Introduction to Logical Reasoning

Further Categorical Inferences

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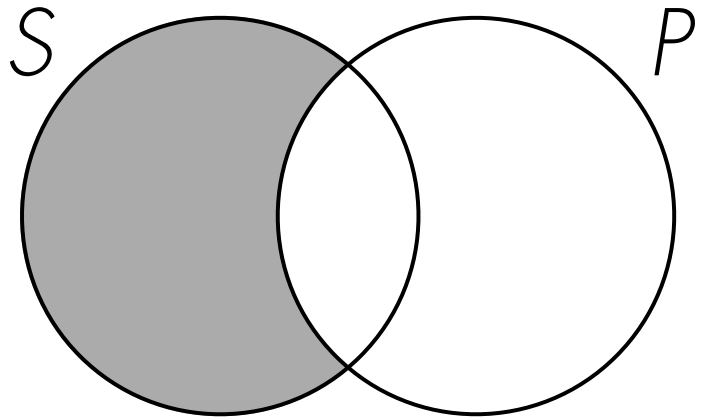
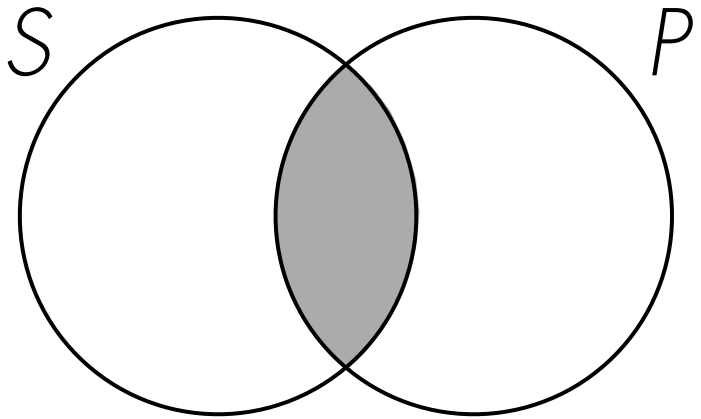
Categorical Statements

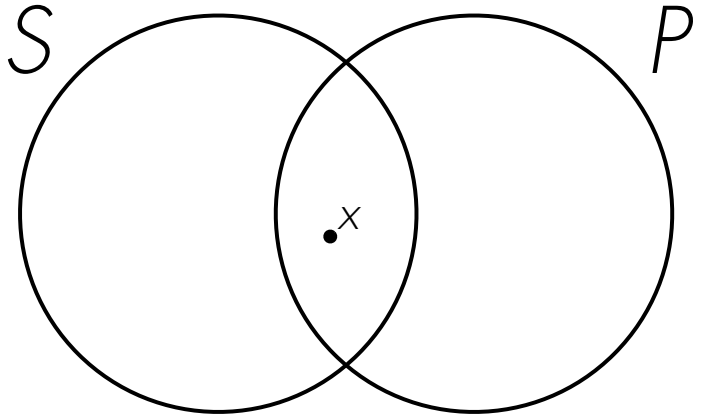
The four standard forms of categorical statements:



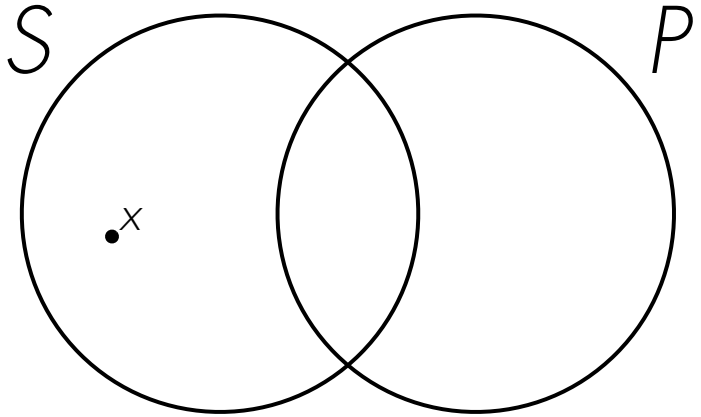
A: All *S* is *P*.



E: No *S* is *P*.



I: Some *S* is *P*.



O: Some *S* is not *P*.

Statement 1

Consider the following categorical statement:

No students are lazy people.

