Introduction to Logical Reasoning

Problem Set #8

Although I strongly suggest that you write out answers to all these problems, *you do not have to turn in any written responses*. You do, however, need to be prepared to do these types of problems, for *questions on the weekly quizzes and exams will primarily be drawn from the problem sets*. The solutions to these problems will be provided, so you can check your own work and seek help from me as necessary.

We will devote considerable time to these problems during the next in-class workshop. In order to make that workshop productive, please make a solid start on them. That way we can use the workshop to look at the problems that presented the most difficulties.

If you do the Extra Credit Logic Puzzle, you must turn in your *type-written* solution at the *beginning* (i.e., within the first ten minutes) of class on Sunday, March 13th.

Part A Instructions

Each of the following problems presents an valid argument. Use natural deduction to construct that argument's formal proof of validity. Each proof will only require **one** step, so essentially the task is to identify the one rule of inference that is enough to justify the conclusion from the stated premise(s).

Part A Problems

1. 1. (A & B) → C.
   ∴ (A & B) → [(A & B) & C].

2. 1. (D ∨ E) & (F ∨ G).
   ∴ D ∨ E.

3. 1. H → I.
   ∴ (H → I) ∨ (H → ~I).

4. 1. ~(J & K) & (L → ~M).
   ∴ ~(J & K).

5. 1. [N → (O & P)] & [Q → (O & R)].
   2. N ∨ Q.
   ∴ (O & P) ∨ (O & R).

6. 1. (X ∨ Y) → ~(Z & ~A).
   2. ~~(Z & ~A).
   ∴ ~(X ∨ Y).

7. 1. (S → T) ∨ [(U & V) ∨ (U & W)].
   2. ~(S → T).
   ∴ (U & V) ∨ (U & W).

8. 1. ~(B & C) → (D ∨ E).
   2. ~(B & C).
   ∴ D ∨ E.

9. 1. (F → G) → ~(G & ~F).
   2. ~(G & ~F) → (G → F).
   ∴ (F → G) → (G → F).

10. 1. ~(H & ~I) → (H → I).
    2. (I → H) → ~(H & ~I).
    ∴ (I → H) → (H → I).

11. 1. (A → B) → (C ∨ D).
    2. A → B.
    ∴ C ∨ D.

12. 1. [E → (F → G)] ∨ (C ∨ D).
    2. ~[E → (F → ~G)].
    ∴ C ∨ D.

13. 1. (C ∨ D) → [(J ∨ K) → (J & K)].
    2. ~[(J ∨ K) → (J & K)].
    ∴ ~(C ∨ D).

14. 1. ~[L → (M → N)] → ~(C ∨ D).
    2. ~[L → (M → N)].
    ∴ ~(C ∨ D).

15. 1. (J → K) & (K → L).
    2. L → M.
    ∴ [(J → K) & (K → L)] & (L → M).

16. 1. N → (O ∨ P).
    2. Q → (O ∨ R).
    ∴ [Q → (O ∨ R)] & [N → (O ∨ P)].

17. 1. (S → T) → (U → V).
    ∴ (S → T) → [(S → T) & (U → V)].

18. 1. (W & ~X) → (Y → Z).
    ∴ [(W & ~X) → (Y → Z)] ∨ (X → ~Z).

19. 1. [(H & ~I) → C] & [(I & ~H) → D].
    2. (H & ~I) ∨ (I & ~H).
    ∴ C ∨ D.

20. 1. [(O → P) → Q] → ~(C ∨ D).
    2. (C ∨ D) → [(O → P) → Q].
    ∴ (C ∨ D) → ~(C ∨ D).

Part B Instructions

Each of the following problems presents a complete and correct formal proof of validity for an argument by natural deduction. For each proof, state the justification for each step within that proof (i.e., the numbered steps that start *after* the conclusion has been indicated by the ∴).

Part B Problems

1. 1. A & B.
   2. (A ∨ C) → D.
   ∴ A & D.

   3. A.
   4. A ∨ C.
   5. D.
   6. A & D.

2. 1. (E ∨ F) & (G ∨ H).
   2. (E → G) & (F → H).
   3. ~G.
   ∴ H.

