Introduction to Logical Reasoning

The Square of Opposition

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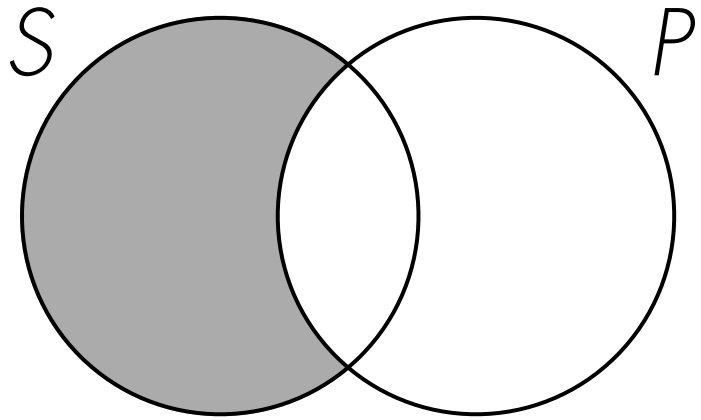
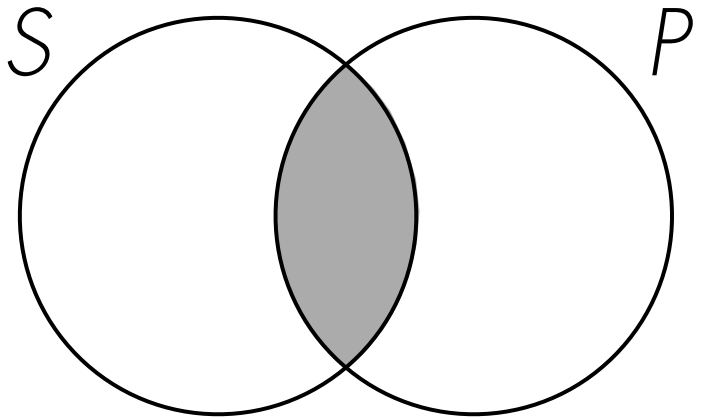
Categorical Statements

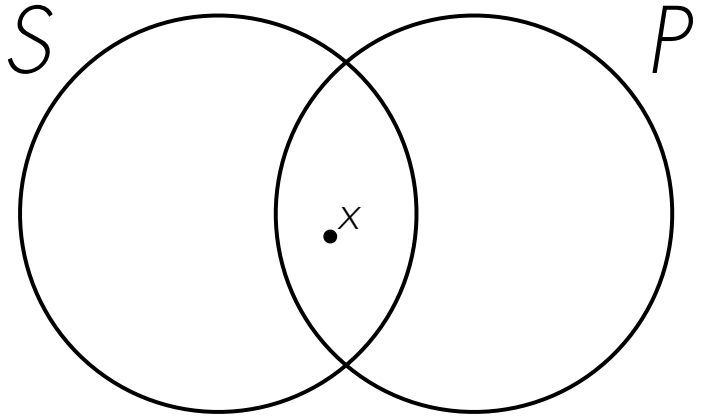
The four standard forms of categorical statements:



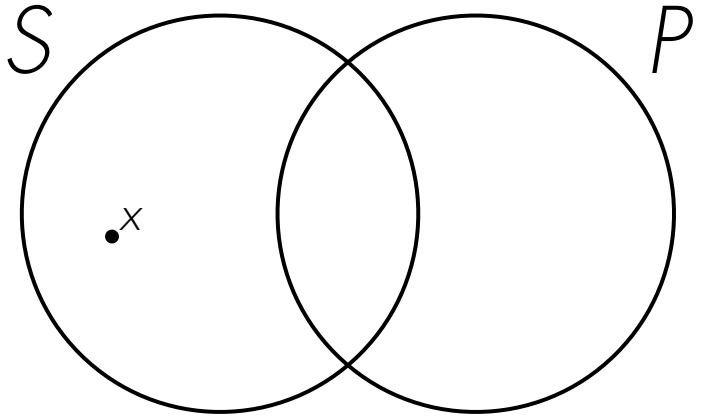
A: All *S* is *P*.



E: No *S* is *P*.



I: Some *S* is *P*.



O: Some *S* is not *P*.

Statement 1

Consider the following categorical statement:

All students are hard workers.

Categorical Inferences

Suppose that statement 1 is *true*.

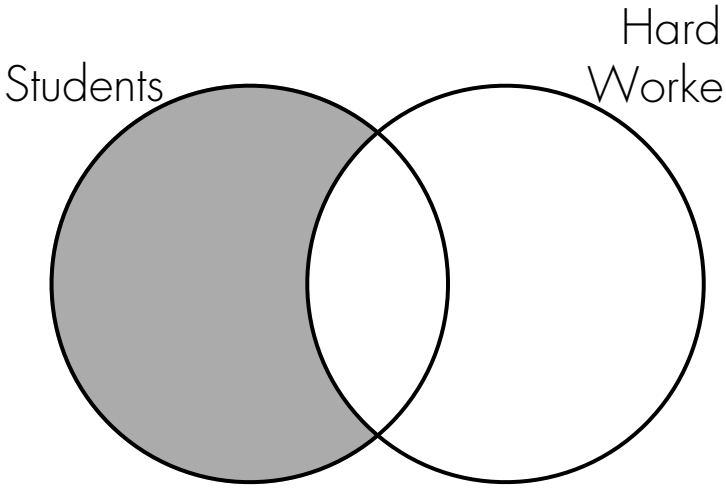
What can we then infer about the claim that "Some students are not hard workers"?

Is it true or false, or its truth/falsity undetermined?

This is an example of making an inference from one categorical statement to another one that involves the same subject (students) and predicate (hard workers).

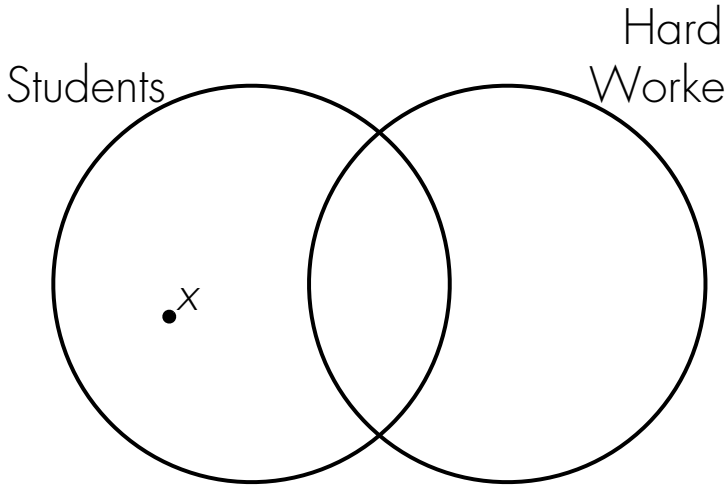
Categorical Inferences

When making a categorical inference, it is good to always draw out the associated Venn diagrams and then compare the two pictures.