Statement 2

Consider the following categorical statement:

Some students are lazy people.

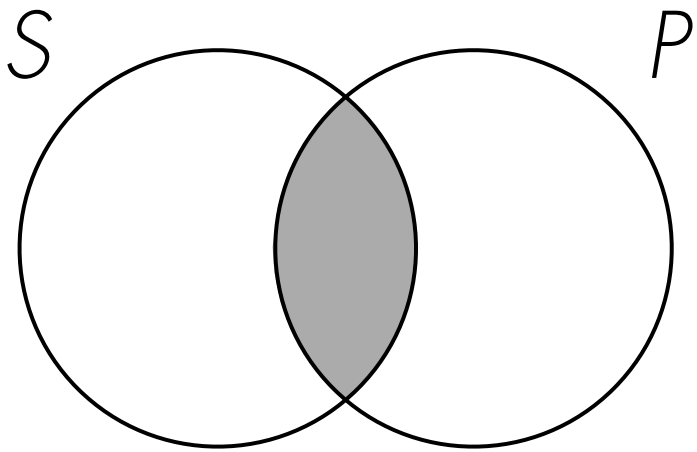
Conversion

The **conversion** of a categorical statement swaps the subject and predicate to create a new categorical statement.

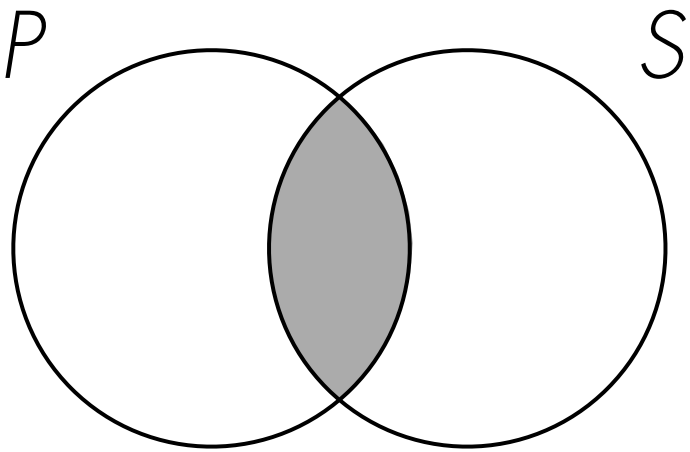
In *some* instances, the new statement will be logically equivalent to the original one. For example, the statement "No students are lazy" (**E**) is logically the same as "No lazy persons are students" (**E**).

Conversion

In general, any **E** statement and its conversion are logically the same, as seen in these Venn diagrams.



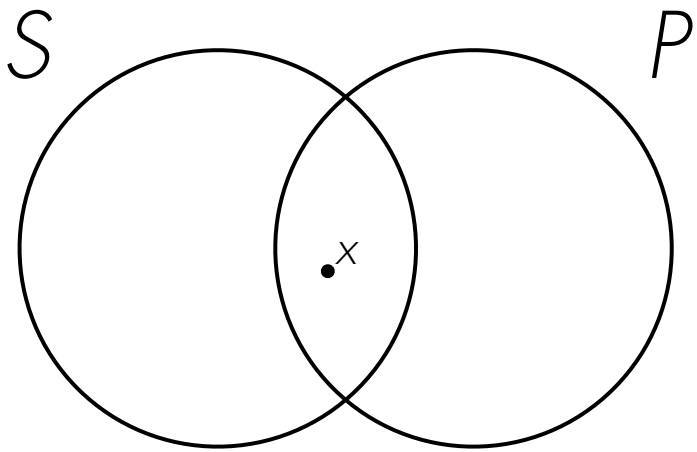
E Statement
(No *S* is *P*)



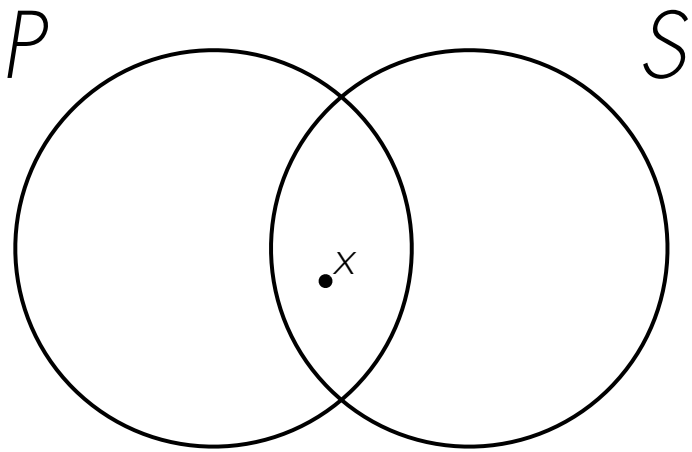
E's Conversion
(No *P* is *S*)

Conversion

Similarly, any **I** statement and its conversion are logically the same, as seen in these Venn diagrams.



