

# Introduction to Logical Reasoning

## *Argument Patterns*

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# Modus Ponens

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Consider the argument:

1. If I study hard, then I pass the class.
  2. I study hard.
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- $\therefore$  I pass the class.

This can be formalized as follows:

1.  $S \rightarrow P$ .
  2.  $S$ .
- 
- $\therefore P$ .

# Modus Ponens

A truth table shows that this argument is valid:

S	P	$S \rightarrow P$
T	T	T
T	F	F
F	T	T
F	F	T

Premise 2      Conclusion      Premise 1

# Modus Ponens

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This argument has the following general form,  
which is known as *modus ponens* (M.P.):

$$\begin{array}{l} 1. \ p \rightarrow q. \\ 2. \ p. \\ \hline \therefore q. \end{array}$$

So any inference that has this form—i.e., affirming (1)  
a hypothetical and (2) its antecedent to imply ( $\therefore$ )  
affirming its consequent—is logically valid.

# Identifying Patterns

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This same pattern may appear in arguments that appear to be more complicated:

1. If I study hard and I attend every class, then I either pass the class or die trying.
  2. I study hard and I attend every class.
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- $\therefore$  I either pass the class or die trying.

Notice this is still just a (1) a hypothetical and (2) its antecedent implying ( $\therefore$ ) affirming its consequent.

# Identifying Patterns

This can be seen more clearly when formalizing the argument:

$$1. (S \& A) \rightarrow (P \vee D).$$

$$2. S \& A.$$

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$$\therefore P \vee D.$$

You can then see the pattern of M.P. emerge:

$$1. \overbrace{(S \& A)}^p \rightarrow \overbrace{(P \vee D)}^q.$$

$$2. \overbrace{S \& A}^p$$

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$$\therefore \overbrace{P \vee D}^q$$

# Argument Patterns

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Knowing commonly used argument patterns is extremely useful. Once you know that a particular pattern is logically valid, if you see that same pattern appear in another argument, you then know right away that this new argument is also logically valid.

# Argument Patterns

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So, for instance, any argument that has the pattern of *modus ponens*—no matter what content statements  $p$  and  $q$  may have, and no matter whether they positive, negative, or compound—is logically valid.

# Argument 1

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Consider the following argument:

If you are eighteen, then you can vote. You are eighteen. Therefore you can vote.

# Argument 2

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Consider the following argument:

If you are eighteen, then you can vote. You not  
eighteen. Therefore you cannot vote.

# Modus Tollens

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Another common argument pattern is known as *modus tollens* (M.T.):

$$\begin{array}{l} 1. \ p \rightarrow q. \\ 2. \ \sim q. \\ \hline \therefore \sim p. \end{array}$$

In this case (1) affirming a hypothetical statement but (2) denying its consequent is said to imply ( $\therefore$ ) denying its antecedent.

# Modus Tollens

And a truth table shows that this form is also valid:

$p$	$q$	$\sim p$	$\sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Conclusion      Premise 2      Premise 1

# Identifying Patterns

So anytime you see an inference where (1) a hypothetical is affirmed, (2) its consequent is denied, it is valid to conclude by ( $\therefore$ ) denying its antecedent. Once again, this is valid even when these three things are more complex:

$$1. (A \rightarrow B) \rightarrow \sim(C \vee D).$$

$$2. \sim\sim(C \vee D).$$

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$$\therefore \sim(A \rightarrow B).$$

$$1. \overbrace{(A \rightarrow B)}^p \rightarrow \overbrace{\sim(C \vee D)}^q.$$

$$2. \overbrace{\sim\sim(C \vee D)}^q.$$

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$$\therefore \overbrace{\sim(A \rightarrow B)}^p.$$

# Argument 3

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Consider the following argument:

If you are eighteen, then you can vote. You cannot vote. Therefore you are not eighteen.

# Fallacy of Affirming the Consequent

Now all argument patterns are good, however.

Consider the following common argument pattern:

$$1. \ p \rightarrow q.$$

$$2. \ q.$$

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$$\therefore p.$$

The pattern here is affirming both (1) a hypothetical and (2) its consequent in order to conclude ( $\therefore$ ) by affirming its antecedent.

# ❧ Fallacy of Affirming the Consequent

A truth table shows that this form is invalid:

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conclusion      Premise 2      Premise 1

# Fallacy of Affirming the Consequent

This is an extremely common fallacy known as **the fallacy of affirming the consequent**. For instance:

If I have good business skills, then I will earn a lot of money. I earn a lot of money. Therefore, I have good business skills.

On a quick read this (rather common) argument may seem logically valid. But on closer inspection, it has the same pattern as this fallacy. So it is invalid!

# 🐷 Fallacy of Denying the Antecedent

Here is another bad argument pattern:

$$1. p \rightarrow q.$$

$$2. \sim p.$$

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$$\therefore \sim q.$$

The pattern here is (1) affirming a hypothetical but (2) denying its antecedent in order to conclude ( $\therefore$ ) by denying its consequent.

# ❧ Fallacy of Denying the Antecedent

And a truth table shows that this form is also invalid:

$p$	$q$	$\sim p$	$\sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Premise 2      Conclusion      Premise 1

# Fallacy of Denying the Antecedent

This is another extremely common fallacy known as **the fallacy of denying the antecedent**. For instance:

If I have good business skills, then I will earn a lot of money. I do not have good business skills.

Therefore, I will not earn a lot of money.

On a quick read this may seem logically valid. But it has the same pattern as this fallacy. So it is invalid!

# Valid vs. Invalid Patterns

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It is sometimes easy to confuse a valid argument with a fallacy, so you need to be on guard!

- Do not confuse M.P. (affirming the *antecedent*) with the fallacy of affirming the *consequent*, and
- Do not confuse M.T. (denying the *consequent*) with the fallacy of denying the *antecedent*.

# Next Class...

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We will do a workshop on using truth tables to assess the validity of arguments.

(We will work more on identifying argument patterns after Spring Break.)