All students are hard workers.
(**A** Statement)

True



Some students are not hard workers.
(**O** Statement)

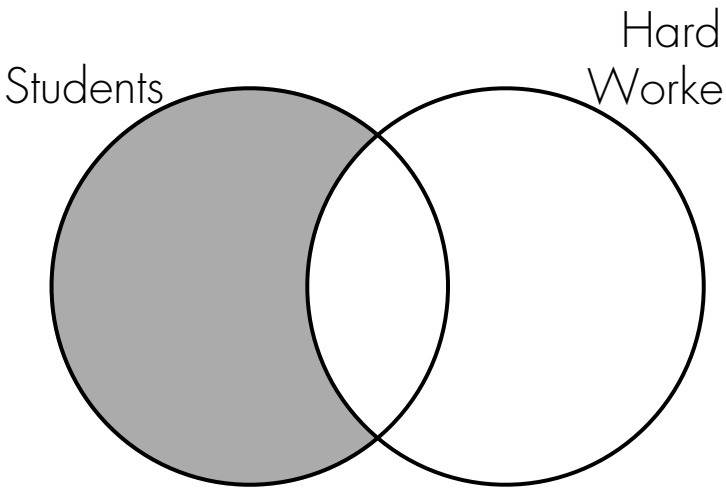
??

Categorical Inferences

The **A** statement says there is *nothing* in the area of *S* outside of *P*, whereas the **O** statement says there is *something* in that same area. So both statements cannot be true!

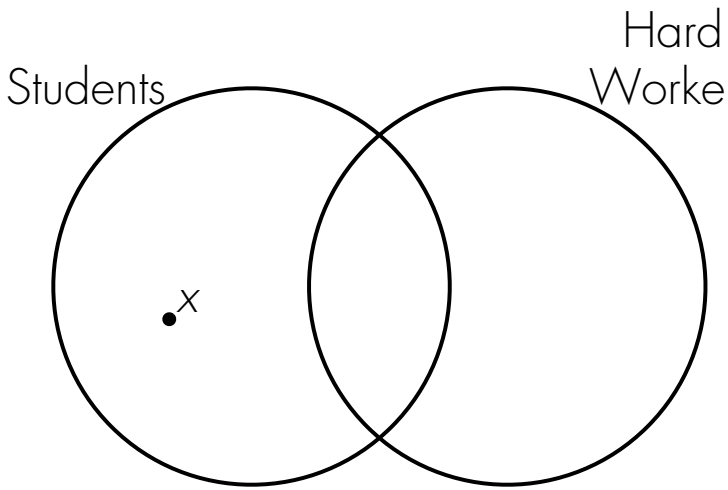
So given that "All students are hard workers" is true, the claim that "Some students are not hard workers" must be false.

Categorical Inferences



All students are
hard workers.
(**A** Statement)

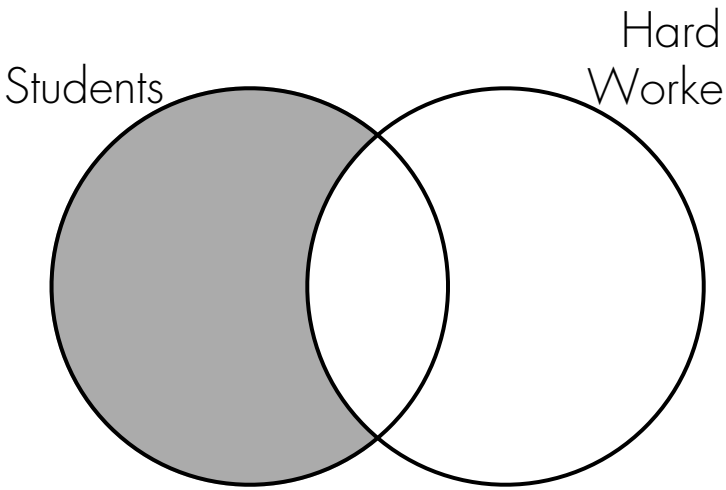
True



Some students are
not hard workers.
(**O** Statement)

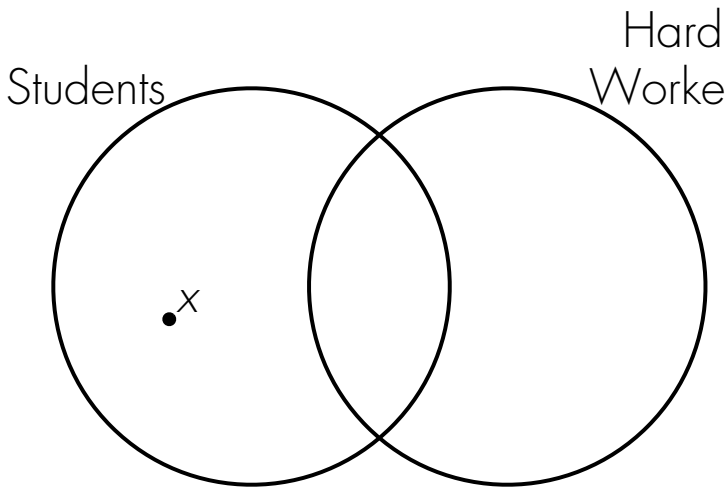
??

Categorical Inferences



All students are
hard workers.
(**A** Statement)

True



Some students are
not hard workers.
(**O** Statement)

False!

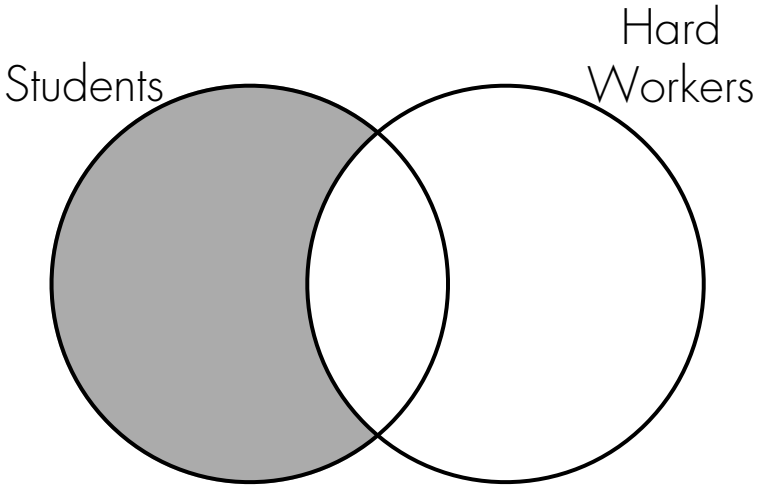
Categorical Inferences

Suppose statement 1 is *false*.