I Statement
(Some *S* is *P*)



I's Conversion
(Some *P* is *S*)

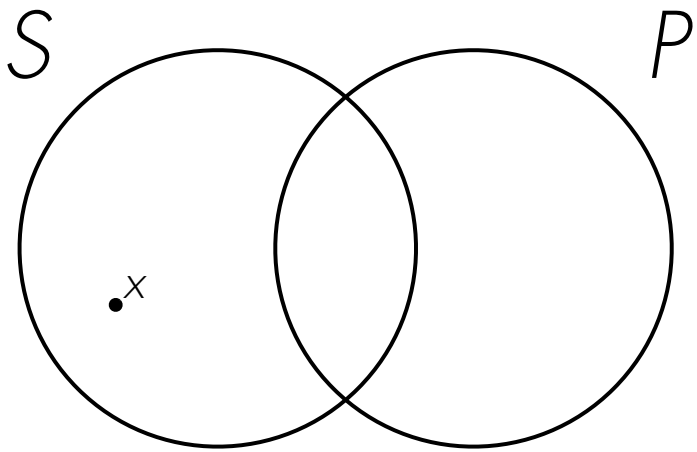
Statement 3

Consider the following categorical statement:

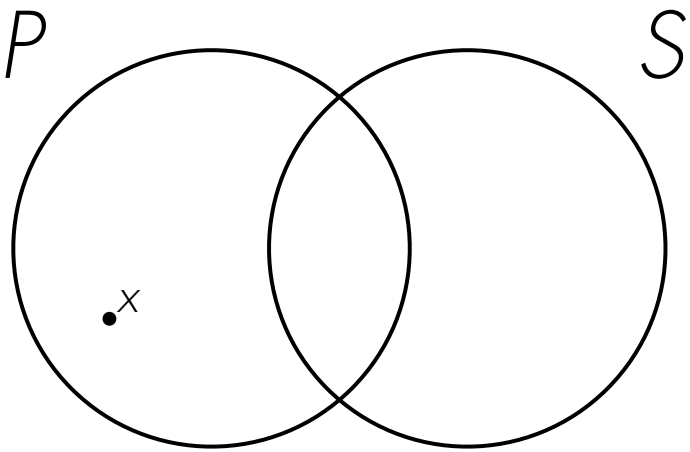
Some students are not lazy people.

Conversion

In general, any **O** statement and its conversion are *not* logically the same, as seen in these Venn diagrams.



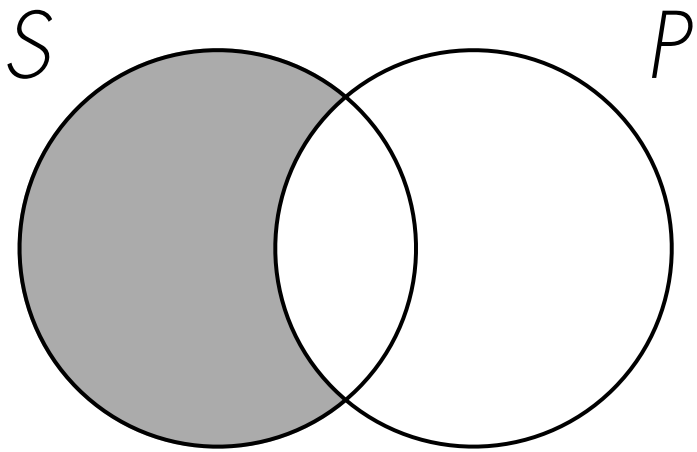
O Statement
(Some *S* is not *P*)



