Rational Choice

The Decision Matrix

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The Components of a Decision

Acts: The decision maker's options (set A).

$$A = \{a_1, a_2, \ldots, a_m\}.$$

States: The different ways the world might be (set Ω).

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}.$$

Outcomes: The possible consequences of your actions based on how the world turns out (set O).

$o_{i,j}$ is the result of choosing a_i when ω_j holds.

The Matrix (Normal Form)

States of Affairs (Ω)

Acts (A)

	ω_1	ω_2	$\dots$	ω_j	$\dots$	ω_n
a_1	$o_{1,1}$	$o_{1,2}$		$o_{1,j}$		$o_{1,n}$
a_2	$o_{2,1}$	$o_{2,2}$		$o_{2,j}$		$o_{2,n}$
$\dots$						
a_i	$o_{i,1}$	$o_{i,2}$		$o_{i,j}$		$o_{i,n}$
$\dots$						
a_m	$o_{m,1}$	$o_{m,2}$		$o_{m,j}$		$o_{m,n}$

The Challenge of Rational Choice

Suppose that you have a ranking of the *consequences*. How do you use this to generate a ranking of the *acts*?

Once you know how to rank the acts, then it is easy to know what to choose: pick the best, top-ranked action that is available! Generating that ranking of acts, however, may prove difficult.

Pascal's Application

What are the options (set A) that Pascal gives us?

What are the possible states of affairs (set Ω) that Pascal considers?

What are the possible consequences (set O)?

How do these come together in a decision matrix?

Pascal's Initial Choice

Acts (A)	States of Affairs (Ω): ω_1 - God exists	States of Affairs (Ω): ω_2 - God doesn't exist
a_1 - Believe in God	Heaven	Nothing gained, nothing lost
a_2 - Don't believe	Hell	Nothing gained, nothing lost

Pascal's Initial Choice

States of Affairs (Ω)

Acts (A)	ω_1 - God exists
a_1 - Believe in God	Heaven
a_2 - Don't believe	Hell

Pascal's Initial Choice

		States of Affairs (Ω)
		ω_2 - God doesn't exist
Acts ($\mathcal{A}$)	a_1 - Believe in God	Nothing gained, nothing lost
	a_2 - Don't believe	Nothing gained, nothing lost

Pascal's Wager

Pascal immediately recognizes a problem with this presentation of the decision:

> *That is wonderful. Yes, I must wager, but perhaps I am wagering too much.*

The point is that the previous decision matrix does not accurately represent the consequences.

Pascal's Wager

Acts (A)	States of Affairs (Ω): ω_1 - God exists	States of Affairs (Ω): ω_2 - God doesn't exist
a_1 - Believe in God	Heaven	Costs of belief
a_2 - Don't believe	Hell	Benefits of disbelief

Pascal’s Wager

States of Affairs (Ω)

		ω_1 - God exists
Acts ($\mathcal{A}$)	a_1 - Believe in God	Heaven
	a_2 - Don’t believe	Hell

Pascal's Wager

States of Affairs (Ω)

Acts (A)	ω_2 - God doesn't exist
a_1 - Believe in God	Costs of belief
a_2 - Don't believe	Benefits of disbelief

Concerns About the Wager

Are the rows an accurate representation of the possible acts (set A)?

Are the columns an accurate representation of the possible states (set Ω)?

Understanding the States

Unless told otherwise, we must always assume **act/state independence**: the state that actually occurs is not influenced of the act chosen.

In addition the states in Ω must be . . .

Mutually exhaustive: There are *no* other relevant states (outside of Ω) to consider.

Mutually exclusive: Only *one* state can occur.

Understanding the States

Do not to confuse events with states. An **event** is something that can happen in the world. A **state** is a set of events. In particular, the states in Ω must cover *all* the possible combinations of the relevant events influencing a decision.

Evaluating Consequences

The "challenge of rational choice" assumes that the decision maker has a ranking of the possible consequences. This can be made formally precise by using a **value function** $v\colon O \rightarrow \mathbf{R}$. In other words $v(o_{i,j})$ is a numerical representation of the "value" of outcome $o_{i,j}$.($\mathbf{R}$ is the set of real numbers.)

