Introduction to Logical Reasoning

Further Categorical Inferences

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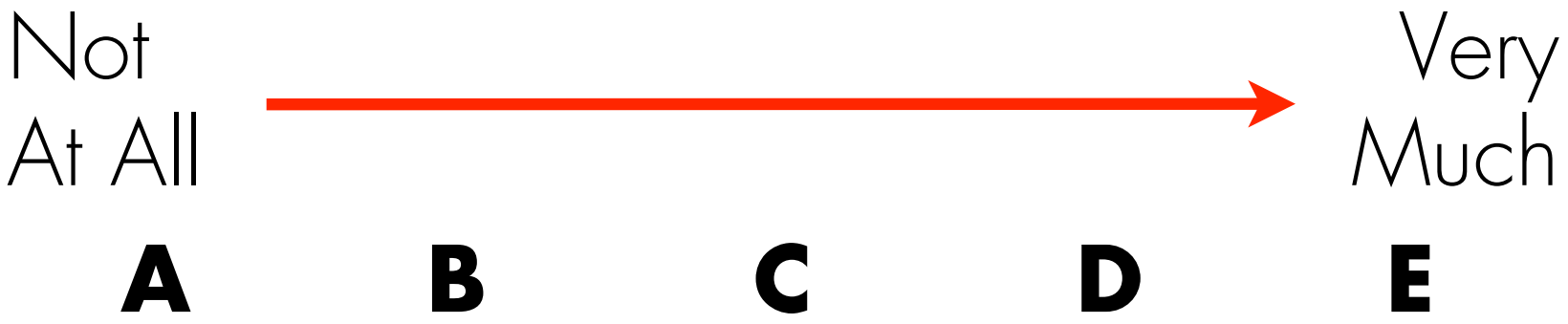
Question 1

Indicate how you felt about solving the problems on today's quiz:

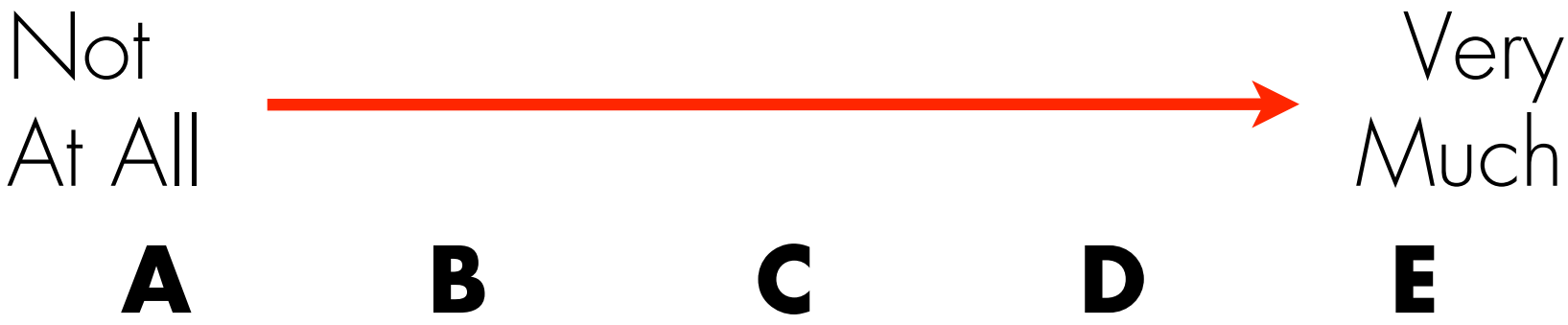
Did you understand what you were supposed to do to solve these problems?



Question 2

Indicate how you felt about solving the problems on today's quiz:

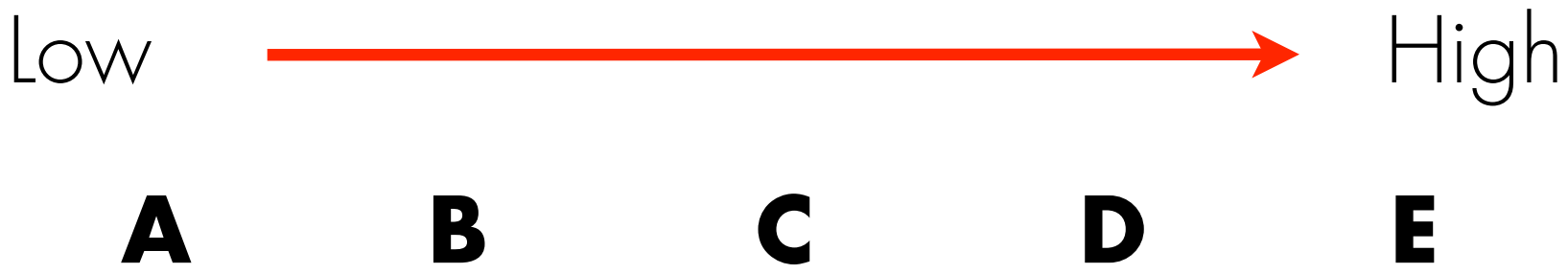
Did you receive adequate feedback for solving these problems?



