Rational Choice

Choice Functions and Revealed Preference

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The Decision Matrix

States of Affairs (Ω)

Acts ($\mathcal{A}$)

	ω_1	ω_2	$\ldots$	ω_j	$\ldots$	ω_n
a_1	$o_{1,1}$	$o_{1,2}$		$o_{1,j}$		$o_{1,n}$
a_2	$o_{2,1}$	$o_{2,2}$		$o_{2,j}$		$o_{2,n}$
$\ldots$						
a_i	$o_{i,1}$	$o_{i,2}$		$o_{i,j}$		$o_{i,n}$
$\ldots$						
a_m	$o_{m,1}$	$o_{m,2}$		$o_{m,j}$		$o_{m,n}$

Choice Under Certainty

Acts (A)		States of Affairs (Ω): ω_1
	a_1	o_1
	a_2	o_2
	…	
	a_i	o_i
	…	
	a_m	o_m

Choice Under Certainty

States of Affairs (Ω)

Acts (A)	ω_1
a_1	o_1
a_2	o_2
$\ldots$	
a_i	o_i
$\ldots$	
a_m	o_m

Notice that choosing an *action* in this situation is identical with choosing an *outcome*. That is, choosing act a_i is equivalent to choosing outcome o_i.

Choice Under Certainty

Recall that the challenge of rational choice is to generate a ranking of *acts* given a ranking of *outcomes*.

Rational choice under certainty is therefore "easy" insofar as the best act to choose will simply be the one leading to the best outcome. Even so, it is also interesting because it reveals a lot about what goes into a ranking of the outcomes.

The Preference Relation

Suppose there is a set of acts A. Evaluation of these actions should generate a ranking of them. In particular, these evaluations should form a set of pairwise judgments like "a_1 is better than a_2", "a_5 is better than a_2", "a_1 is better than a_3", and so on.

The Preference Relation

Strict preference ("x is better than y"): $x \succ y$.

Weak preference ("x is at least as good as y"): $x \succcurlyeq y$.

This holds if and only if $x \nsucc y$ ("x is not better than y").

Indifference ("x and y are equally valuable"): $x \sim y$.

This holds if and only if $x \nsucc y$ and $y \nsucc x$ ("x is not better than y and y is not better than x").

The Preference Relation

Most economists assume that these judgments of acts making up $\succ$ are rational just when they come together in what is known as a "preference relation".

(Similarly, economists maintain that judgments of outcomes must also have this form.)

The Preference Relation

Definition: $\succ$ is a **preference relation** if and only if $\succ$ is both asymmetric and negatively transitive.

$\succ$ is **asymmetric** if and only if (for all x and y) $x \succ y$ implies $y \nsucc x$ (i.e., "if x is better than y then y is not better than x").

$\succ$ is **negatively transitive** if and only if (for all x, y, and z) $x \nsucc y$ and $y \nsucc z$ together imply $x \nsucc z$ (i.e., "if x is not better than y and y is not better than z, then x is not better than z").

The Preference Relation

Theorem: $\succ$ is a **preference relation** if and only if $\succ$ is complete and its associated $\succcurlyeq$ is transitive.

$\succ$ is **complete** if and only if (for all x and y) either $x \succ y$, $y \succ x$, or $x \sim y$ (i.e., "either x is better than y, y is better than x, or they are equally valuable").

$\succcurlyeq$ is **transitive** if and only if (for all x, y, and z) $x \succcurlyeq y$ and $y \succcurlyeq z$ implies $x \succcurlyeq z$ ("if x is at least as good as y and y at least as good as z, then x is at least as good as z").

Example

The Preference Relation

1. Veggie is better than lamb ($v \succ l$),
2. Veggie is better than chicken ($v \succ c$),
3. Veggie is better than beef ($v \succ b$),
4. Lamb is equally good as chicken ($l \sim c$),
5. Lamb is better than beef ($l \succ b$), and
6. Chicken is better than beef ($c \succ b$).