What can we then infer about the claim that "Some students are not hard workers"? Is it true or false, or its truth/falsity undetermined?

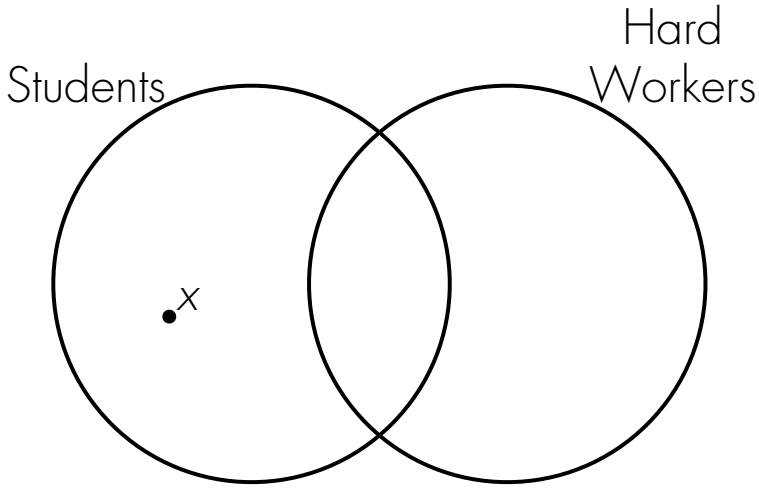
Categorical Inferences

We need to draw the Venn diagrams, but how do we understand the Venn diagram for a false statement?



All students are hard workers.
(**A** Statement)

False

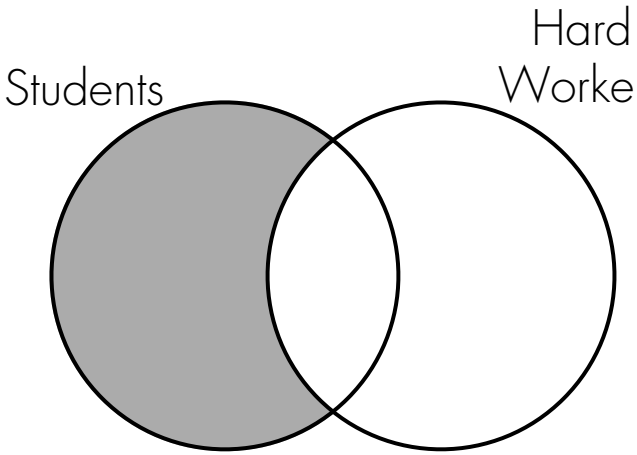


Some students are not hard workers.
(**O** Statement)

??

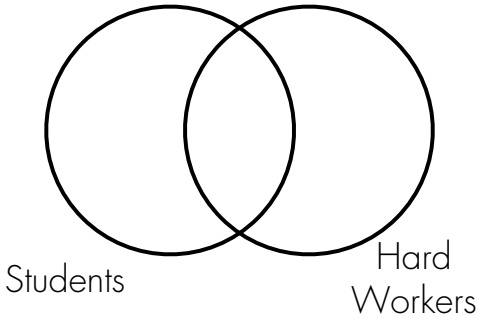
Categorical Inferences

We solve this by creating an intermediary Venn diagram that shows us what is truly going on.

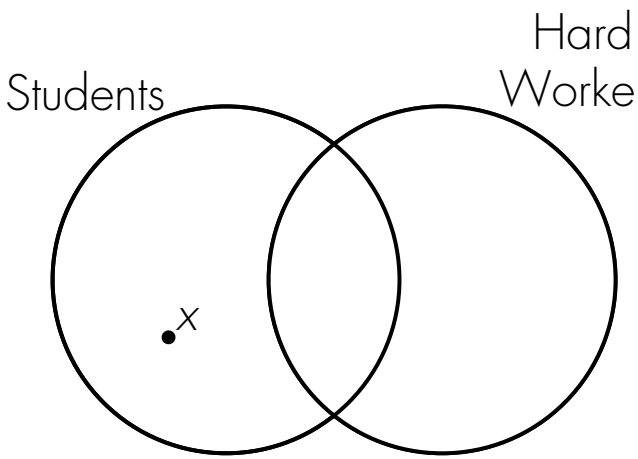


All students are hard workers.
(**A** Statement)

False



True

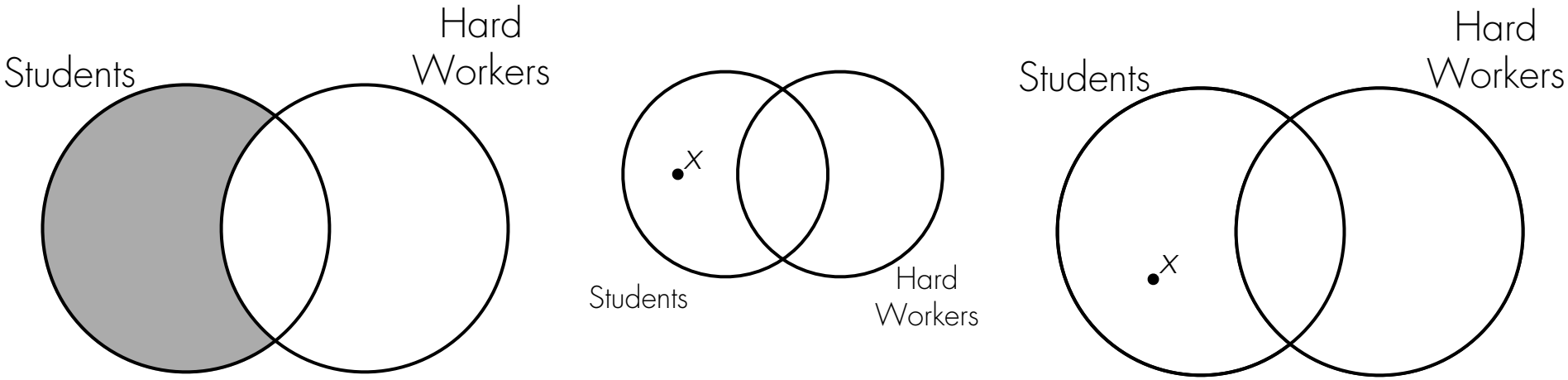


Some students are not hard workers.
(**O** Statement)

??

Categorical Inferences

Well it is given that it is incorrect to claim the area of students outside of hard workers is empty. So there must be something there after all!