Question 3

Indicate how you felt about solving the problems on today's quiz:

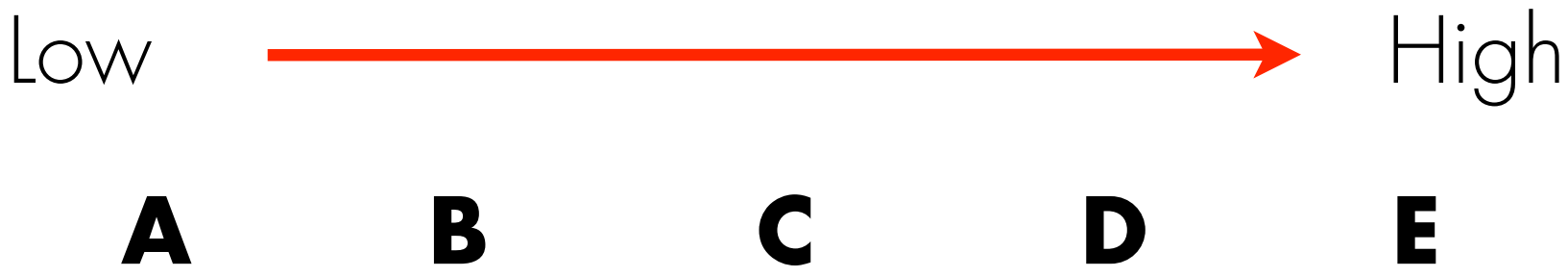
The challenges of solving these problems.



Question 4

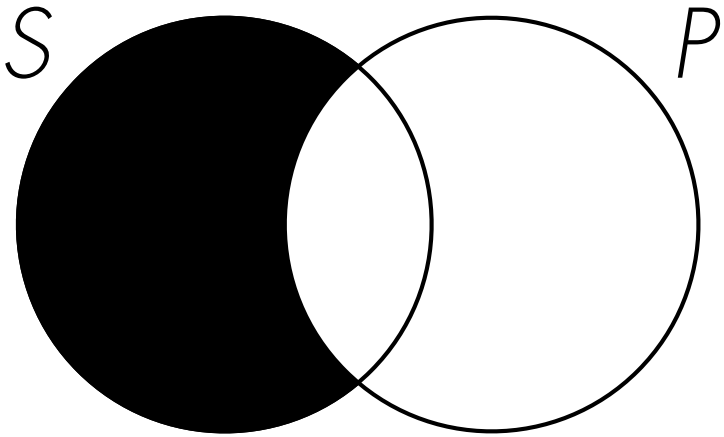
Indicate how you felt about solving the problems on today's quiz:

Your skills in solving these problems.