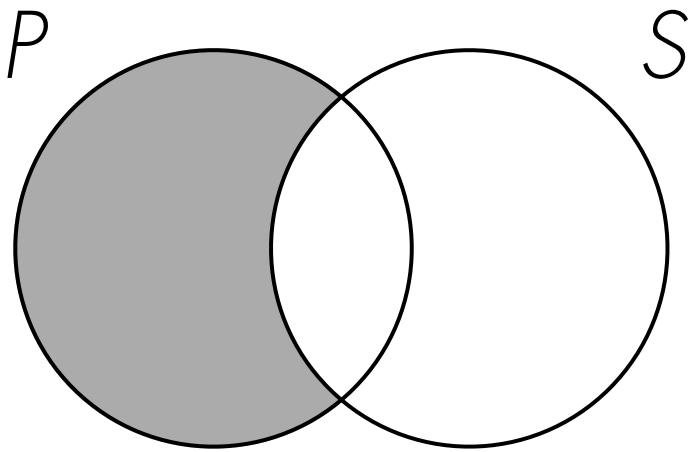
O's Conversion
(Some *P* is not *S*)

Conversion

Similarly, any **A** statement and its conversion are *not* logically the same, as seen in these Venn diagrams.



A Statement
(All *S* is *P*)



A's Conversion
(All *P* is *S*)

Complement

Recall that for any subject (S) or predicate (P) in a categorical statement, we may consider its **complement**. The complement of the subject is denoted as non-S; the complement of the predicate is denoted by non-P.

Statement 4

Consider the following categorical statement:

All students are lazy people.

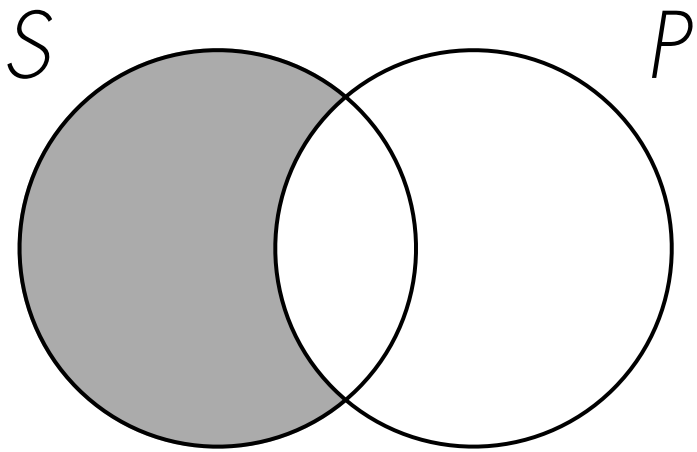
Obversion

The **obversion** of a categorical statement comes from flipping its quality and replacing the predicate with that predicate's complement.

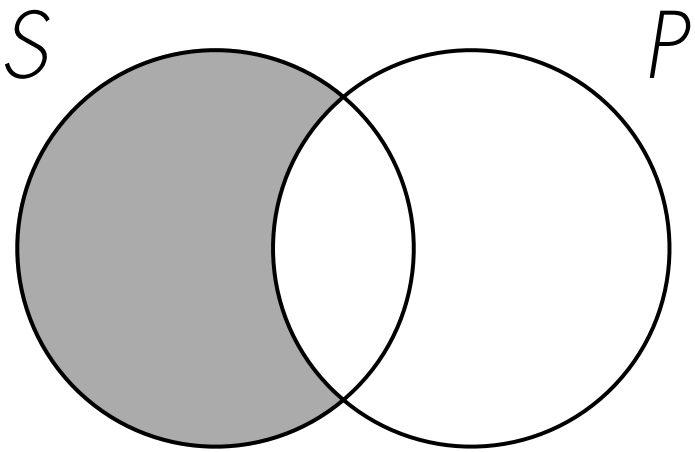
It turns out that the obversion of each of the standard four categorical statements is logically equivalent to the original statement. So, for instance, "All students are lazy" (**A**) is logically equivalent to its obversion: "No students are non-lazy" (**E**).

Obversion

In general, any **A** statement and its obversion (an **E** statement) are logically the same.



