Rational Choice

Basic Set Theory

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Sets and their Contents

Set: A collection of "things". A set is denoted by italicized capital letters, e.g., $A, B, C, \ldots$.

Element: A "thing" that is in a set. An element is denoted by italicized lowercase letters, e.g., $x, y, z, \ldots$.

> $x \in S$ means that "x is an element of set S", while
> $x \notin S$ means that "x is not an element of set S".

Empty set: The set that contains no elements. The empty set is denoted by $\varnothing$.

Sets and their Contents

The contents of a set can be shown in two ways. The first is the **roster** method, where each element of the set is explicitly written out (the order of elements does not matter) without duplicates. For example:

$S = \{2, 4, 6, 8\}$, or

$T = \{a, e, i, o, u\}$.

But what if the set has a hundred elements? Or what if it has an infinite number of elements?

Sets and their Contents

The second way for showing the contents of a set is the **rule** method, where a rule specifies the elements of the set. For example:

$S = \{x \mid x \text{ is even and } 2 \leq x \leq 8\}$, or

$T = \{x \mid x \text{ is a letter of the English alphabet and } x \text{ is a vowel (excluding the letter 'y')}\}$.

Examples

How would you write out the set (call it A) of all the professors who teach business at CMU-Q?

How would you write out the set (call it B) of all prime numbers less than 5 ?

Comparing Sets

Subset: For any sets S and T, $S \subseteq T$ (S is a subset of T) if, and only if, for every $x \in S$, $x \in T$. In English, this just means that everything is S is also in T.

Proper Subset: For any sets S and T, $S \subset T$ (S is a proper subset of T) if, and only if, $S \subseteq T$ and there exists at least one x, such that $x \in T$ but $x \notin S$.
In English, this just means that S is "contained" within T, but there is at least one "extra" thing in T.

Examples

Consider the following sets:

$$A = \{2, 4, 6, 8\},$$
$$B = \{4, 6, 2\},$$
$$C = \{1, 3\}, \text{ and}$$
$$D = \{x \mid x \text{ is even and } 2 \leq x \leq 8\}.$$

Write out all the subsets of B, labeling them B_1, B_2, etc.