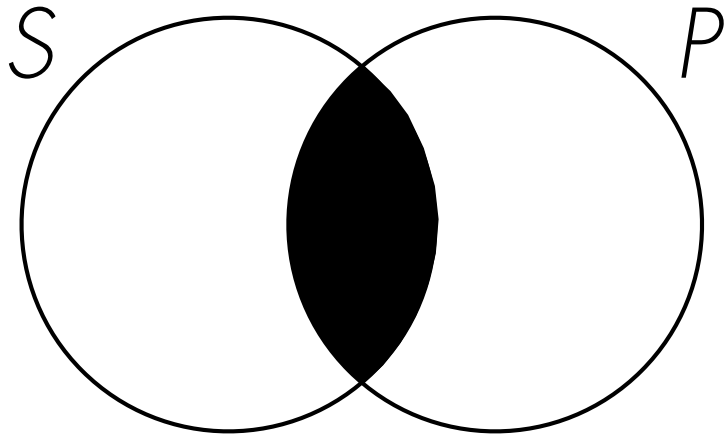


Categorical Statements

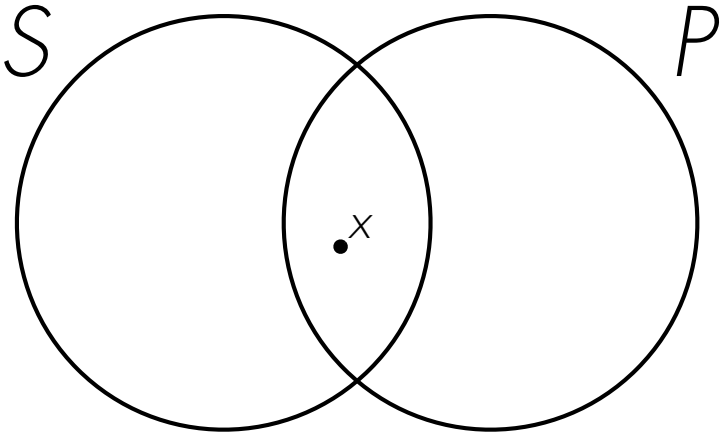
Recall the four standard forms of categorical statements:



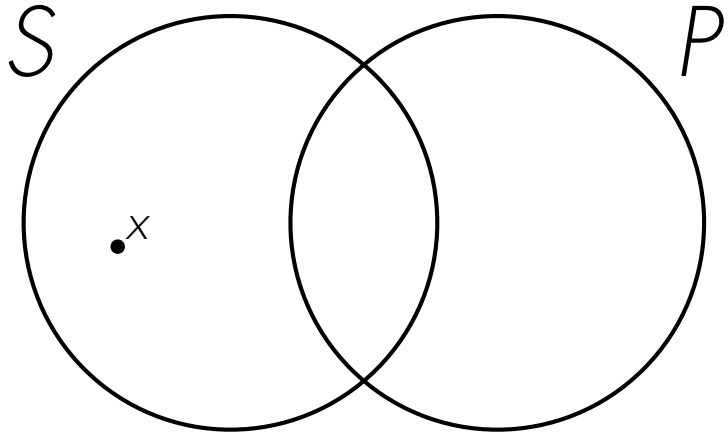
A: All *S* is *P*.



E: No *S* is *P*.



I: Some *S* is *P*.



O: Some *S* is not *P*.

Statement 1

Consider the following categorical statement:

No students are lazy people.

Question 5

Suppose statement 1 is *true*.

What can we then infer about the claim that "No lazy people are students"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 1 tells us nothing about the truth or falsity of the above claim).

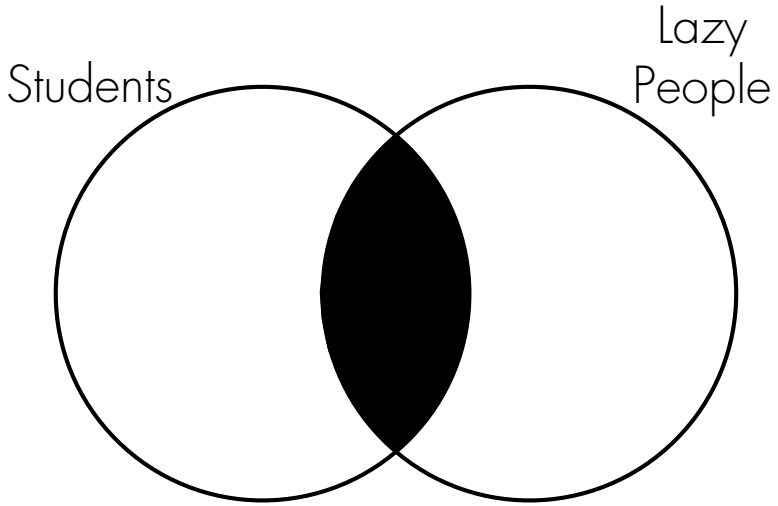
Conversion

The **conversion** of a categorical statement swaps the subject and predicate to create a new categorical statement.

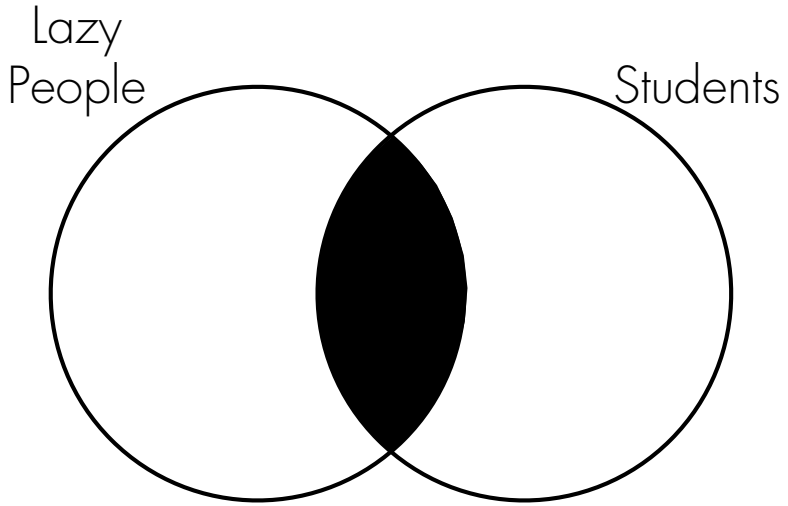
In some instances, the new statement will be logically equivalent to the original one. For example, the statement "No students are lazy" (**E**) is logically the same as "No lazy persons are students" (**E**).