   4. E ∨ F.
   5. G ∨ H.
   6. H.

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3.
1. $I \to J$.
2. $J \to K$.
3. $L \to M$.
4. $I \lor L$.

$\therefore$ $K \lor M$.

5. $I \to K$.
6. $(I \to K)$ & $(L \to M)$.
7. $K \lor M$.

4.
1. $N \to O$.
2. $(N$ & $O) \to P$.
3. $\sim(N$ & $P)$.

$\therefore$ $\sim N$.

4. $N \to (N$ & $O)$.
5. $N \to P$.
6. $N \to (N$ & $P)$.
7. $\sim N$.

5.
1. $Q \to R$.
2. $\sim S \to (T \to U)$.
3. $S \lor (Q \lor T)$.
4. $\sim S$.

$\therefore$ $R \lor U$.

5. $T \to U$.
6. $(Q \to R)$ & $(T \to U)$.
7. $Q \lor T$.
8. $R \lor U$.

6.
1. $W \to X$.
2. $(W \to Y) \to (Z \lor X)$.
3. $(W$ & $X) \to Y$.
4. $\sim Z$.

$\therefore$ X.

5. $W \to (W$ & $X)$.
6. $W \to Y$.
7. $Z \lor X$.
8. X.

7.
1. $(A \lor B) \to C$.
2. $(C \lor B) \to [A \to (D \to E)]$.
3. A & D.

$\therefore$ $D \to E$.

4. A.
5. $A \lor B$.
6. C.
7. $C \lor B$.
8. $A \to (D \to E)$.
9. $D \to E$

8.
1. $F \to \sim G$.
2. $\sim F \to (H \to \sim G)$.
3. $(\sim I \lor \sim H) \to \sim\sim G$.
4. $\sim I$.

$\therefore$ $\sim H$.

5. $\sim I \lor \sim H$.
6. $\sim\sim G$.
7. $\sim F$.
8. $H \to \sim G$.
9. $\sim H$.

9.
1. $I \to J$.
2. $I \lor (\sim\sim K$ & $\sim\sim J)$.
3. $L \to \sim K$.
4. $\sim(I$ & $J)$.

$\therefore$ $\sim L \lor \sim J$.

5. $I \to (I$ & $J)$.
6. $\sim I$.
7. $\sim\sim K$ & $\sim\sim J$.
8. $\sim\sim K$.
9. $\sim L$.
10. $\sim L \lor \sim J$.

10.
1. $(L \to M) \to (N \to O)$.
2. $(P \to \sim Q) \to (M \to \sim Q)$.
3. $\{[(P \to \sim Q) \lor (R \to S)]$ & $(N \lor O)\} \to [(R \to S) \to (L \to M)]$.
4. $(P \to \sim Q) \lor (R \to S)$.
5. $N \lor O$.

$\therefore$ $(M \to \sim Q) \lor (N \to O)$.

6. $[(P \to \sim Q) \lor (R \to S)]$ & $(N \lor O)$.
7. $(R \to S) \to (L \to M)$.
8. $(R \to S) \to (N \to O)$.
9. $[(P \to \sim Q) \to (M \to \sim Q)]$ & $[(R \to S) \to (N \to O)]$.
10. $(M \to \sim Q) \lor (N \to O)$.

Note: There may a lot of exercises here. Do not feel obligated to do all of them. I often assign many exercises so that you have plenty of opportunities to practice the skills these exercises are trying to impart. I suggest doing just enough of them so that you are confident that you could use these skills on a quiz or an exam.

Extra Credit Logic Puzzle (Hard)

In Washington, D.C., politicians never ever tell the truth, and all non-politicians always tell the truth. Last week I met three people in Washington, D.C. I asked the first one, "Are you a politician?" He said his answer so quickly that I could not hear him. The second person then told me that the first denied being a politician. The third person then said that the first person is a politician.

Question: How many of these three people are politicians: 0, 1, 2, or 3?

To receive any credit you must justify your answer with a logical argument showing why you are 100% right. That is to say, this question has a definitive answer that can be justified without *any* guessing on your part.