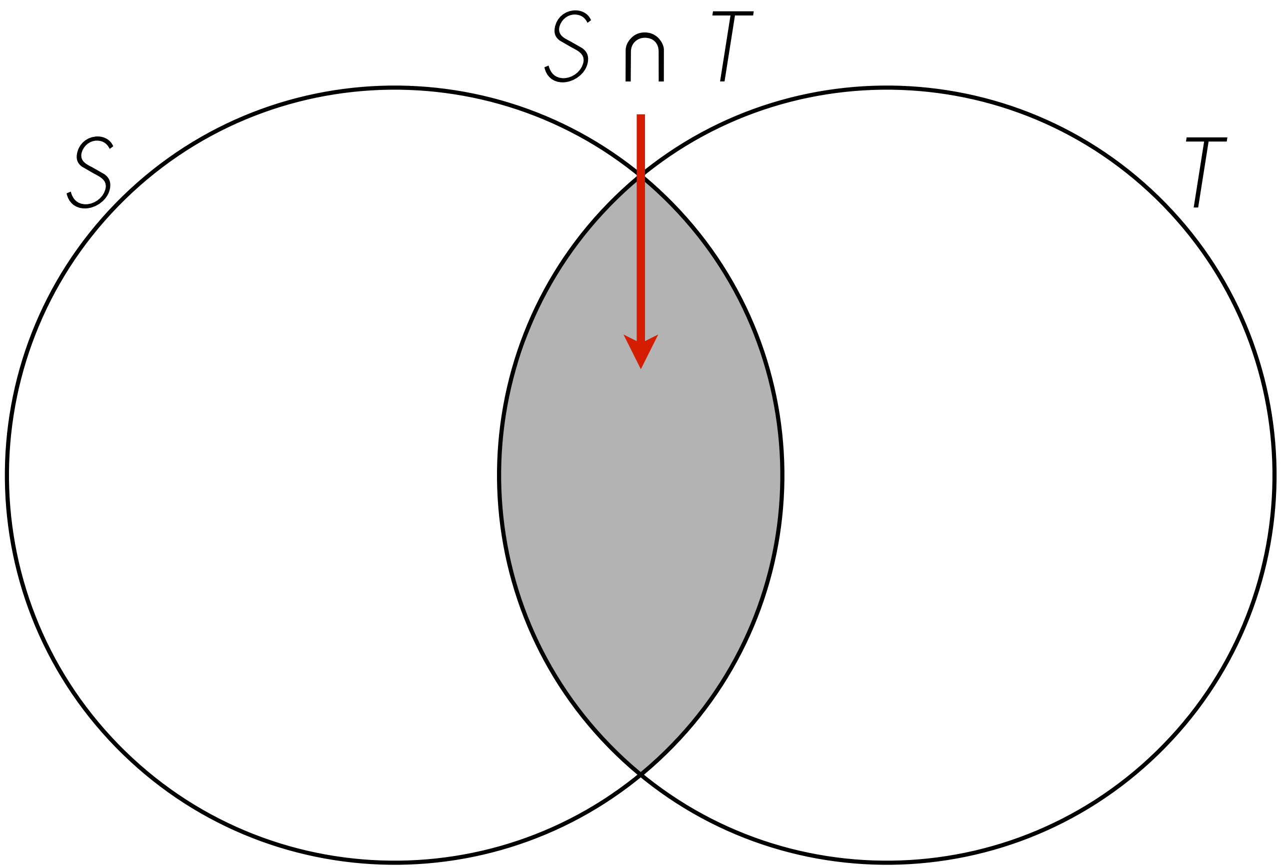
Is $B \subseteq C$? Is $B \subset A$? Is $A \subset D$? Is $A \subseteq D$?

Set Operations

Set Intersection: For any sets S and T, there exists the set $S \cap T$ (the intersection of S and T), such that for any x, $x \in S \cap T$ if, and only if, $x \in S$ and $x \in T$. In English, the intersection is just the set of things that S and T have in common.

Set Disjointness: For any sets S and T, S and T are disjoint if, and only if, $S \cap T = \varnothing$. In English, S and T are disjoint just when they have absolutely nothing in common.