Conversion

The Venn diagrams for **E** and its conversion confirm that they are logically the same.



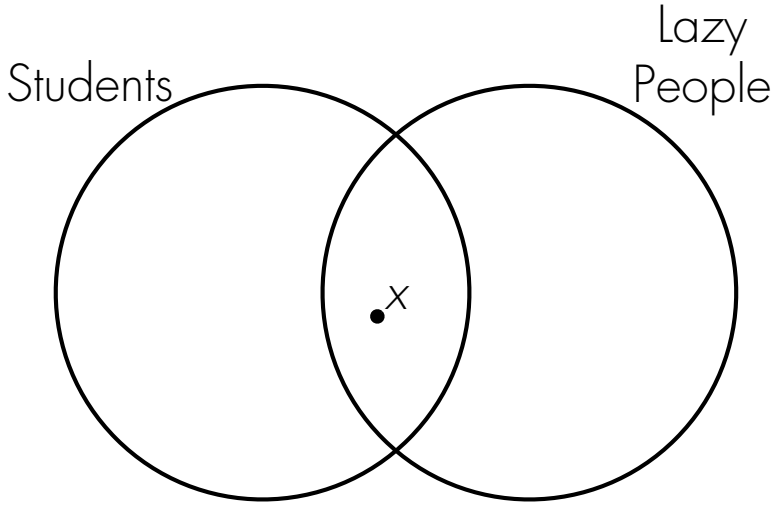
E Statement
(No *S* is *P*)



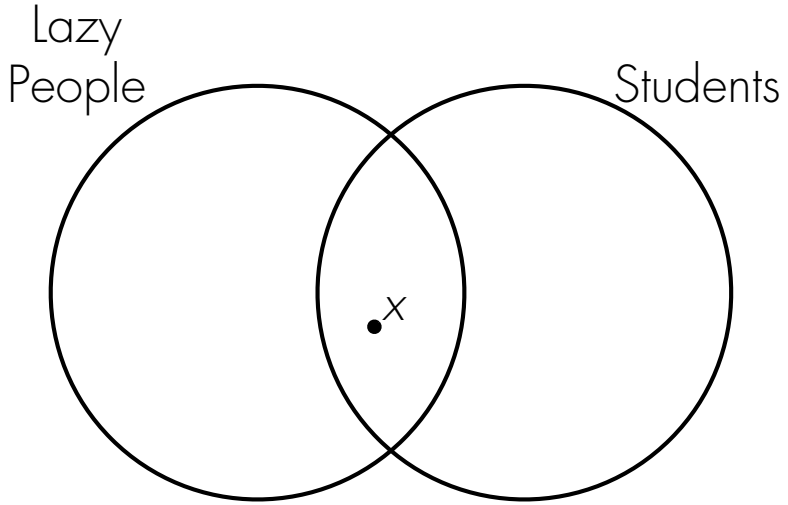
E's Conversion
(No *P* is *S*)

Conversion

Similarly, the Venn diagrams for **I** and its conversion confirm that they are also logically the same.



I Statement
(Some *S* is *P*)



I's Conversion
(Some *P* is *S*)

Statement 2

Consider the following categorical statement:

Some students are not lazy people.

Question 6

Suppose statement 2 is *true*.

What can we then infer about the claim that "Some lazy people are not students"?

