Rational Choice

The Basic Rationality Postulates

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The Decision Matrix

States of Affairs (Ω)

$$
\text{Acts } (A) \quad
\begin{array}{c|cccccc|}
 & \omega_1 & \omega_2 & \dots & \omega_j & \dots & \omega_n \\
\hline
a_1 & o_{1,1} & o_{1,2} & & o_{1,j} & & o_{1,n} \\
a_2 & o_{2,1} & o_{2,2} & & o_{2,j} & & o_{2,n} \\
\dots & & & & & & \\
a_i & o_{i,1} & o_{i,2} & & o_{i,j} & & o_{i,n} \\
\dots & & & & & & \\
a_m & o_{m,1} & o_{m,2} & & o_{m,j} & & o_{m,n} \\
\hline
\end{array}
$$

Choice Under Certainty

Acts ($\mathcal{A}$)	States of Affairs (Ω): ω_1
a_1	o_1
a_2	o_2
$\dots$	
a_i	o_i
$\dots$	
a_m	o_m

Notice that choosing an *action* in this situation is identical with choosing an *outcome*. That is, choosing act a_i is equivalent to choosing outcome o_i.

Choice Under Certainty

Recall that the challenge of rational choice is to generate a ranking of *acts* given a ranking of *outcomes*.

Rational choice under certainty allows us to understand with more formal precision the conditions that our judgments must satisfy in order to possess the ordinal information necessary for making these types of choice.

The Preference Relation

Strict preference ("x is better than y"): $x \succ y$.

Weak preference ("x is at least as good as y"): $x \succcurlyeq y$.

This holds if and only if $x \not\succ y$ ("x is not better than y").

Indifference ("x and y are equally valuable"): $x \sim y$.

This holds if and only if $x \not\succ y$ and $y \not\succ x$ ("x is not better than y and y is not better than x").

The Preference Relation

Definition: $\succ$ is a **preference relation** if and only if $\succ$ is both asymmetric and negatively transitive.

$\succ$ is **asymmetric** if and only if (for all x and y) $x \succ y$ implies $y \nsucc x$ (i.e., "if x is better than y then y is not better than x").

$\succ$ is **negatively transitive** if and only if (for all x, y, and z) $x \nsucc y$ and $y \nsucc z$ together imply $x \nsucc z$ (i.e., "if x is not better than y and y is not better than z, then x is not better than z").

The Preference Relation

Theorem: $\succ$ is a **preference relation** if and only if $\succ$ is complete and its associated $\succcurlyeq$ is transitive.

$\succ$ is **complete** if and only if (for all x and y) either $x \succ y$, $y \succ x$, or $x \sim y$ (i.e., "either x is better than y, y is better than x, or they are equally valuable").

$\succcurlyeq$ is **transitive** if and only if (for all x, y, and z) $x \succcurlyeq y$ and $y \succcurlyeq z$ implies $x \succcurlyeq z$ ("if x is at least as good as y and y at least as good as z, then x is at least as good as z").

The Preference Relation

Version #1

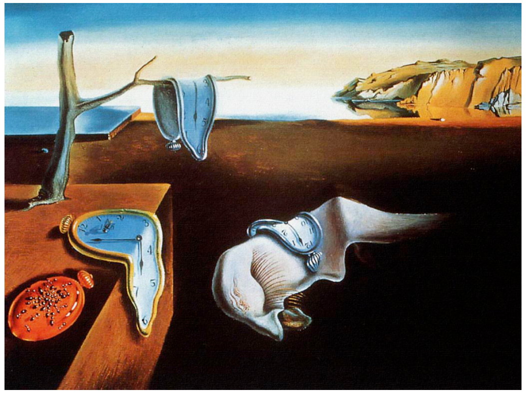
$\succ$ is asymmetric, and

$\succ$ is negatively transitive.