Pascal's Initial Choice

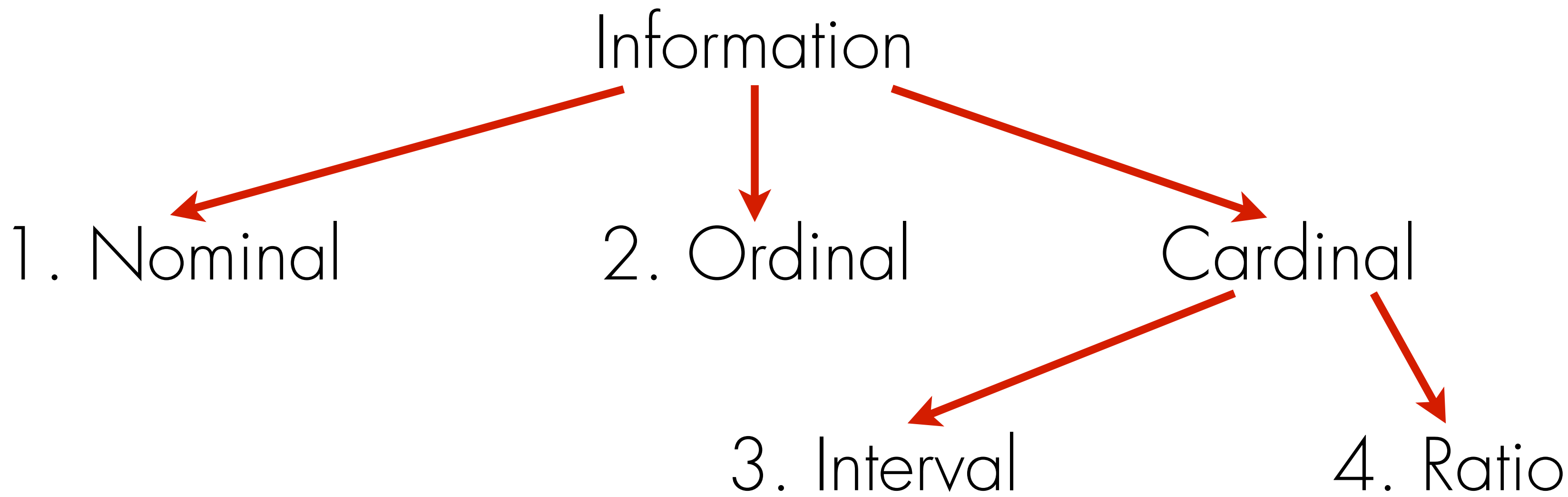
		States of Affairs (Ω)	
		ω_1 - God exists	ω_2 - God doesn't exist
Acts (A)	a_1 - Believe in God	10	0
	a_2 - Don't believe	-10	0

Pascal's Wager

Acts (A)	States of Affairs (Ω): ω_1 - God exists	States of Affairs (Ω): ω_2 - God doesn't exist
a_1 - Believe in God	10	-2
a_2 - Don't believe	-10	1

Evaluating Consequences

This leads to questions about measurement: what do the numbers returned by a value function actually mean? What information do the numbers reveal? How are the numbers on an evaluative scale related to each other?