All students are hard workers. (**A** Statement)

False

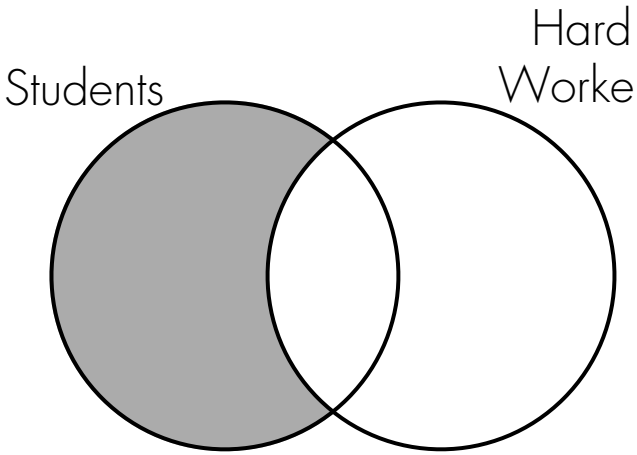
True

Some students are not hard workers. (**O** Statement)

??

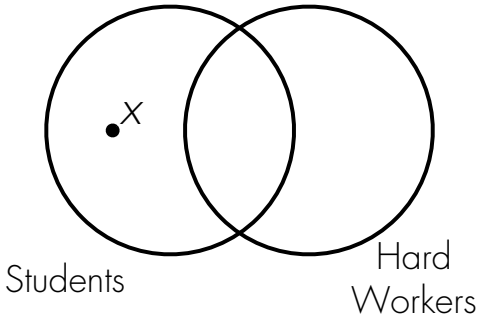
Categorical Inferences

Now we see quite clearly that the **O** statement must be true. After all, it has the exact same diagram that we know already to be true.

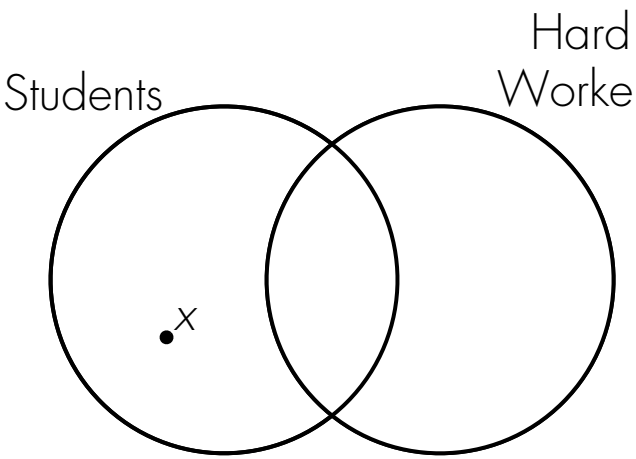


All students are hard workers.
(**A** Statement)

False



True



Some students are not hard workers.
(**O** Statement)

True

Statement 2

Consider the following categorical statement:

No students are hard workers.

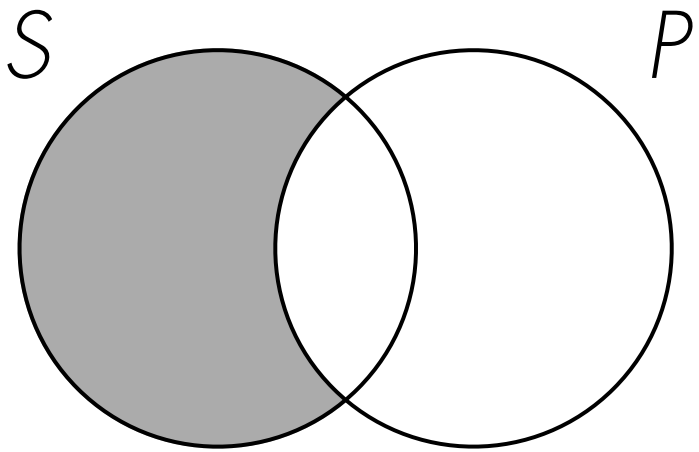
Contradictories

In general, two statements are **contradictories** if they both cannot be true and both cannot be false.

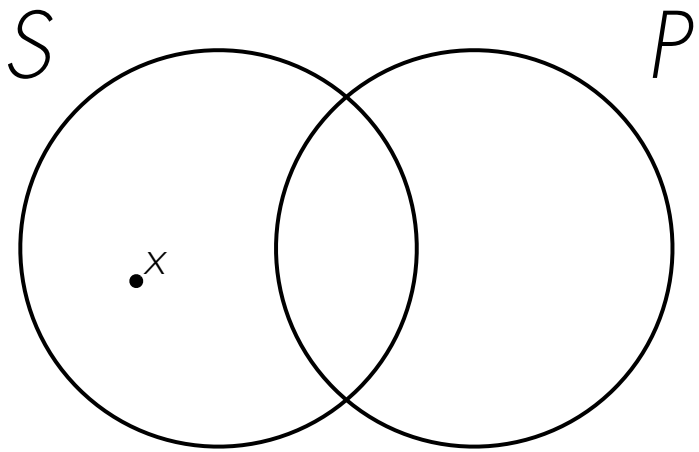
For instance, the statements "All students are hard workers" (**A**) and "Some students are not hard workers" (**O**) are contradictories. Both cannot be true and both cannot be false. So if you know one is true, the other must be false, and vice versa.

Contradictories

In general, **A** and **O** statements on the same *S* and *P* are always contradictories, as seen in these diagrams.



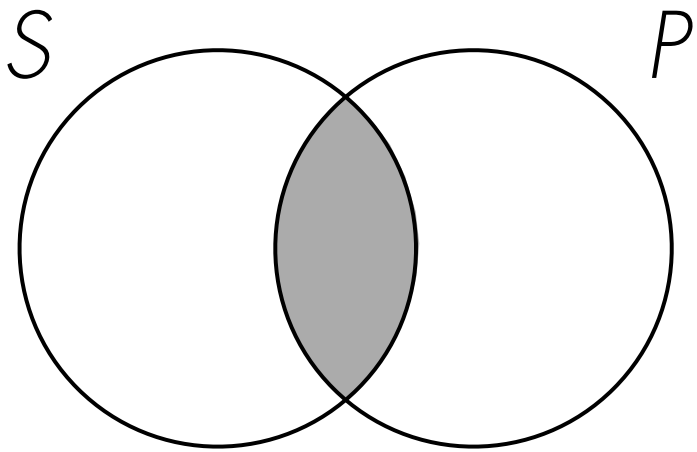
A Statement
(All *S* is *P*)



