Introduction to Logical Reasoning

Advanced Natural Deduction

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The Nine Rules of Inference

1. *Modus Ponens* (M.P.)

   1. $p \rightarrow q$.
   2. p.

   $\therefore\ q$.

2. *Modus Tollens* (M.T.)

   1. $p \rightarrow q$.
   2. $\sim q$.

   $\therefore\ \sim p$.

3. Hypothetical Syllogism (H.S.)

   1. $p \rightarrow q$.
   2. $q \rightarrow r$.

   $\therefore\ p \rightarrow r$.

4. Disjunctive Syllogism (D.S.)

   1. $p \vee q$.
   2. $\sim p$.

   $\therefore\ q$.

5. Constructive Dilemma (C.D.)

   1. $(p \rightarrow q)\ \&\ (\mathrm{r} \rightarrow \mathrm{s})$.
   2. $p \vee r$.

   $\therefore\ q \vee s$.

6. Absorption (Abs.)

   1. $p \rightarrow q$.

   $\therefore\ p \rightarrow (p\ \&\ q)$.

7. Simplification (Simp.)

   1. $p\ \&\ q$.

   $\therefore\ p$.

8. Conjunction (Conj.)

   1. p.
   2. q.

   $\therefore\ p\ \&\ q$.

9. Addition (Add.)

   1. p.

   $\therefore\ p \vee q$.

Natural Deduction

Today we finally bring all of our skills in natural deduction together. We look at proofs where we do not know in advance how many steps it will take to solve. However, the process remains the same.

Argument 1

1. A → B.
2. A ∨ (C & D).
3. ~B & ~E.

∴ C.

Argument 1

1. $A \rightarrow B$.
2. $A \vee (C \,\&\, D)$.
3. $\sim B \,\&\, \sim E$.

∴ C.

4. ~B.	3; Simp.
5. ~A.	1, 4; M.T.
6. C & D.	2, 5; D.S.
7. C.	6; Simp.

Argument 2

1. $(\sim M \& \sim N) \to (O \to N)$.
2. $N \to M$.
3. $\sim M$.

$\therefore \sim O$.

Argument 2

1. $(\sim M \& \sim N) \rightarrow (O \rightarrow N)$.
2. $N \rightarrow M$.
3. $\sim M$.

∴ $\sim O$.

4. $\sim N$.	2, 3; M.T.
5. $\sim M \& \sim N$.	3, 4; Conj.
6. $O \rightarrow N$.	1, 5; M.P.
7. $\sim O$.	6, 4; M.T.

Argument 3

If Ayah is present then Bilal is happy. If Ayah is present and Bilal is happy, then Cala is pleased. If Ayah wins and Cala is pleased, then Dirran is pleased. Therefore, if Ayah is present then Dirran is pleased.

Argument 3

1. $A \rightarrow B$.
2. $(A \& B) \rightarrow C$.
3. $(A \& C) \rightarrow D$.

∴ $A \rightarrow D$.

Argument 3

1. $A \rightarrow B$.
2. $(A \& B) \rightarrow C$.
3. $(A \& C) \rightarrow D$.

$\therefore A \rightarrow D$.

4. $A \rightarrow (A \& B)$. 1; Abs.
5. $A \rightarrow C$. 4, 2; H.S.
6. $A \rightarrow (A \& C)$. 5; Abs.
7. $A \rightarrow D$. 6, 3; H.S.

Natural Deduction

So far we have just seen formal proofs that only required being able to recognize the rules of inference (the "patterns") being applied. Now we can begin to begin to construct proofs where the steps are not given to us. To introduce you to this process, today we will look at arguments whose proofs can be done in just two steps.

Next Class…

We will have a review for Exam #2, which is Thursday.