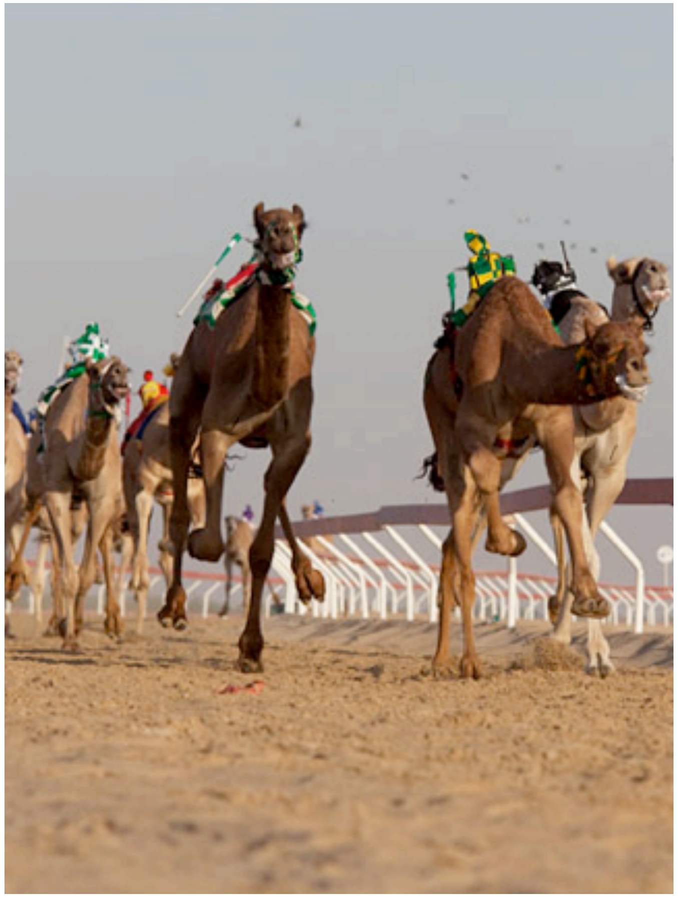


Evaluating Consequences

According to a **nominal** scale, numbers only reveal that things are different, but they are otherwise essentially arbitrary in terms of making comparisons.

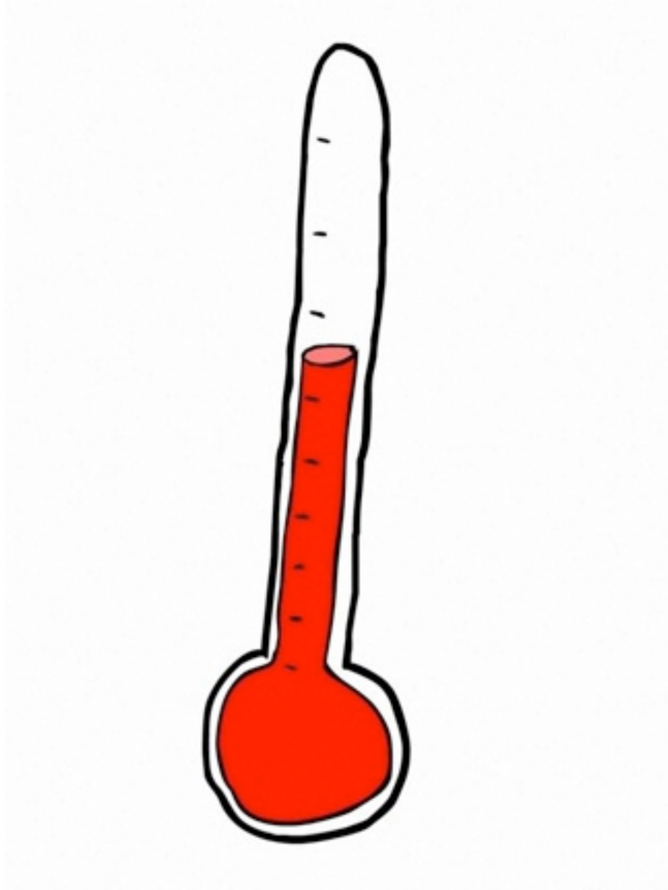
Evaluating Consequences

According to an **ordinal** scale, numbers do allow for comparisons, but they do not say anything about the relative distance between them.



Evaluating Consequences

According to an **interval** scale, not only are numbers ordered but also the distances (intervals) between the numbers are meaningful.



Evaluating Consequences

According to a **ratio** scale, numbers are ordered and intervals are meaningful. In addition, there is a true "zero" point (i.e., 0) meaing the absence of the quantity.

Examples

Which type of scale best describes each of these?

The elimination order of Arab Idol.

Centimeters (as measured by a ruler).

The amount of Qatar riyals in my wallet.

The chapter numbers in a book.

Mathematic Equivalences

Scales v and v', are equivalent according to each scale when …

Nominal scale: $v(x) = v(y)$ if and only if $v'(x) = v'(y)$.

Ordinal scale: $v(x) \geq v(y)$ if and only if $v'(x) \geq v'(y)$ (i.e., equivalent under positive monotone transformations).

Interval scale: $v(x) = \alpha \times v'(x) + \beta$, where $\alpha > 0$ (i.e., equivalent under positive affine transformations).

Ratio scale: $v(x) = \alpha \times v'(x)$, where $\alpha > 0$ (i.e., equivalent under positive linear transformations).

Study Tip

The end of each chapter in the textbook has exercises and solutions. Practice on them. See the TAs or me if you are having problems or confusions with them.

Next Class…

We will begin looking at choice under certainty.

The Kreps reading on this topic is probably the most difficult one we will do in this course. Keep in mind that while I expect you to understand (in English) the claims being made, I do *not* expect you to understand the details of the formal proofs.