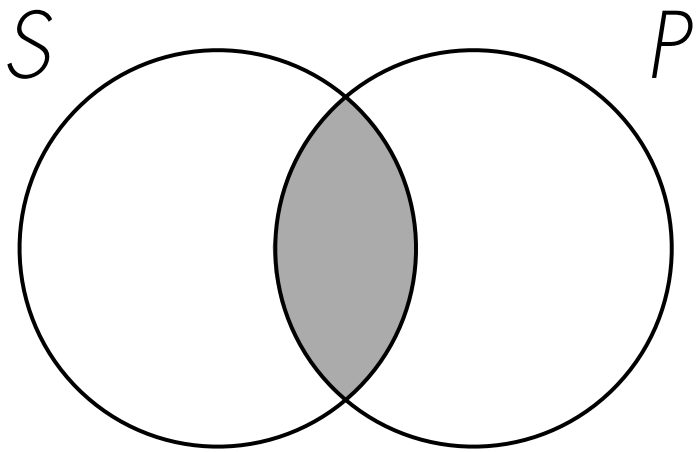
A Statement
(All *S* is *P*)



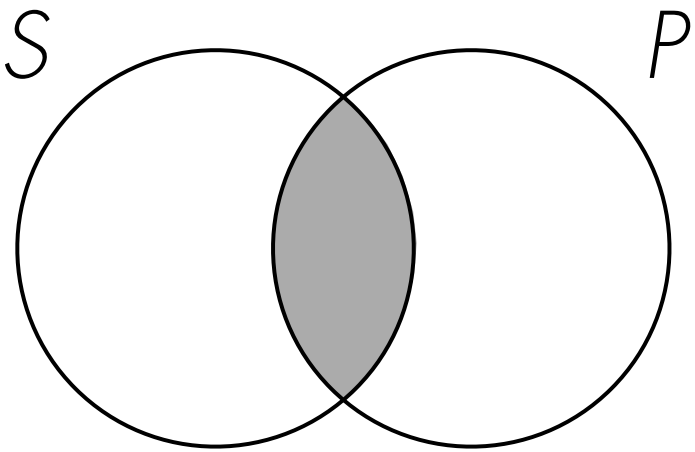
A's Obversion (**E**)
(No *S* is non-*P*)

Obversion

Similarly, any **E** statement and its obversion (an **A** statement) are logically the same.



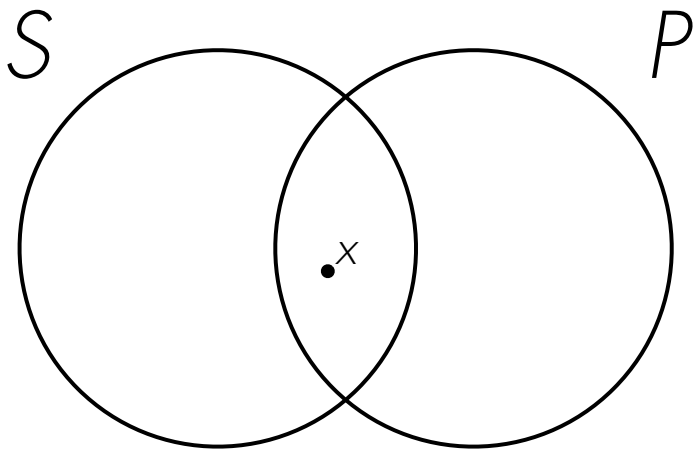
E Statement
(No *S* is *P*)



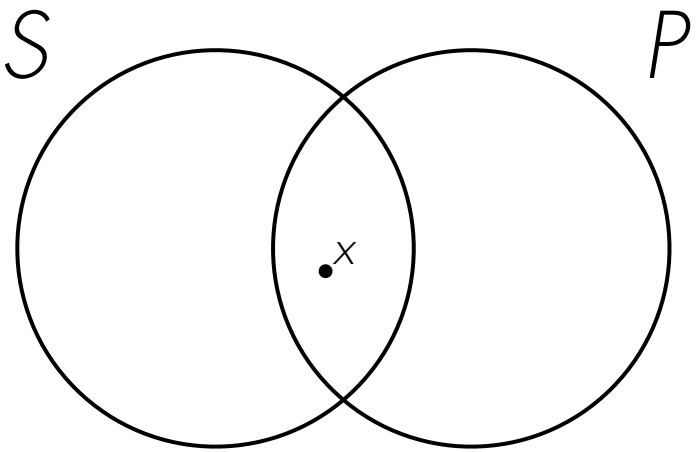
E's Obversion (**A**)
(All *S* is non-*P*)

Obversion

And so for any **I** statement and its obversion (an **O** statement).