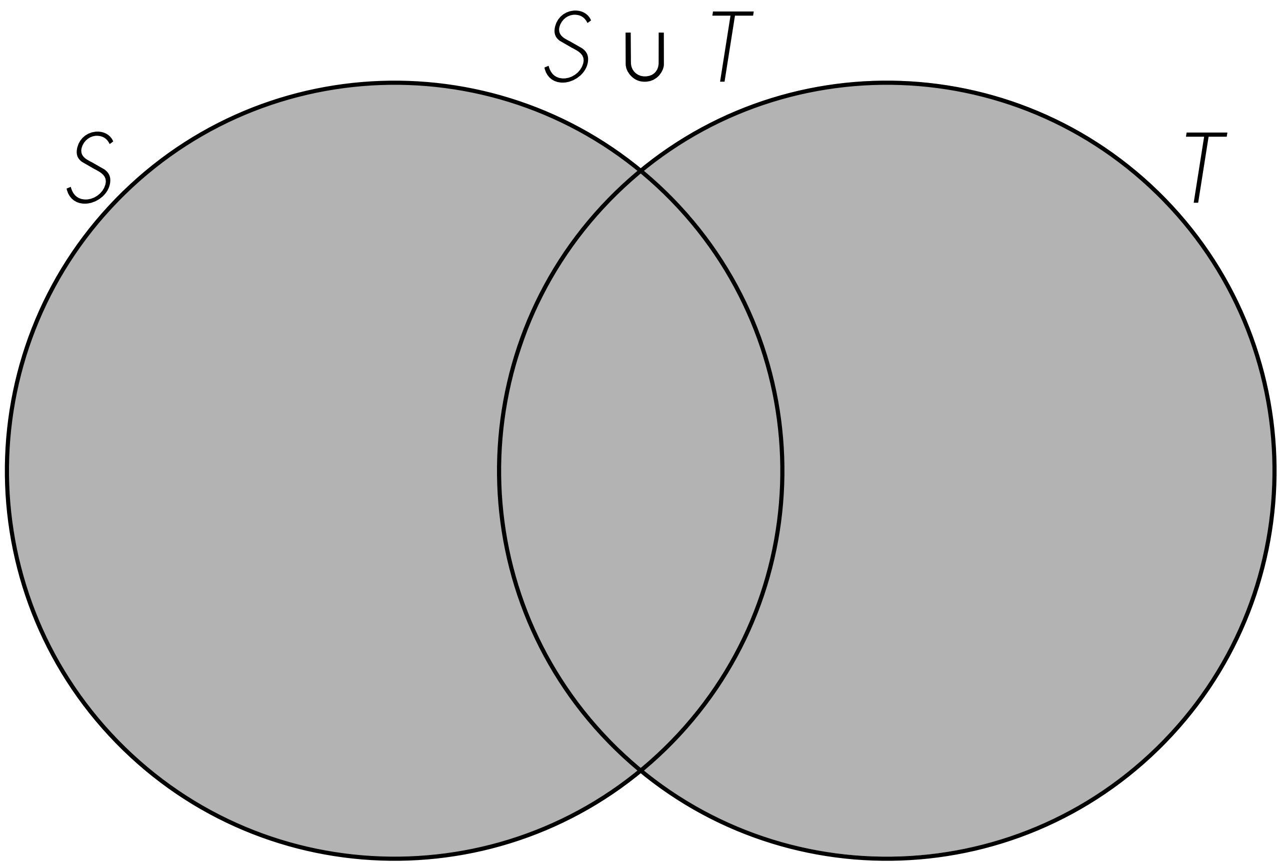
Set Intersection



Set Operations

Set Union: For any sets S and T, there exists the set $S \cup T$ (the union of S and T), such that for any x, $x \in S \cup T$ if, and only if, $x \in S$ or $x \in T$. In English, the union is just the combination of the contents of both S and T.

Set Union



Examples

Consider the following sets:

$$A = \{2, 4, 6, 8\},$$
$$B = \{4, 6, 2\},$$
$$C = \{1, 3\}, \text{and}$$
$$D = \{x \mid x \text{ is even and } 2 \le x \le 8\}.$$