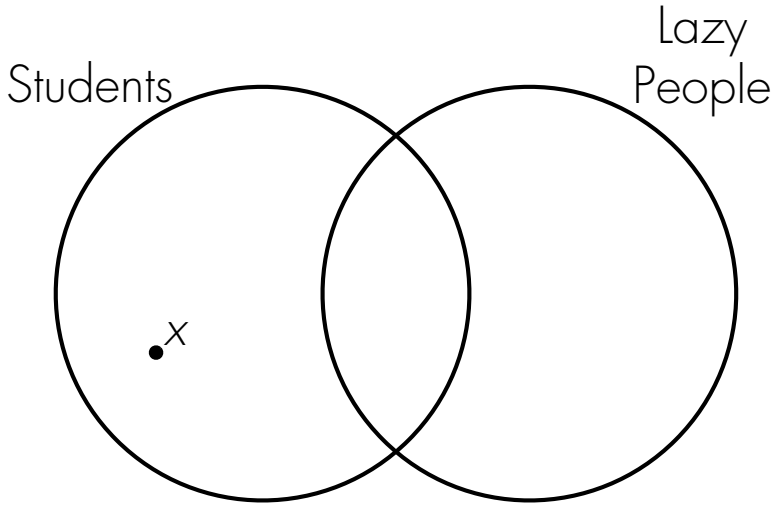
(**A**) It is true,

(**B**) It is false, or

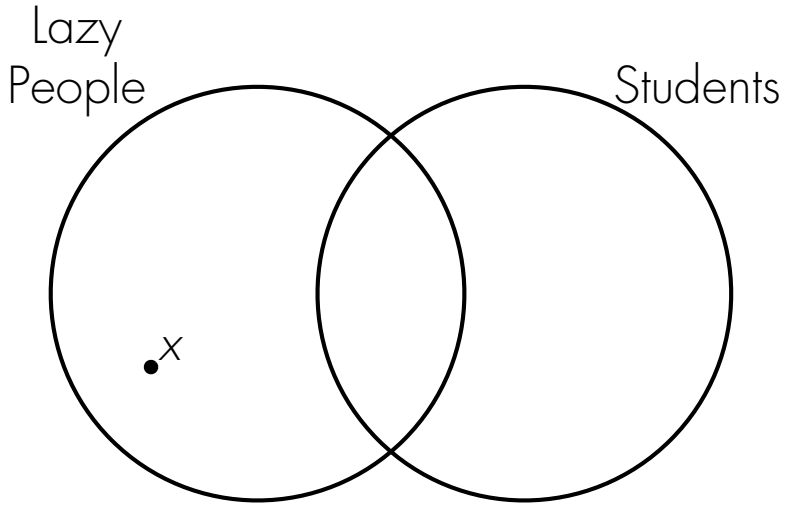
(**C**) Nothing (statement 2 tells us nothing about the truth or falsity of the above claim).

Conversion

The Venn diagrams for **O** and its conversion reveal that they are *not* logically the same.



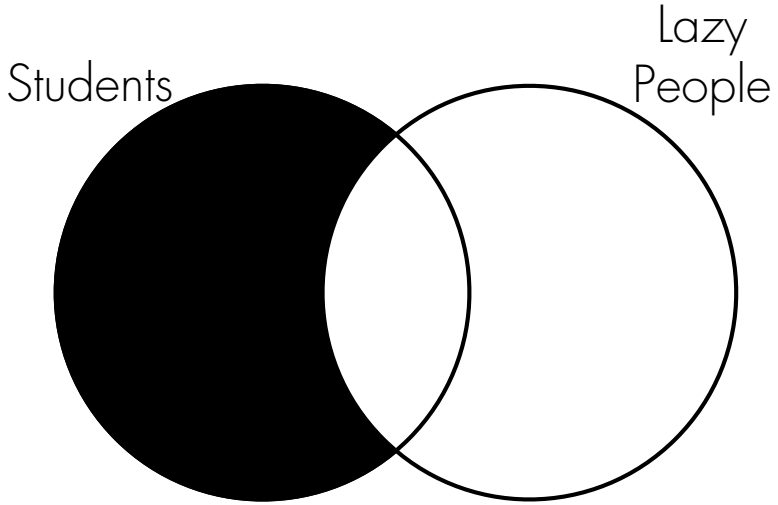
O Statement
(Some *S* is not *P*)



