Rational Choice

Optimization and Maximization

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Choice Based on Preference

A **choice function** $C(\cdot)$ takes a non-empty subset S (the "menu") of a fixed universe $\mathcal{U}$ and returns a non-empty subset of S.

Optimization: $B(S, \succ) = \{x \in S \mid \text{for all } y \in S\colon x \succcurlyeq y\}$.

Revealed Preference

Weak axiom of revealed preference (WARP)*:
For all $x \in S$ and all $y \in S$, if $x \in C(S)$ but $y \notin C(S)$ then $x \succ y$.

Theorem: Choices obey Sen's rules alpha (α) and beta (β) if and only if WARP identifies (through the results of an exhaustive search of all choice situations) an underlying preference relation $\succ$.

Example

PARTY HOST *puts out a bowl with two mangos and two apples*

PROFESSOR GRAY [*to his* WIFE] Honey, please get me the first mango.

ECONOMIST *takes the second mango out of the bowl*

PROFESSOR GRAY [*quickly to his* WIFE] Oh wait! Now I want the first apple instead!

Professor Gray's WIFE *rolls her eyes in embarrassment*

Example

Jim finds himself in a small South American town. Tied up against the wall is a row of twenty Native American women and children. The captain in charge of the town tells Jim that these prisoners are to all be shot because their husbands protested the colonial government, and those men are now hiding in the forest. However, in celebration of Jim's arrival, Jim has the honor to kill just one of the Native Americans while the other nineteen are set free. Should Jim refuse, the executioner will kill them all. Jim refuses and all twenty prisoners are shot.

Revealed Preference

Amartya Sen believes that by focusing exclusively on a person's *choices* to reveal that person's preferences, the theory of revealed preference misses out on important information: the person's *intentions* behind making those particular choices.

As a result, what looks irrational according to WARP may actually be perfectly rational behavior.

Describing Outcomes

Culmination descriptions of outcomes: Describes what was chosen in very narrow terms. For instance, in terms of the resulting commodity vector.

Comprehensive descriptions of outcomes: Describes what was chosen, who chose it, what other choices were available, the relevant social norms involved in the each choice, and so on.

Describing Outcomes

Four explanations for a person's choices besides a simple preference for one thing over another:

1. Reputation and indirect effects,
2. Social commitment and moral imperatives,
3. Direct welfare effects, and
4. Conventional rule following.

These shift focus to the intensions of the person.

Incommensurability

$\succ$ is **complete** if and only if (for all x and y) either $x \succ y$, $y \succ x$, or $x \sim y$.

Sen is skeptical of rationality requiring completeness because sometimes judgments may be incomplete:

1. **Tentative Incompleteness:** The decision maker has not yet definitively compared two options.
2. **Assertive Incompleteness:** The decision maker explicitly denies that two options are comparable.

Incommensurability

Sen is therefore believes that incommensurability is perfectly rational in common situations.

x and y are **incommensurable** ($x <> y$) if and only if $x \not\succ y$, $y \not\succ x$, and $x \nsim y$.

Example

Consider the following options:

x = save a human life,

y = win QR 1,000,000, and

y^+ = win QR 1,000,001.

Suppose that I judge $x <> y$, $x <> y^+$, and $y^+ > y$.

Is this complete? Is it transitive? Is is acyclic?

What are the optimal options? I.e., $B(\{x, y, y^+\}, >) =$

Maximization

In order to accommodate incommensurability, Sen offers another form of choice:

Maximization: $M(S, \succ) = \{x \in S \mid \text{for all } y \in S\text{: } y \nsucc x\}$.

According to this, an option is maximal provided that there is no available option strictly better than it.

Example

Going back to the case of x, y, and y^+. What are the maximal options there? I.e., $M(\{x, y, y^+\}, \succ) =$

Maximization

Theorem (Sen's 5.2): $M(S, \succ) \neq \varnothing$ if and only if $\succ$ is acyclic.

So neither completeness nor transitivity is required for there to be maximal options. Compare this to the fact that if $\succ$ is complete then $B(S, \succ) \neq \varnothing$ if and only if $\succ$ is acyclic. Optimality is therefore quite dependent on completeness.

Maximization

Theorem (Sen's 5.1): $B(S, \succ) \subseteq M(S, \succ)$.

That is, an optimal option is always maximal, but a maximal option need not be optimal. When the two sets of options differ, exactly one of the following cases must hold:

1. $B(S, \succ) = \varnothing$ but $M(S, \succ) \neq \varnothing$, or
2. $\varnothing \subset B(S, \succ) \subset M(S, \succ)$.

Example

Case 1 of Theorem 5.1 is seen in the earlier example of x, y, and y^+.

Case 2 of Theorem 5.1 is seen with the following judgments: $a \sim b$, $b \sim c$, and $a <> c$.

Is this complete? Is this transitive? Is this acyclic?

What are the optimal options? I.e., $B(\{x, y, y^+\}, \succ) =$

What are the maximal ones? I.e., $M(\{x, y, y^+\}, \succ) =$

Maximization

Theorem (Sen's 5.3): $B(S, \succ) = M(S, \succ)$ if either of the following cases holds:

1. $\succ$ is complete, or

2. $\succcurlyeq$ is transitive and $B(S, \succ) \neq \varnothing$.

So the contrapositive of this theorem reveals the two conditions under which maximization and optimization will differ.

Example

Case 1 of theorem 5.3 is seen in the earlier example of x, y, and y^+.

Case 2 of theorem 5.3 is seen in the earlier example of a, b, and c.

Maximization

Sen's Theorem 5.4 reveals that all instances of maximization can be turned into optimization by treating incommensurability as indifference.

However, Sen's Theorem 5.5 shows that some instances of optimization cannot be turned into optimization. For instance, if there are no optimal options, then maximization cannot mimic that. In general, optimization may be restrictive in ways that it is impossible to represent with maximization.

Revealed Preference

Notice, however, that a maximizer's preferences cannot be revealed by their choices. The example of using maximization in the case of x, y, and y^+ does not reveal what WARP thinks it does. When the economist watches me choose between saving a life or winning a lot of money, he or she mistakenly believes I revealed a preference.

Next Class…

We go back to standard economic theory, looking at how to construct a numerical value function over outcomes, $v(\cdot)$ when your judgments in $\succ$ do satisfy the requirements of a preference relation.