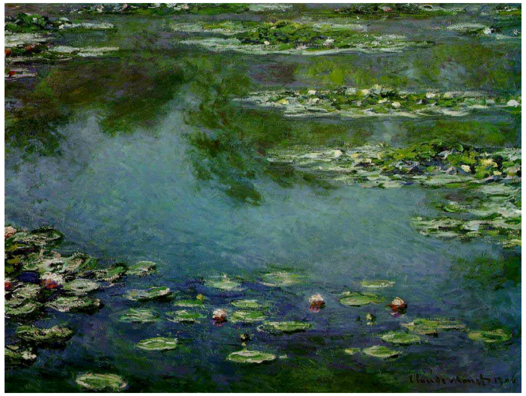
Version #2

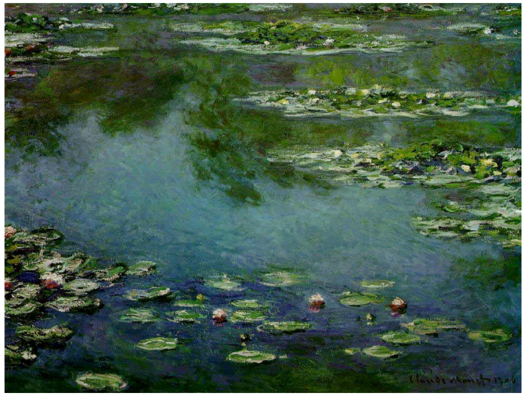
$\succ$ is complete, and

$\succcurlyeq$ is transitive.



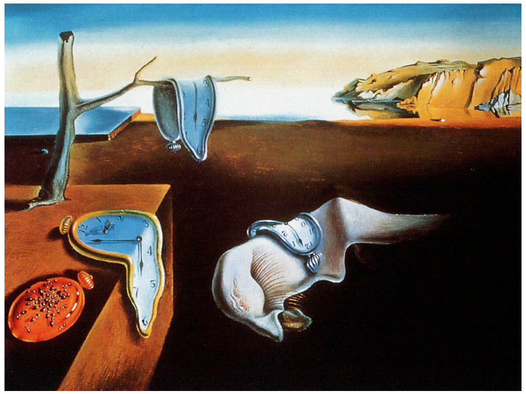
$\succcurlyeq$





$\succcurlyeq$

$\succcurlyeq$



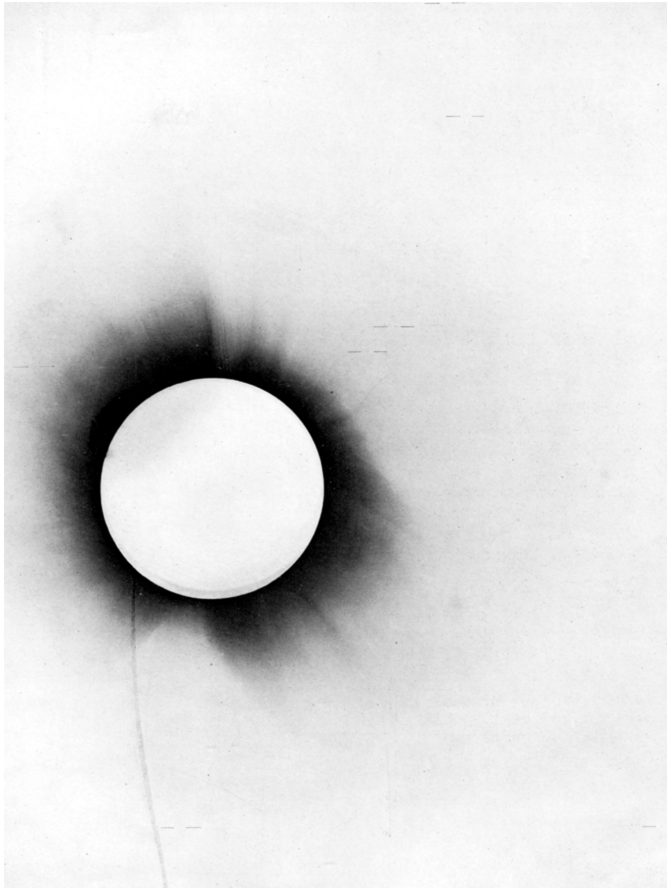
Transitivity

There are two common arguments in favor of transitivity.* The first is the **epistemic argument**, which claims that it is a basic conceptual truth that judgments must be transitive. This is because we all grasp the truth of this claim immediately.