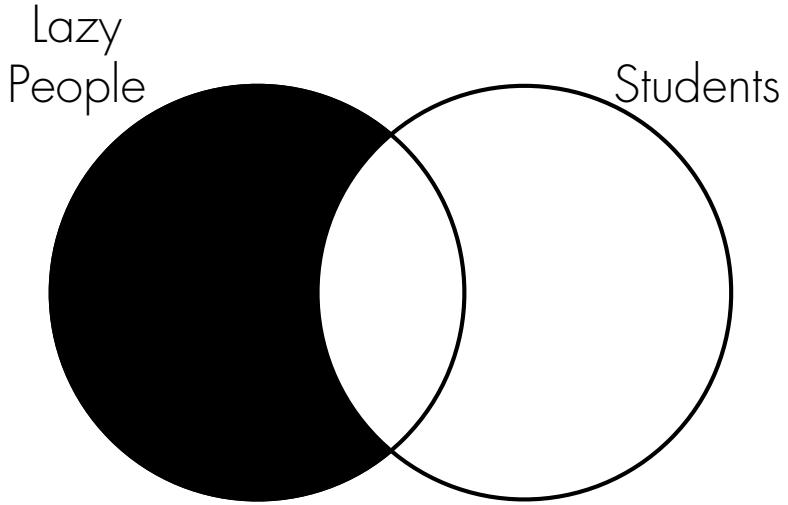
O's Conversion
(Some *P* is not *S*)

Conversion

Similarly, the Venn diagrams for **A** and its conversion reveal that they are not logically the same either.



A Statement
(All *S* is *P*)



A's Conversion
(All *P* is *S*)

Complement

For any subject (S) or predicate (P) in a categorical statement, we may consider its **complement**. The The complement of the subject is denoted as non-S (or $\overline{S}$); the complement of the predicate is denoted by non-P (or $\overline{P}$).

In English, for example, the complement of "students" is "non-students", while the complement of "lazy people" is "non-lazy people".

Statement 3

Consider the following categorical statement:

All students are lazy people.

Question 7

Suppose statement 3 is *true*.

What can we then infer about the claim that "No students are non-lazy"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 3 tells us nothing about the truth or falsity of the above claim).

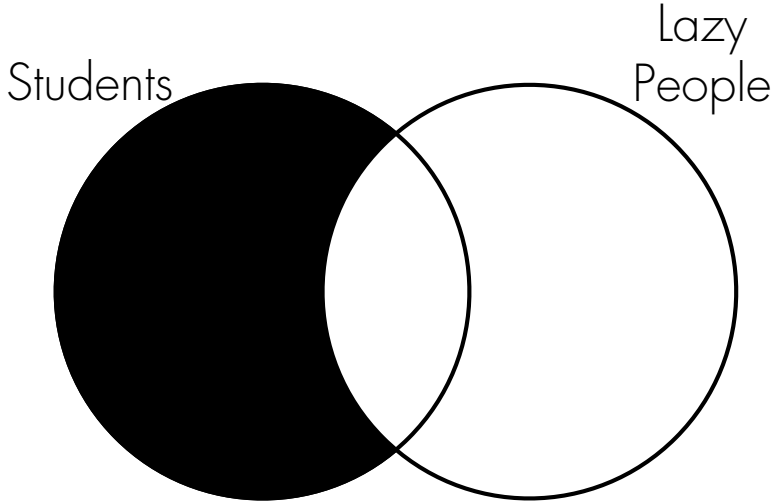
Obversion

The **obversion** of a categorical statement comes from flipping its quality and replacing the predicate with that predicate's complement.

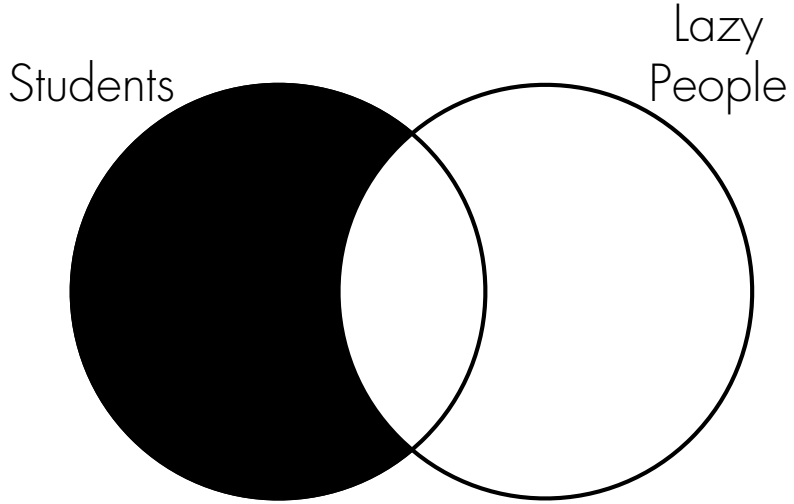
It turns out that the obversion of each of the standard four categorical statements is logically equivalent to the original statement. So, for instance, "All students are lazy" (**A**) is logically equivalent to its obversion: "No students are non-lazy" (**E**).

Obversion

The Venn diagrams for **A** and its obversion (an **E** statement) reveal that they are logically the same.