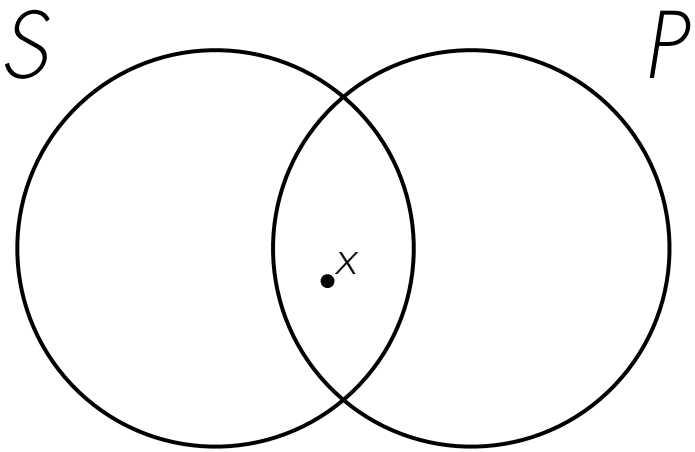
O Statement
(Some *S* is not *P*)

Contradictories

Similarly, **E** and **I** statements on the same *S* and *P* are also always contradictories, as seen in these diagrams.



E Statement
(No *S* is *P*)



I Statement
(Some *S* is *P*)

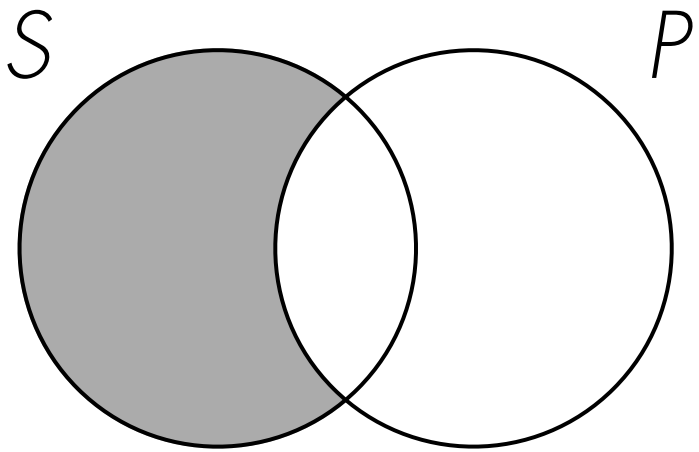
Contraries

Two statements are **contraries** if they both cannot be true, though both may be false.

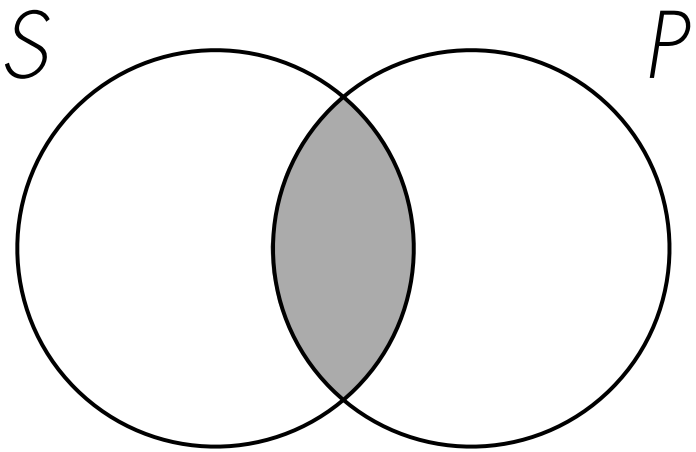
For instance, the statements "All students are hard workers" (**A**) and "No students are hard workers" (**E**) are contraries. Both cannot be true: if one is true, the other must be false. However, both positions could, in fact, be false. There might be some students who are hard workers and some others who are not.

Contraries

In general, **A** and **E** statements on the same *S* and *P* cannot both be true,* as seen in these diagrams.



A Statement
(All *S* is *P*)

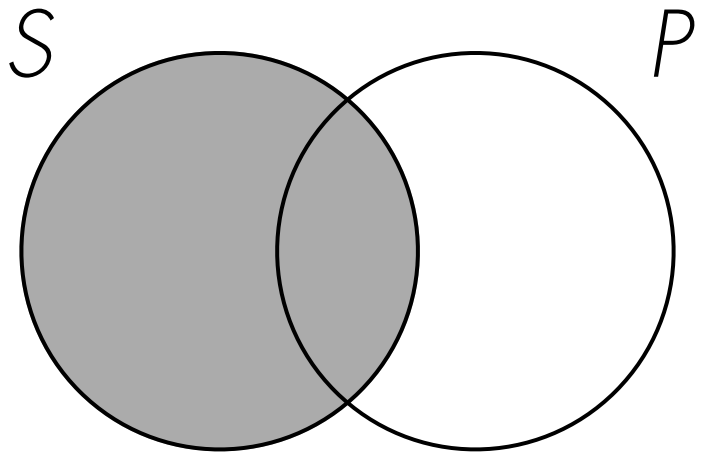


E Statement
(No *S* is *P*)

*This only works as long at the subject category is not empty! For this course, we will *always* assume each category is non-empty.

Contraries

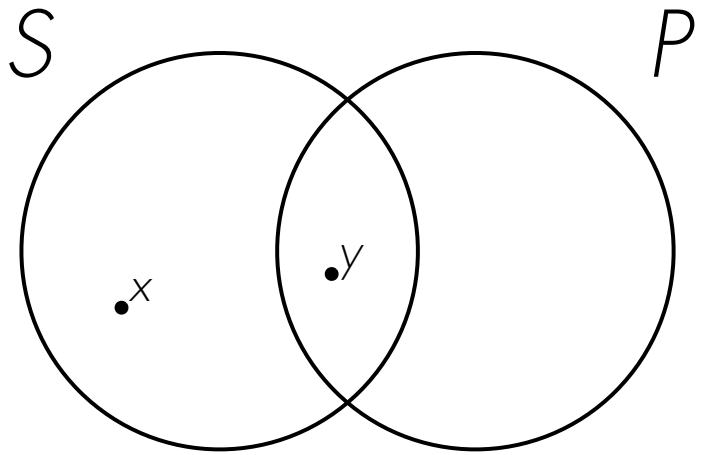
This Venn diagram shows what happens when the corresponding **A** and **E** statements are both *true*. This cannot happen since it means that *S* is empty!*



*For this course, we will *always* assume each category is non-empty.

Contraries

However, this Venn diagram shows a situation in which the corresponding **A** and **E** statements are both *false*. Nothing problematic about this.