*The clever student will notice how both arguments might also justify the asymmetry of $>$.

Transitivity

The problem with this epistemic argument is that it is extremely difficult to know what is and what is not a conceptual truth.



Transitivity

The second argument in favor of transitivity is a **pragmatic argument** that it is in a person's own self-interest to have transitive judgments. This is because intransitive preferences are susceptible to a "money pump" leading to certain loss.

Transitivity

One problem with this pragmatic argument is that a clever person would figure out what is going on with the money pump and stop swapping. An economist might respond, however, that it is unclear how or when the person should know to stop swapping.

Transitivity

A second problem with the money pump argument is that is does not demand transitivity but only that $>$ is acyclic.

> $>$ is **acyclic** if and only if $x_1 > x_2, x_2 > x_3, \ldots,$ and $x_{n\text{-}1} > x_n$ implies $x_1 \neq x_n$.

Transitivity

It turns out that if two options are incommensurable, then transitivity can be violated while acyclicity prevents a money pump.

> x and y are **incommensurable** ($x <> y$) if and only if $x \not> y$, $y \not> x$, and $x \not\sim y$.

For instance, suppose that $x > y$ and $y > z$ but x and z are incommensurable. $>$ is not transitivity but it is acyclic. So a money pump does not work here.

Transitivity

The typical economist would not be impressed by talk of incommensurability. The suspicion is that when someone says $x <> y$, they really mean $x \sim y$. This is because the economist challenges that person to explain how incommensurability is practically different (that is, when you make an actual choice) from indifference.

Completeness

The completeness axiom is a complete rejection of the possibility of incommensurable options.

One defense of this might be to make another epistemic argument about completeness as a conceptual truth, but it is not clear that is any more successful that it is when defending transitivity. Indeed, the possibility of incommensurable things may seem quite realistic to many people.

Completeness

A second, pragmatic argument argues that incommensurability leads to a money pump.

The problem with this argument is that it seems to assume that incommensurability is the same as indifference, and the "small improvements argument" attempts to show that these two things are different.

Small Improvements Argument

Consider the following options:

x = save a human life,

y = win QR 1,000,000, and

y^+ = win QR 1,000,001.

Suppose I say:

J_1: $x <> y$,

J_2: $x <> y^+$, and

J_3: $y^+ \succ y$.

The economist hears:

P_1: $x \not\succ y$ and $y \not\succ x$,

P_2: $x \not\succ y^+$ and $y^+ \not\succ x$, and

P_3: $y^+ \succ y$.

Small Improvements Argument

In addition, the economist would probably accept this weak form of transitivity:

P$_4$: $x \sim y$ and $y^+ \succ y$ imply that $y^+ \succ x$.

Now I can use the economist's own assumptions (P$_1$–P$_4$) to show that he or she must accept the existence of at least two incommensurable things.

Small Improvements Argument

C_1: $x \sim y$ and $y^+ \succ y$ cannot both be true, which follows from P_2 and P_4 (via *modus tollens*).

C_2: $x \nsim y$ or $y^+ \nsucc y$, which follows immediately from C_1 (by De Morgan's theorem).

C_3: $x \nsim y$, which follows from C_2 and P_3 (by elimination or the disjunctive syllogism).

C_4: $x \nsucc y$ and $y \nsucc x$ and $x \nsim y$ by putting together P_1 and C_3.

C_5: $x \prec\succ y$, following from C_4.

Completeness

"Fine!" says the economist, "I'll let you talk all you want about 'incommensurable' outcomes. But remember, I am watching the decisions you make. Someday, when you have to choose between money and saving a human life, I'll be paying attention. And whatever you choose, I will then know what your real preference is. You cannot hide behind the language of incommensurability forever, MWAHAHAHA . . . !"

Next Class…

The “rebel” economist Amartya Sen argues that revealed preference may not always reveal what the typical economist thinks it reveals.