The Ranking

1ST	veggie (v)
2ND	lamb, chicken (l, c)
3RD	beef (b)

Choice Based on Preference

Definition: A **choice function** $C(\cdot)$ takes a non-empty subset S (the "menu") of a fixed universe $\mathcal{U}$ and returns a non-empty subset of S.

In particular, consider the universe as all possible acts in A. Call this $\mathcal{U}_A$, where $\mathcal{U}_A = A$. So $C(\cdot)$ is a choice function if and only if, for all sets $S \subseteq \mathcal{U}_A$, $S \neq \emptyset$ implies that $\emptyset \subset C(S) \subseteq S$.

Choice Based on Preference

Optimization: $B(S, \succ) = \{x \in S \mid \text{for all } y \in S\text{: } x \succcurlyeq y\}$.

An option on a menu S is optimal according to $\succ$ just when it is at least as good as everything else in S. More simply put: optimization picks the highest ranked option available on the menu.

Theorem: If $\succ$ is a preference relation then optimization is a choice function.

Example

Using the earlier example of $\succ$ over food …

Suppose $S_1 = \{b, c, l, v\}$. What is (are) the optimal option(s) of S_1?

$$B(S_1, \succ) = \{$$

Suppose $S_2 = \{c, l\}$. What is (are) the optimal option(s) of S_2?

$$B(S_2, \succ) = \{$$

Choice Based on Preference

According to standard economics, optimization is the rational way to go from preference judgments to making decisions. Economists also want to go the *other* direction by using the decisions people make to determine their underlying preference judgements.

Revealed Preference

The typical economist is often suspicious of what other people claim to value. So instead of *asking* someone for his or her preference judgments, the economist will just look at what that person *actually chooses* in various situations and use these decisions to determine the person's underlying judgments.

Definition: Weak axiom of revealed preference (WARP)[*]**:** For all $x \in S$ and all $y \in S$, if $x \in C(S)$ but $y \notin C(S)$ then $x \succ y$.

In other words, if x and y are on the menu and x is chosen while y is not, then this choice has revealed that x is better than y.

*N.B. Technically this is not the standard definition of WARP, but for our purposes it will suffice.

Example

Let b, c, and v represent the food choices from earlier and suppose $S = \{b, c, v\}$ and $C(S) = \{v\}$. What has been revealed about $\succ$ according to WARP?

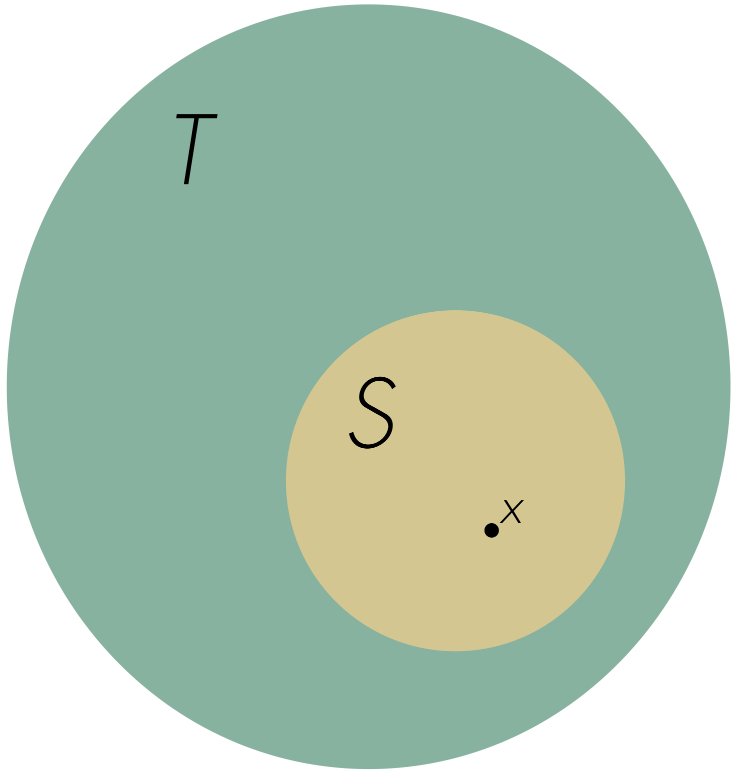
What remains unknown about $\succ$?