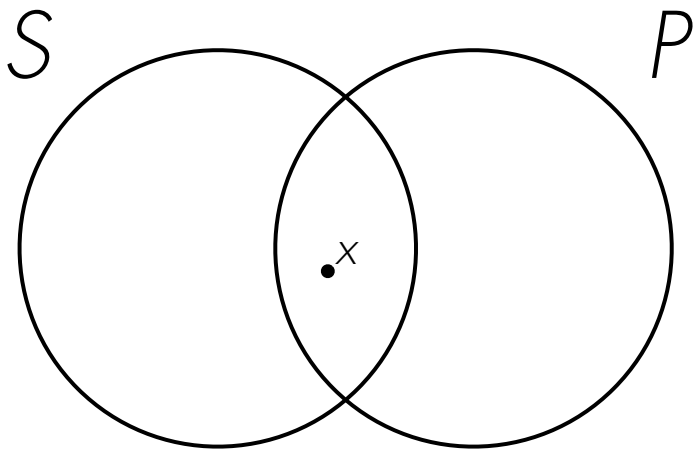
Subcontraries

Two statements are **subcontraries** if they both cannot be false, though they both may be true.

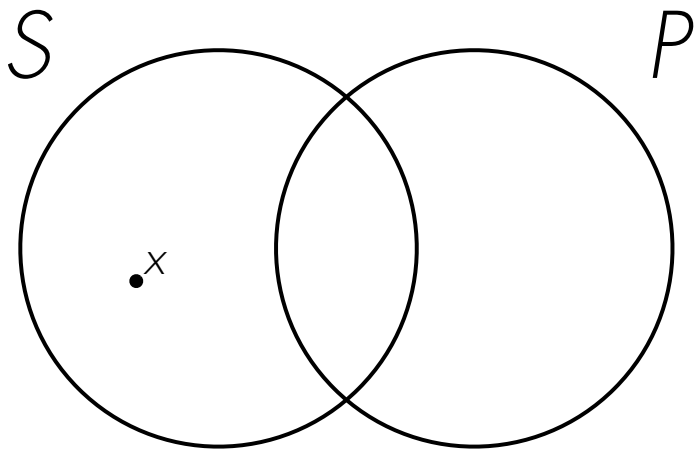
For instance, the statements "Some students are hard workers" (**I**) and "Some students are not hard workers" (**O**) are subcontraries. Both cannot be false: if one is false, the other must be true. However, both statements could be true. As already noted, there might be some students who are hard workers and some others who are not.

Subcontraries

In general, **I** and **O** statements on the same *S* and *P* cannot both be false,* as seen in these diagrams.



I Statement
(Some *S* is *P*)

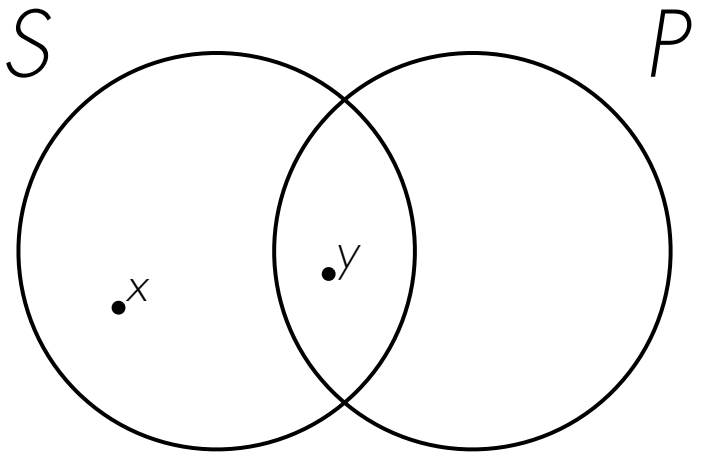


O Statement
(Some *S* is not *P*)

*This also only works as long at the subject category is not empty!

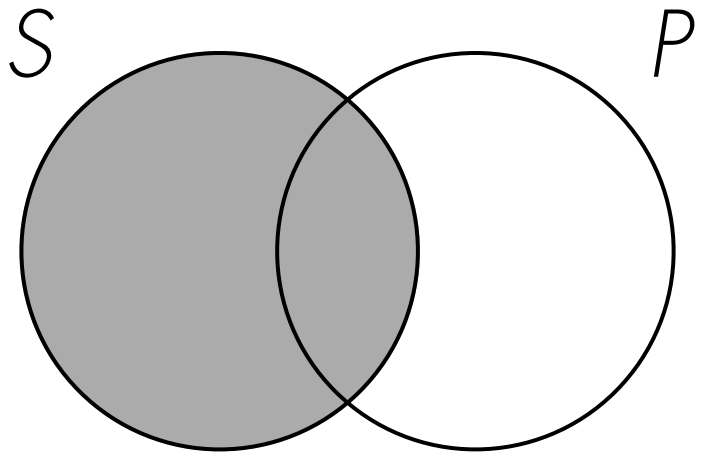
Subcontraries

This Venn diagram shows what happens when the corresponding **I** and **O** statements are both *true*. Nothing problematic about this.



Subcontraries

However, this Venn diagram shows a situation in which the corresponding **I** and **O** statements are both *false*. This cannot happen since *S* is not empty!*



*Again, we will *always* assume each category is non-empty.

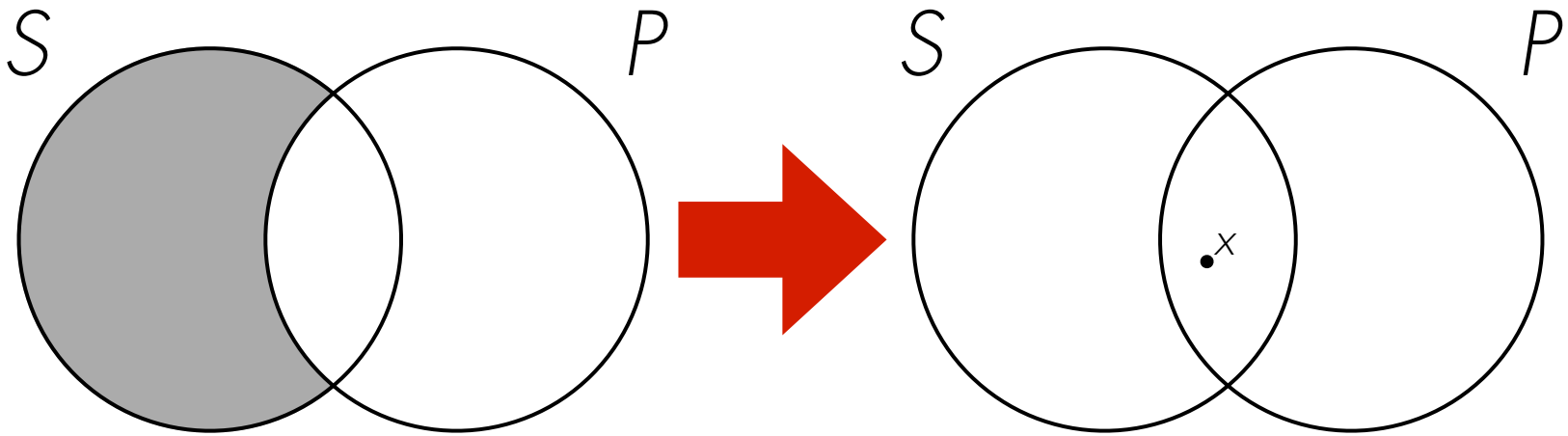
Subalternation

According to **subalternation**, any true *universal* categorical statement may be transformed into a true *particular* one. Going the other direction, subalternation says that any false particular categorical statement may be transformed into a false universal one.