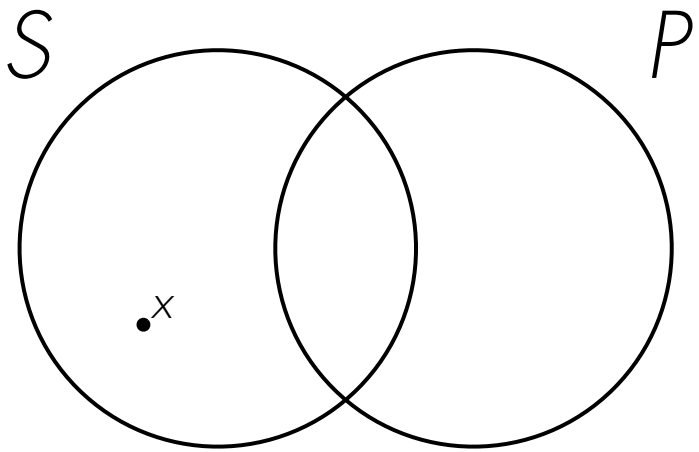
I Statement
(Some *S* is *P*)



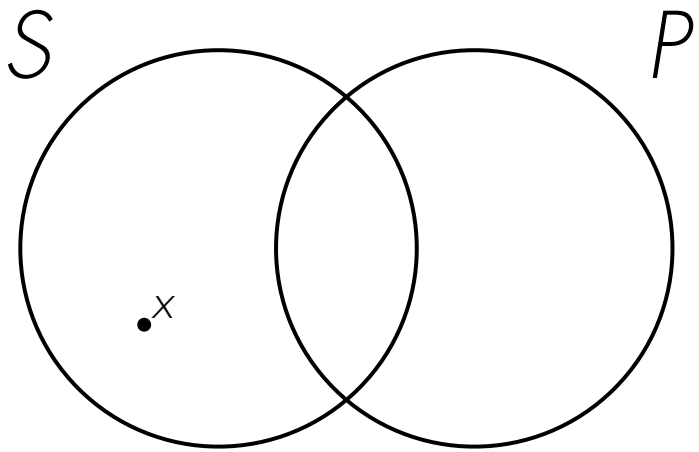
I's Obversion (**O**)
(Some *S* is not non-*P*)

Obversion

And finally for any **O** statement and its obversion (an **I** statement).



O Statement
(Some *S* is not *P*)



O's Obversion (**I**)
(Some *S* is non-*P*)

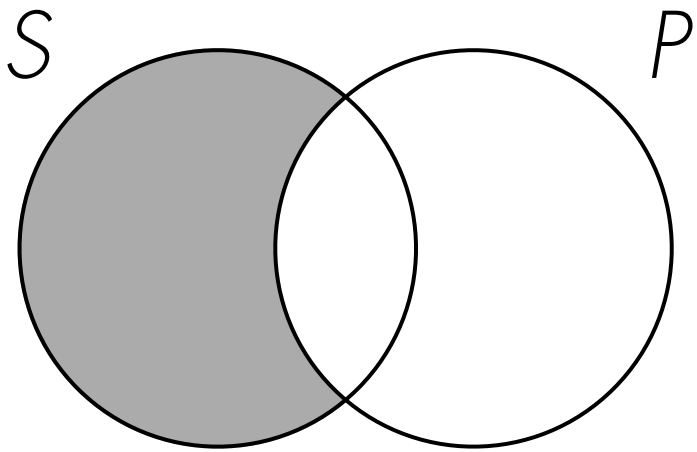
Contraposition

According to **contraposition**, a categorical statement is changed by (1) replacing its subject with that subject's complement, (2) replacing its predicate with that predicate's complement, and (3) swapping this new subject and new predicate.

