Introduction to Logical Reasoning

Symbolic Logic and Natural Language

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Symbolizing English

Taking a sentence in English and translating it to its logical form involves a two-step process:

1. Identify all the simple *positive* statements involved in the sentence or in the passage (labeling each with a capital letter), and

2. Identify the logical structure of any compound statements (using the symbols representing the various logical operators).

The Logic "Alphabet"

Specific **positive statements** are represented by the upper case, upright letters A, B, C, D, . . . , Z.

Generic **statements** (that is, statements that are either positive, negative, or compound) are represented by the lower case, italic letters p, q, r, . . . , z.

Five **logical operators** are represented by &, ~, ∨, →.

Grouping punctuation is represented by (,), [,], {, }.

Conjunction

Recall that a **conjunctive statement** asserts the truth of *all* its statements. It is symbolized using & (called "ampersand").

So the conjunctive statement *p* & *q* asserts that statements *p* and *q* are both true. In this example, *p* and *q* are known as **conjuncts**.

Conjunction: Basic Example

Consider the following conjunctive statement:

Logic is fun **and** logic is hard.

This is made up of two positive statements, which may be symbolized:

1. Logic is **f**un. (F)
2. Logic is **h**ard. (H)

So the entire statement is symbolized as F & H.

Conjunction: Further Examples

However, there are other ways to express the same logical claim symbolized by F & H:

Logic is fun **and** hard.

Logic is **both** fun **and** hard.

Logic is fun, **also** it is hard.

Logic is fun **but** hard.

Logic is fun, **yet** it is hard.

Logic is fun, **though** it is hard.

These certainly have different *connotations*, but they all have the same *logical* content.

Negation

Recall that a **negative statement** asserts that a given statement is false. It is symbolized using ~ (called "tilde").

So the negative statement ~*p* asserts that statement *p* is false.

Negation: Basic Example

Consider the following negative statement:

> Logic is **not** fun.

This is made up of one positive statement, which may be symbolized as:

> Logic is fun. (F)

So the entire statement is symbolized as ~F.

Negation: Further Examples

However, there are other ways to express the same logical claim symbolized by ~F:

It is false that logic is fun.

It is not the case that logic is fun.

It is not true that logic is fun.

Disjunction

Recall that a **disjunctive statement** asserts the truth of *at least one* of its statements. It is symbolized using ∨ (called "wedge").

So the disjunctive statement $p \vee q$ asserts that at least one of statements p and q is true. In this example, p and q are known as **disjuncts**.

Disjunction: Basic Example

Consider the following disjunctive statement:

> Logic is fun **or** logic is hard.

The two positive statements may be symbolized (as before) by F and H.

So the entire statement is symbolized as $F \vee H$.

Disjunction: Further Examples

However, there are other ways to express the same logical claim symbolized by $F \vee H$:

Logic is fun **or** hard.

Logic is **either** fun **or** hard.

Logic is fun **unless** it is hard.

As before, these certainly have different connotations, but they all are all logically identical.

Inclusive vs. Exclusive

The word 'or' (and the word 'unless') can be used in two slightly different, but significant, ways.

Logic is fun **or** hard.

This is ***inclusive*** **disjunction**, where the claim is that *at least one* of the statements is true. Notice that this claim is still true when logic is both fun and hard. This is the type of disjunction represented by $\vee$.

So this is symbolized as $L \vee H$.

Inclusive vs. Exclusive

I will pass **or** fail logic.

This is ***exclusive*** **disjunction**, where the claim is that *exactly one* of the statements is true. This statement is more precisely stated as:

I will pass **or** fail logic, **but not both**.

This is then symbolized differently:

$$(P \vee F)\ \&\ {\sim}(P\ \&\ F).$$

or   but   not   both

Implication

Recall that a **hypothetical statement** has the form of "if… then", asserting that whenever the "if" part is true, the "then" part must be true as well. It is symbolized using $\rightarrow$ (called "arrow").

So the hypothetical statement $p \rightarrow q$ asserts that if statement p is true, then statement q is true. In this example, p is known as the **antecedent**, and q is known as the **consequent**.

Implication: Basic Example

Consider the following hypothetical proposition:

If I study hard **then** I pass the class.

The two positive statements may be symbolized as:

1. I **s**tudy hard. (S)
2. I **p**ass the class. (P)

So the entire statement is symbolized as $S \rightarrow P$.

Implication: Further Examples

However, there are other ways to express the same logical claim symbolized by $S \rightarrow P$:

If I study hard I pass the class.

My studying hard will **cause** me to pass the class.

I study hard **only if** I pass the class.

Studying hard is a **sufficient condition** for passing the class.

I pass the class **if** I study hard.

Passing the class is a **necessary condition** for studying hard.

These are the tricky ones to remember!

Sufficient vs. Necessary

Notice that "p is a ***sufficient*** **condition** for q" is symbolized as $p \rightarrow q$. Sufficiency means that p is *enough* (but may not be required) to get q.

However, "p is a ***necessary*** **condition** for q" is symbolized as $q \rightarrow p$. Necessary means that p is *required* (but may not be enough) to get q.

Sufficient vs. Necessary

Passing **l**ogic is a sufficient condition for fulfilling the journalism **m**ath requirement.

$L \rightarrow M.$

Passing logic is *enough* to fulfill the requirement. But passing logic is *not required* to do so: passing statistics is an alternative for the requirement.

The idea is if you pass logic (L), then you have fulfilled the math requirement (M). So $L \rightarrow M$.

Sufficient vs. Necessary

Passing a **h**istory course is a necessary condition for earning a **d**egree in journalism.

$D \rightarrow H$.

Passing a history course is *required* to earn the degree, but passing it is *not enough* to do so: you have to pass a lot of other courses as well.

The idea is that if you earn the degree (D), then you passed a history course (H). So $D \rightarrow H$.

Next Class…

We will learn how take apart statements and assess their truth value by using truth tables.