So, for instance, if the statement "All students are hard workers" (**A**) is true, then "Some students are hard workers" (**I**) is trivially true as well.

Subalternation

In general, any true **A** statement may be transformed into a true **I** statement about the same *S* and *P*.*



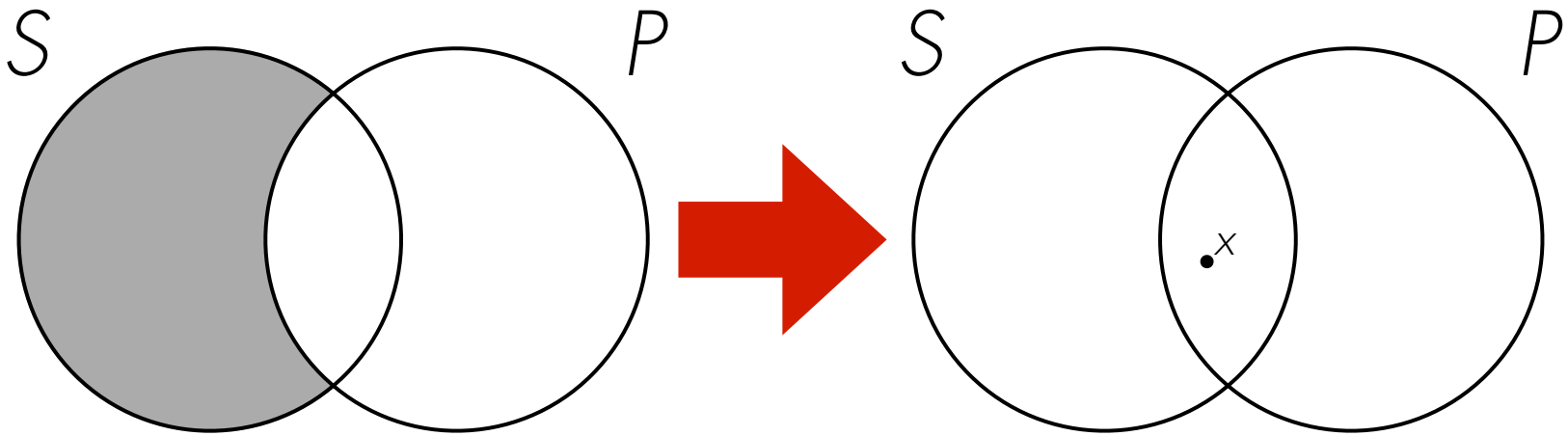
A Statement
(All *S* is *P*)

I Statement
(Some *S* is *P*)

*This also only works as long at the subject category is not empty!

Explanation

The idea is that there are no *S*'s outside of *P*, but those *S*'s have to be somewhere!* So we know that there is at least one *S* (call it *x*) in the area of overlap between *S* and *P*.



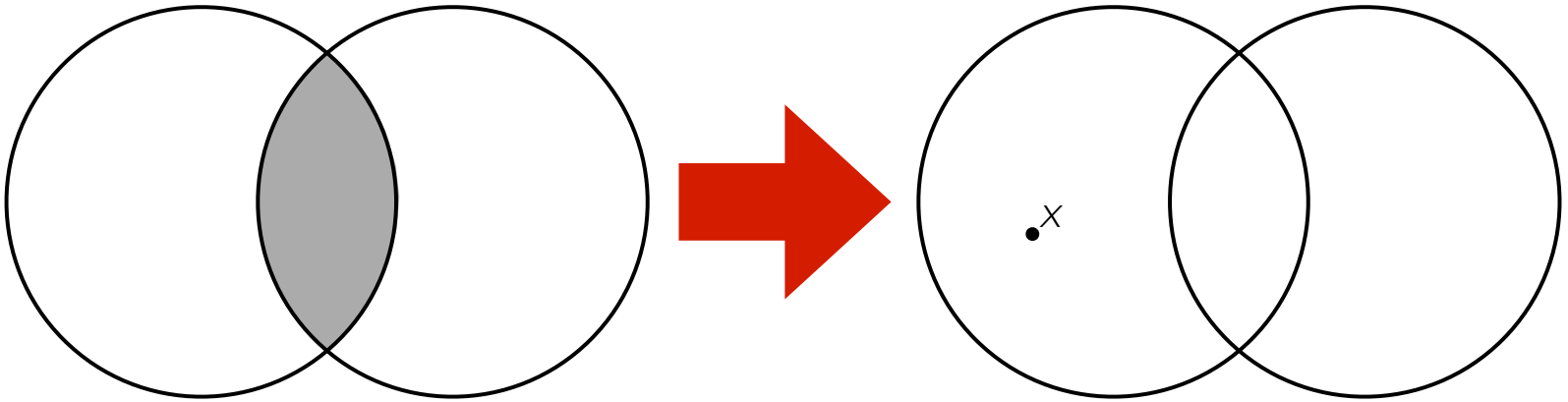
A Statement
(All *S* is *P*)

I Statement
(Some *S* is *P*)

*This also only works as long at the subject category is not empty!