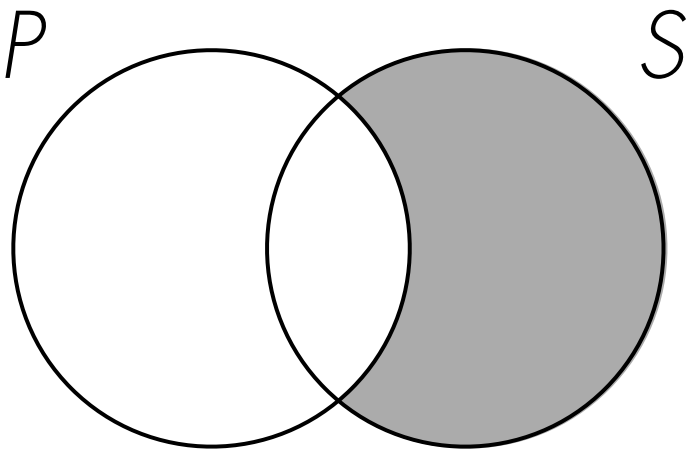
In some instances, the new statement will be logically equivalent to the original one. For example, the proposition "All students are lazy" (**A**) is logically the same as "All non-lazy people are non-students" (**A**).

Contraposition

In general, any **A** statement and its contrapositive (also an **A** statement) are logically the same.



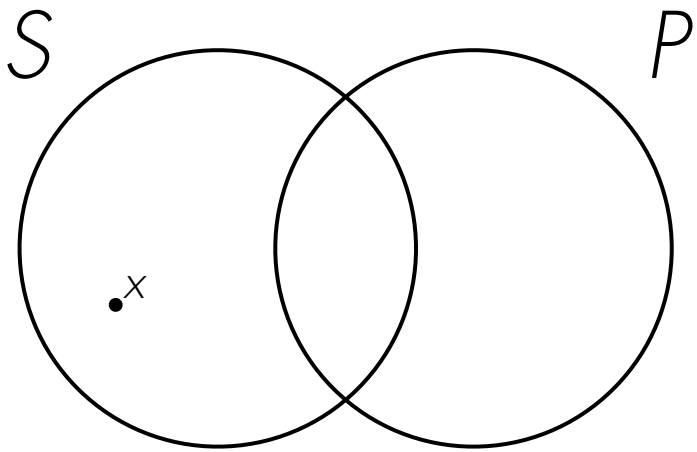
A Statement
(All *S* is *P*)



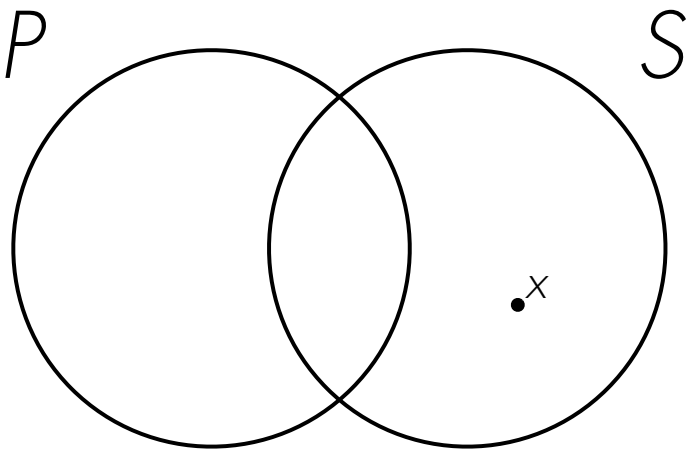
A's Contrapositive
(All non-*P* is non-*S*)

Contraposition

Similarly, any **O** statement and its contrapositive (also an **O** statement) are logically the same.



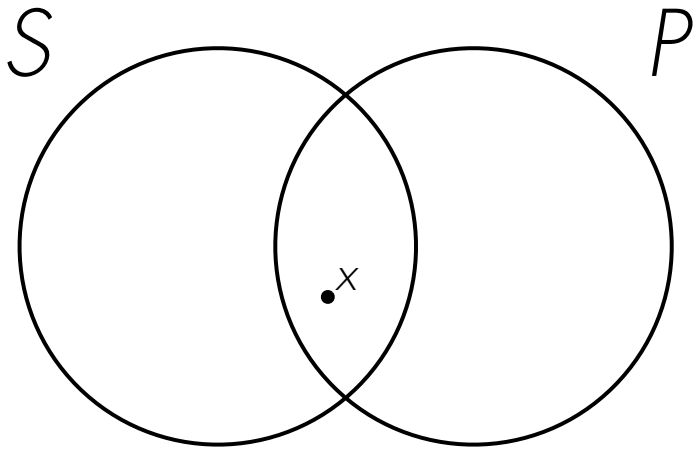
O Statement
(Some *S* is not *P*)