Revealed Preference

Theorem: Choices obey Sen's rules alpha (α) and beta (β) if and only if WARP identifies (through the results of an exhaustive search of all choice situations) an underlying preference relation $\succ$.

Definition: Sen's Property Alpha (α): If $x \in S \subseteq T$, and x is in $C(T)$, then x is also in $C(S)$.

In other words, you cannot turn a winner into a loser by taking away other options.



Example

PROFESSOR GRAY What shawarma do you have today?

BATEEL STAFF We have veggie, chicken, and lamb.

PROFESSOR GRAY I'll have the chicken shawarma.

BATEEL STAFF [*not hearing*] Oh, I forgot, we are out of lamb.

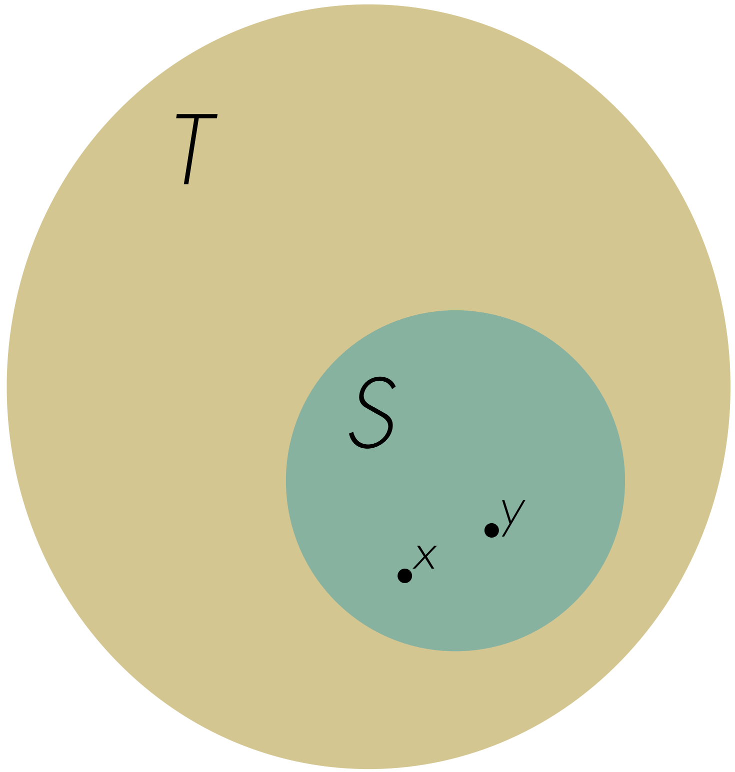
PROFESSOR GRAY In that case, I'll have the veggie.

Professor Gray's WIFE *rolls her eyes in embarrassment*

Revealed Preference

Definition: Sen's Property Beta (β)**:** If $x \in C(S)$ and $y \in C(S)$, $S \subseteq T$, and $y \in C(T)$, then $x \in C(T)$.

In other words, you cannot split winners by adding more options. Both options remain winners or both options become losers.



Example

Top Sellers:

Sony Bravia XBR HX929, 65 in.

QR 22,000

LG 65LW6500, 65 in.

QR 18,000

Example

Example

Sony Bravia XBR HX929, 65 in.

QR 22,000

Sony Bravia LED HX729, 65 in.

QR 17,000

LG 65LW6500, 65 in.

QR 18,000

Example

Sony Bravia XBR HX929, 65 in.

QR 22,000

Sony Bravia LED HX729, 65 in.

QR 17,000

LG 65LW6500, 65 in.

QR 18,000

Suddenly the high end Sony is the sole top seller!

Next Class…

We will consider whether it is really rational to have your evaluations of options satisfy the requirements of a preference relation.