Subalternation

In general, any true **E** statement may be transformed into a true **O** statement about the same *S* and *P*.*



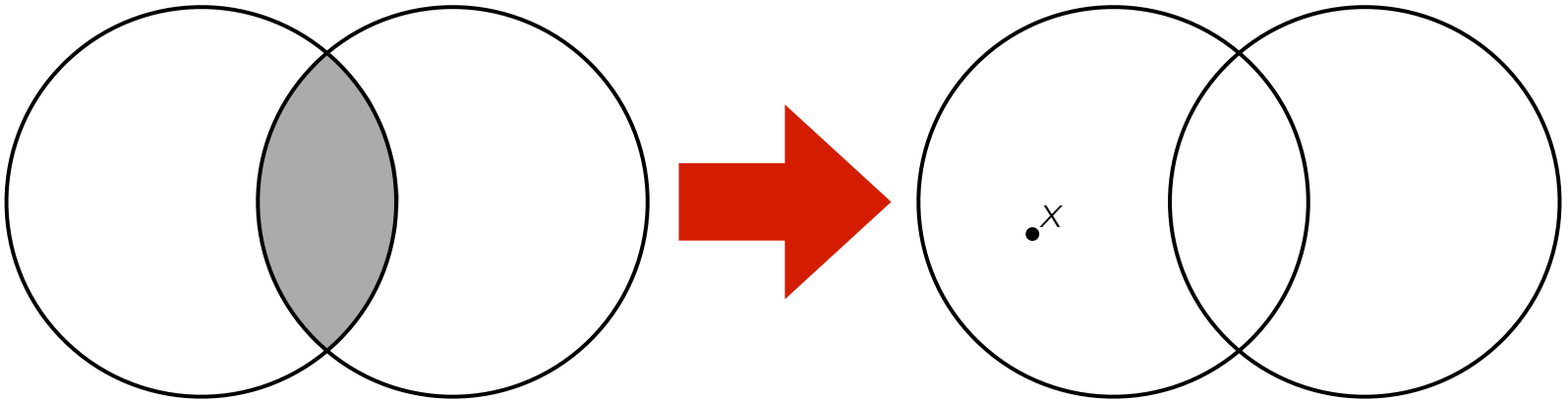
E Statement
(No *S* is *P*)

O Statement
(Some *S* is not *P*)

*This also only works as long at the subject category is not empty!

Explanation

The idea is that there are no *S*'s also inside of *P*, but those *S*'s have to be somewhere!* So we know that there is at least one *S* (call it *x*) in the area of *S* outside of *P*.

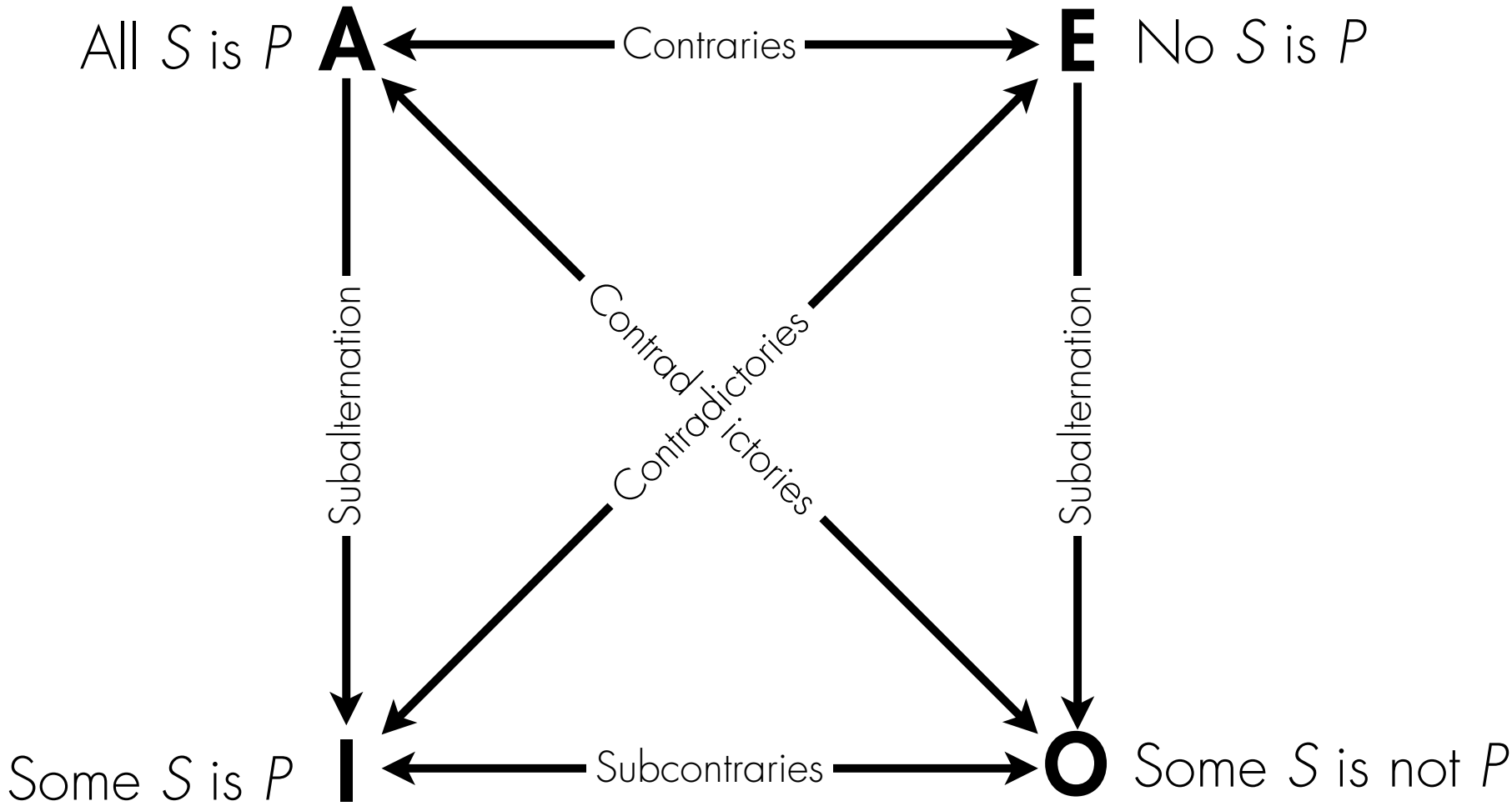


E Statement
(No *S* is *P*)

O Statement
(Some *S* is not *P*)

*This also only works as long at the subject category is not empty!

The Square of Opposition



Inferences from the Square

Fix the subject (S) and the predicate (P). Then the square of oppositions reveals these inferences:

– If **A** is true: **E** is false; **I** is true; **O** is false.
– If **A** is false: **O** is true; **E** and **I** are undetermined.

– If **E** is true: **A** is false; **I** is false; **O** is true.
– If **E** is false: **I** is true; **A** and **O** are undetermined.

– If **I** is true: **E** is false; **A** and **O** are undetermined.
– If **I** is false: **A** is false; **E** is true; **O** is true.

– If **O** is true: **A** is false; **E** and **I** are undetermined.
– If **O** is false: **A** is true; **E** is false; **I** is true.

Inferences from the Square

The square of opposition contains a lot of useful information concerning what you can infer from a single categorical statement, but Venn diagrams provide intuitive ways to figure out these inferences.

Next Class…

We will look at some further inferences that can be made from a single categorical statement.