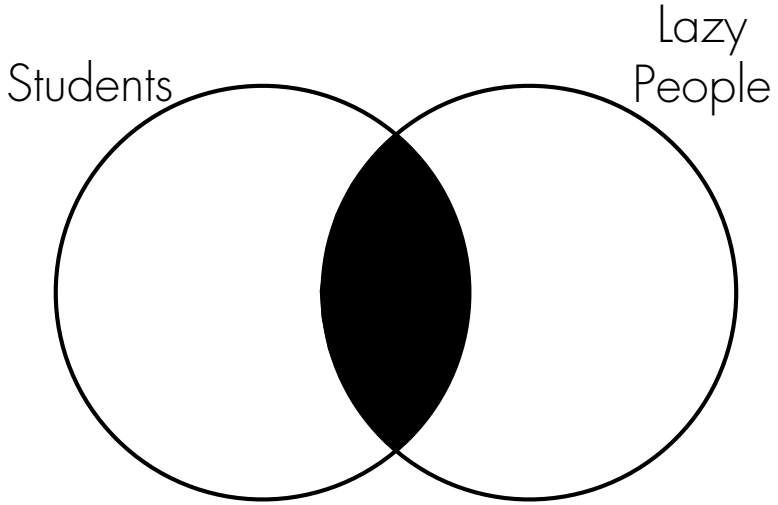
A Statement
(All *S* is *P*)



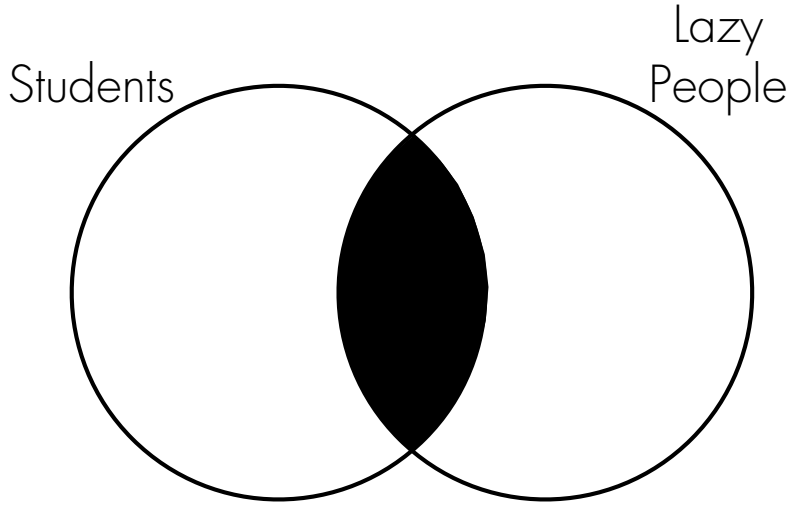
A's Obversion (**E**)
(No *S* is non-*P*)

Obversion

Similarly, **E** and its obversion (an **A** statement) are also logically the same.



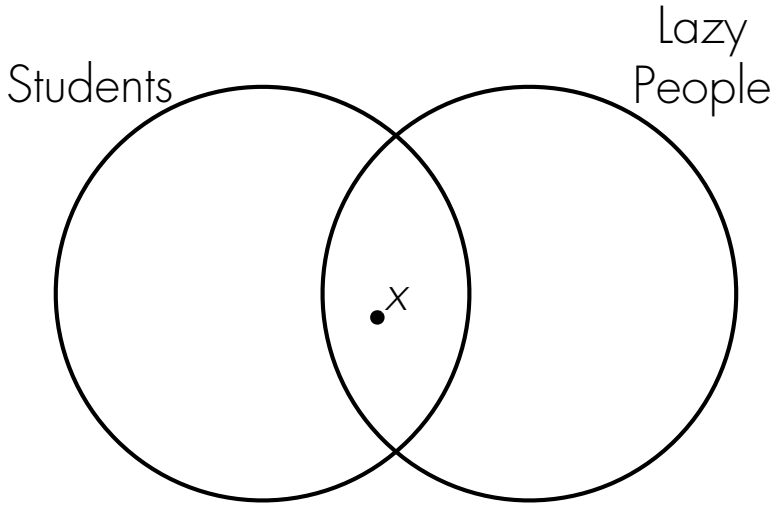
E Statement
(No *S* is *P*)



