Introduction to Logical Reasoning

Workshop on Basic Set Theory

David Emmanuel Gray

Northwestern University in Qatar
Carnegie Mellon University in Qatar

Find Your Group

These are the groups (based on the most visited blogs on the Internet because everyone likes to read blogs):

Boing Boing

- Mohammed Hammouda
- Alanna Alexander

Engadget

- Sara Al-Thani
- Maha Al-Ansari

Mashable!

- Gena El Aker
- Saoud Faiadh

TechCrunch

- Ola Diab
- Penny Wang

The Daily Beast

- Amit Chowdhury
- Marie-Joe Khachan

The Huffington Post

- Muneera Al-Buainain
- Maha Al-Moghany

Think Progress

- Marium Saeed
- Ridin Balakrishnan

TMZ | Celebrity Gossip

- Aamena Ahmed
- Angel Polacco

Gizmodo

- Jaimee Haddad
- Amna Elsaka

Question 1

Are you done with Part I of the workshop?

(**A**) No.

(**B**) Yes.

(**C**) Yes!

(**D**) Yes!!

(**E**) Yes yes yes, already!

Part 1, Problem 1 Solution

Is watermelon $\in A$?

No.

$A = \{\text{apple, orange}\}$, and so watermelon is not in set A.

Part 1, Problem 2 Solution

Is watermelon $\in B$?

Yes.

$B = \{$apple, guava, watermelon$\}$, and so watermelon is in set B.

Part 1, Problem 3 Solution

How many subsets does A have? Name them all, labeling them $A1$, $A2$, etc.

A has $2^2 = 4$ subsets. They are:

$A1 = \varnothing$,

$A2 = \{\text{apple}\}$,

$A3 = \{\text{orange}\}$, and

$A4 = \{\text{apple, orange}\}$.

Part 1, Problem 4 Solution

How many proper subsets does *A* have? Name them all, feeling free to refer to those sets already labeled from problem 3.

A has $2^2 - 1 = 3$ proper subsets. They are A_1, A_2, and A_3 from problem 3. A_4 is not included because it is the same as *A* and so not a *proper* subset of *A*.

Part 1, Problem 5 Solution

Is $A \subseteq B$?

No.

While A and B both have apple in them, A also has orange, which does not appear in B. So A cannot be a subset of B.

Part 1, Problem 6 Solution

Is $A \subseteq \mathcal{U}_2$?

Yes.

Everything in A is also in $\mathcal{U}_2$. That is, A contains apple and orange, and both of these are also in $\mathcal{U}_2$. So A is a subset of $\mathcal{U}_2$.

Part 1, Problem 7 Solution

Is $B \subseteq \mathcal{U}_2$?

No.

While B and $\mathcal{U}_2$ both have apple and watermelon in them, B also has mango, which does not appear in $\mathcal{U}_2$. So B cannot be a subset of $\mathcal{U}_2$.

Part 1, Problem 8 Solution

Is $A \subset C$?

No.

While A and C both have apple and orange in them, there is nothing “extra” in C that is not also in A. So A cannot be a proper subset of C.

[Just because apple is repeated three times in C does not make any difference.]

Part 1, Problem 9 Solution

Is $A = B$?

No.

A and B are clearly do not have the same contents. A has orange, which B does not have; and B has watermelon and mango, which A does not have. So A and B are not equivalent to each other.

Part 1, Problem 10 Solution

Is $A = C$?

Yes.

Both A and C only have apple and orange, so they have the same contents. Therefore, they are equivalent to each other.

[Just because apple is repeated three times in C does not make any difference.]

Part 1, Problem 11 Solution

Is $\mathcal{U}_1 \subseteq \mathcal{U}_2$?

No.

$\mathcal{U}_1$ contains all fruit in it, whereas $\mathcal{U}_2$ contains a precise list of fruits. So, for instance, $\mathcal{U}_1$ would contain mango, while this is not in $\mathcal{U}_2$.

Part 1, Problem 12 Solution

Is $E \subseteq \mathcal{U}_2$?

I answer this yes, because I had an apple for breakfast. That is, for me (Professor Gray) $E = \{\text{apple}\}$, and apple also appears in $\mathcal{U}_2$. Therefore, E is a subset of $\mathcal{U}_2$.

Your answer may differ based on what fruit (if any) you had for breakfast this morning.

Part 1, Problem 13 Solution

Specify the intersection of A and B:

$A \cap B = \{\text{apple}\}$.

This is because apple is the only thing the two sets have in common.

Part 1, Problem 14 Solution

Specify the union of A and B:

$A \cup B$ = {apple, orange, mango, watermelon}.

This is because the union is simply the combination of both sets. Notice that apple is not repeated (since it appears in both sets). It should only appear once.

Part 1, Problem 15 Solution

Specify the intersection of A and D:

$A \cap D = \varnothing$.

This is because the sets have nothing in common, so their intersection is the empty set.

Part 1, Problem 16 Solution

Specify the union of A and D:

$A \cup D$ = {apple, orange, mango, watermelon}.

This is because the union is simply the combination of both sets.

Part 1, Problem 17 Solution

Are A and B disjoint sets?

No.

As seen in the solution to problem 13, the intersection between the two sets is not empty. So they are not disjoint sets.

Part 1, Problem 18 Solution

Are *A* and *D* disjoint sets?

Yes.

As seen in the solution to problem 15, the intersection between the two sets is empty. So they are disjoint sets.

Part 1, Problem 19 Solution

Let $\mathcal{U}_1$ be the universal set for E, specify $\bar{E}$ by the rule method:

$\bar{E} = \{x \mid x$ is a fruit that you did not have for breakfast this morning$\}$.

Part 1, Problem 20 Solution

Let $\mathcal{U}_2$ be the universal set for A, specify $\overline{A}$ by the roster method:

$\overline{A} = \{$ guava, watermelon, kiwi, banana$\}$.

This is the list of fruit in $\mathcal{U}_2$ that are not present in A.

Question 2

Are you done with Part II of the workshop?

(**A**) No.

(**B**) Yes.

(**C**) Yes!

(**D**) Yes!!

(**E**) Yes yes yes, already!

Part 2 Solution

1. $(\overline{S} \cap \overline{T}) \cap \overline{W}$.
2. $(S \cap \overline{T}) \cap \overline{W}$.
3. $(S \cap T) \cap \overline{W}$.
4. $(\overline{S} \cap T) \cap \overline{W}$.
5. $(S \cap \overline{T}) \cap W$.
6. $(S \cap T) \cap W$.
7. $(\overline{S} \cap T) \cap W$.
8. $(\overline{S} \cap \overline{T}) \cap W$.

Next Class…

We will begin to use Venn diagrams for visualizing the logical structure of categorical claims.

Also, please don't forget to turn in your response to the Workshop #9 Questionnaire on your way out.