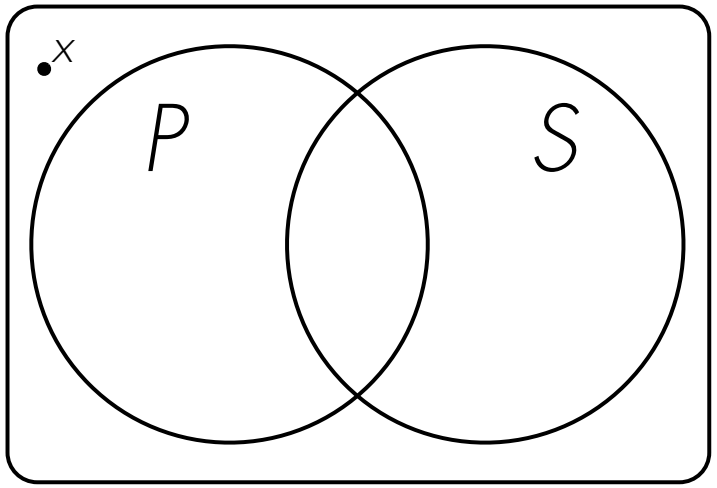
O's Contrapositive
(Some non-*P* is not non-*S*)

Contraposition

However, any **I** statement and its contraposition (also an **I** statement) are *not* logically the same.



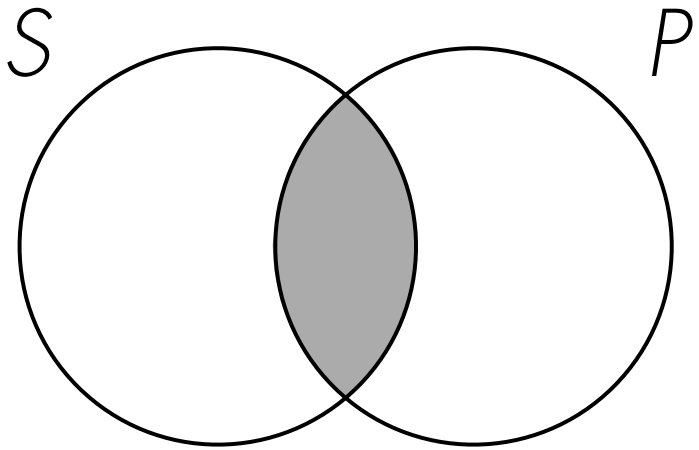
I Statement
(Some *S* is *P*)



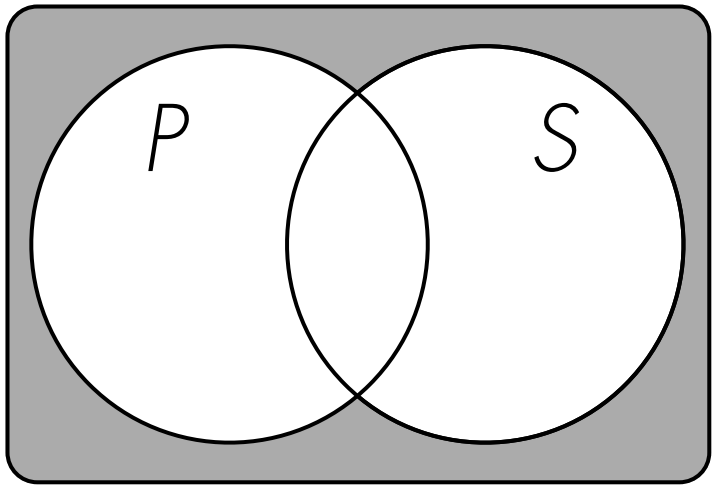
I's Contrapositive
(Some non-*P* is non-*S*)

Contraposition

Similarly, any **E** statement and its contraposition (also an **E** statement) are *not* logically the same.



E Statement
(No *S* is *P*)



E's Contrapositive
(No non-*P* is non-*S*)

Categorical Inferences

Don't let this table overwhelm you. Never forget: if you ever get lost, make a Venn diagram.

From this simple picture, you should be able verify any of these inferences.

Next Class…

We will have a workshop on using Venn diagrams for making inferences from one categorical statement to another.