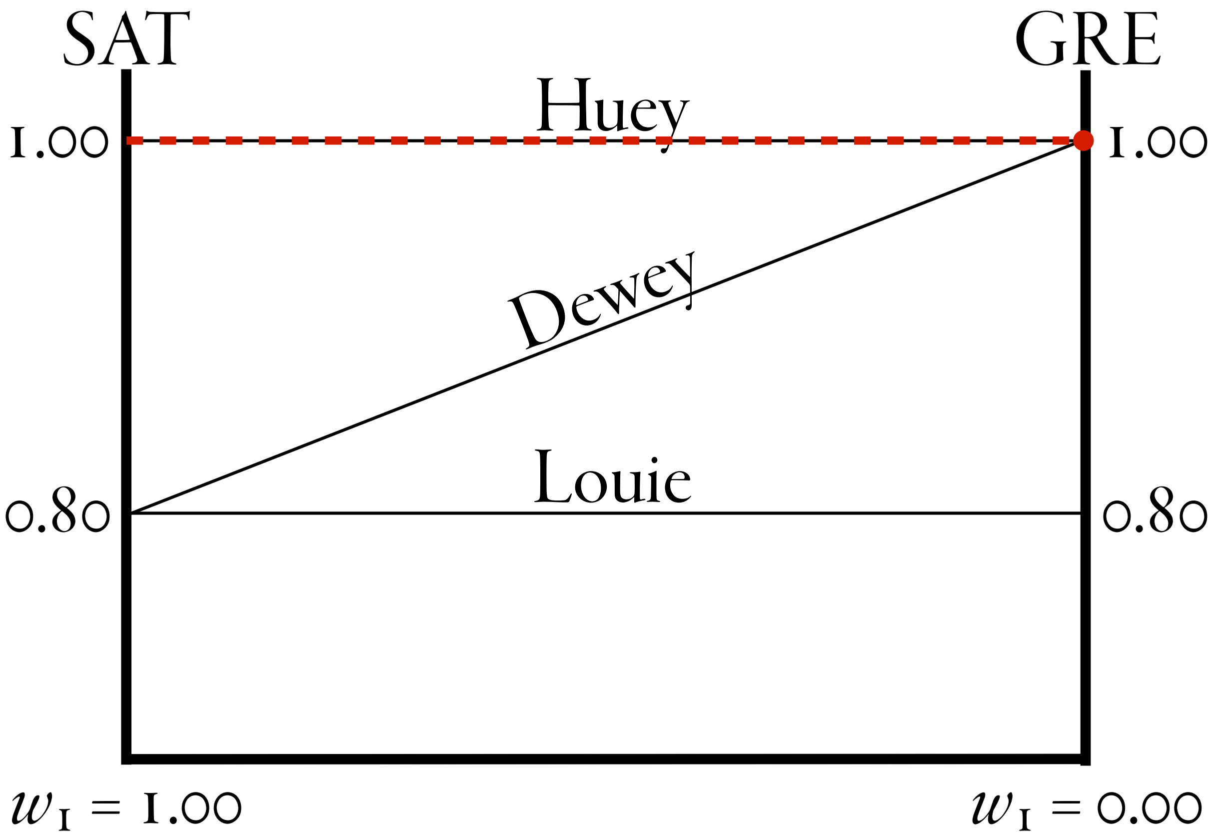
❧ Example



Example



Example



Security (Maximin)

Maximin: A "worse-case scenario" utility function u may be constructed from a set of utility functions $U = \{u_1, u_2, \ldots, u_l\}$ by ranking each outcome according to the lowest value it receives:

$$u(x) = \mathrm{Min}^{l}_{j=1}[u_j(x)].$$

Example

Options (O)	Utility Functions (U): SAT	Utility Functions (U): GPA	u
Huey	1.00	0.80	0.80
Dewey	0.80	1.00	0.80
Louie	0.85	0.85	0.85

Accept only Louie.

Optimism (Maximax)

Maximax: A "best-case scenario" utility function u may be constructed from a set of utility functions $U = \{u_1, u_2, \ldots, u_l\}$ by ranking each outcome according to the highest value it receives:

$$u(x) = \mathrm{Max}^{l}_{j=1}[u_j(x)].$$

Example

Options (O)	SAT	GPA	u
	Utility Functions (U)		
Huey	1.00	0.80	1.00
Dewey	0.80	1.00	1.00
Louie	0.85	0.85	0.85

Accept either Huey and Dewey.

Lexicography

According to a **lexicographic rule**, one utility function from *U* (the first-tier) is used to decide the issue. In the event of a tie, then use a second utility function from *U* (the second-tier) to break that tie. Repeat until either there is a single outcome left, or all utility functions from *U* have been exhausted.

Example

Options (O)	Utility Functions (U): SAT	Utility Functions (U): GPA
Huey	0.85	1.00
Dewey	0.80	1.00
Louie	0.95	0.95
Lexical Priority	Second-Tier	First-Tier

Accept only Huey.

Thresholds

According to a **threshold**, there a value that a utility function must exceed in order to be acceptable.

> **Conjunctive thresholds:** An outcome is acceptable if it passes *all* the thresholds.
>
> **Disjunctive thresholds:** An outcome is acceptable if it passes *at least one* threshold.

Example

Options (O)	Utility Functions (U): SAT	Utility Functions (U): GPA
Huey	1.00	0.80
Dewey	0.80	1.00
Louie	0.85	0.85
Threshold	0.85	0.85

If thresholds are *conjunctive*, then only accept Louie.

Example

Options (O)	SAT	GPA
	Utility Functions (U)	
Huey	1.00	0.80
Dewey	0.80	1.00
Louie	0.85	0.85
Threshold	0.90	0.90

If thresholds are *disjunctive*, then only accept Huey or Dewey.

Problem

A concern with thresholds is that they sometimes may judge that *nothing* is acceptable. I.e., nothing passes the requisite thresholds.

The Problem of Attribute Selection

One concern with multi-attribute decision making is that these rules tend to be extremely sensitive to the attributes chosen.

Example

Options (O)	Utility Functions (U): SAT	Utility Functions (U): GPA	u
Huey	1.00	0.80	0.90
Dewey	0.80	1.00	0.90
Louie	0.85	0.85	0.85
Weight	$w_1 = 0.5$	$w_2 = 0.5$	

Accept either Huey and Dewey.

Example

Options (O)	Utility Functions (U): SAT-R	SAT-M	SAT-W	GPA	u
Huey	1.00	1.00	1.00	0.80	0.95
Dewey	1.00	1.00	0.40	1.00	0.85
Louie	0.85	0.85	0.85	0.85	0.85
Weight	$w_1 = 0.25$	$w_2 = 0.25$	$w_3 = 0.25$	$w_4 = 0.25$	

Accept only Huey.

The Problem of Plurality

A major concern with multi-attribute decision making is that there is little consensus on which rule is the correct one for rational choice. Each rule has its own benefits and burdens. How would you choose between them?

Next Class…

We begin to discuss choice under ignorance. It shares some similarities to multi-attribute decision making, and so it may give us insights into which rule is the proper rule of rational choice in both contexts.