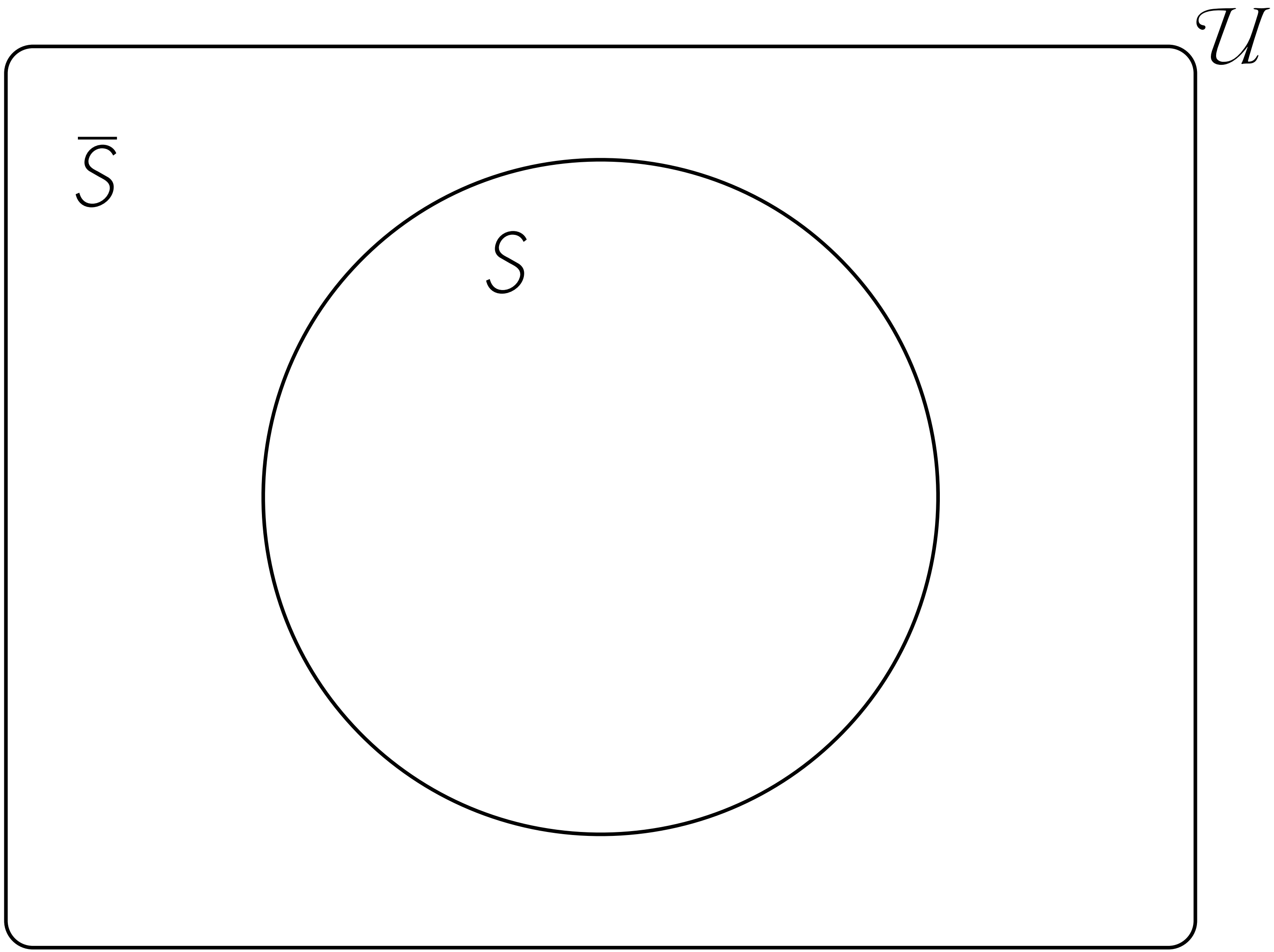
Write out the following sets: $A \cap B$, $A \cup B$, $B \cap C$, $B \cup C$, and $A \cap D$, $A \cup D$.

Set Operations

Set Complement: For any set S, there exists the set $\overline{S}$ (the complement of S), such that $S \cap \overline{S} = \varnothing$ and $S \cup \overline{S} = \mathcal{U}$. In English, the complement of S is just the collection of items *not* in S.

Universal Set: For any set S, there exists a set $\mathcal{U}$ (the universal set) for S, such that $S \subseteq \mathcal{U}$ and $S \cup \overline{S} = \mathcal{U}$. In English, the universal set for S provides the context for understanding the items in S and outside of S.

Set Complement



Examples

Let $\mathcal{U} = \{1, 2, 3, 4, 5\}$ be the universal set for $F = \{1, 2, 3\}$. Specify $\overline{F}$ by the roster method.

Let $\mathcal{U} = \{x \mid x \text{ is a letter of the English alphabet}\}$ be the universal set for $G = \{\text{a, e, i, o, u}\}$. Specify $\overline{G}$ by the rule method.

Next Class…

We will apply set theory, showing how to formalize aspects of a decision problem while putting this information into a decision matrix.