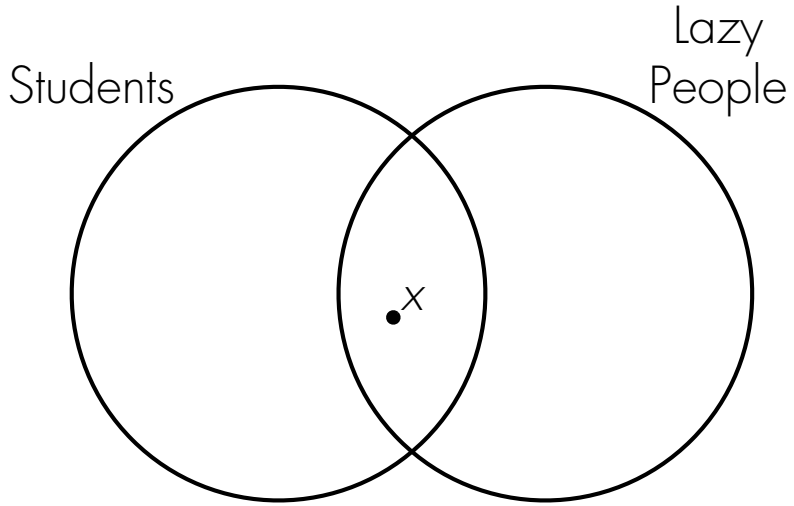
E's Obversion (**A**)
(All *S* is non-*P*)

Obversion

And so for **I** and its obversion (an **O** statement).



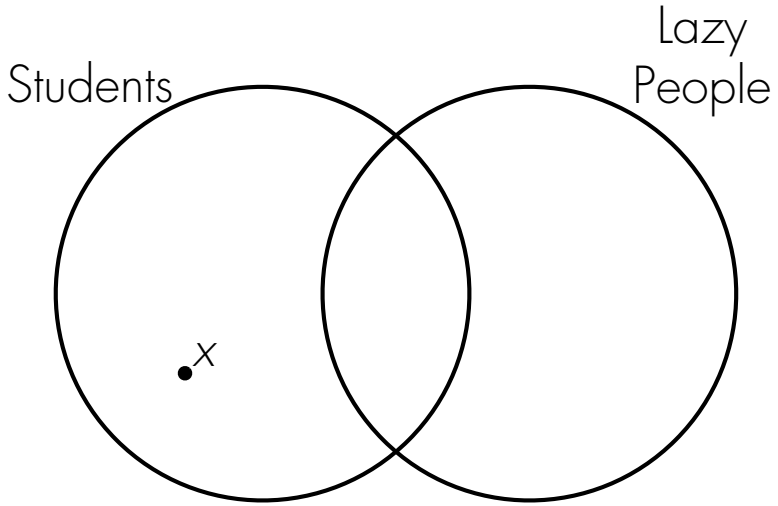
I Statement
(Some *S* is *P*)



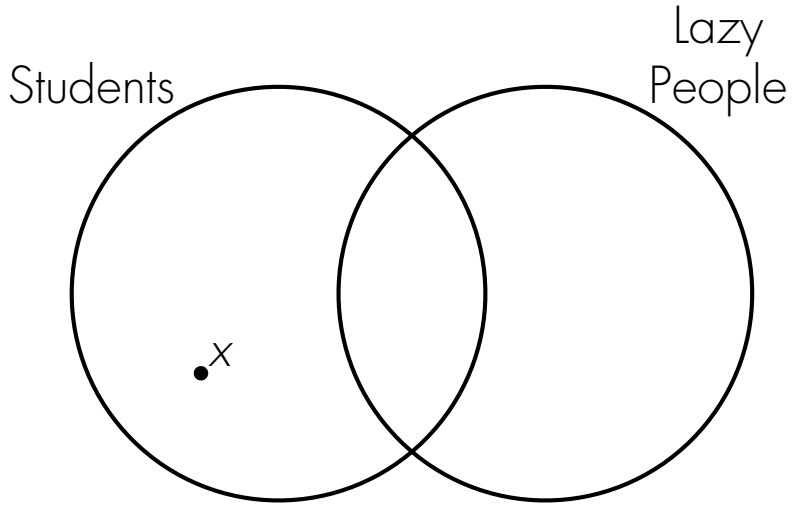
I's Obversion (**O**)
(Some *S* is not non-*P*)

Obversion

And finally for **O** and its obversion (an **I** statement).



O Statement
(Some *S* is not *P*)



O's Obversion (**I**)
(Some *S* is non-*P*)

Question 8

Suppose statement 3 is *true*.

What can we then infer about the claim that "All non-lazy people are non-students"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 3 tells us nothing about the truth or falsity of the above claim).

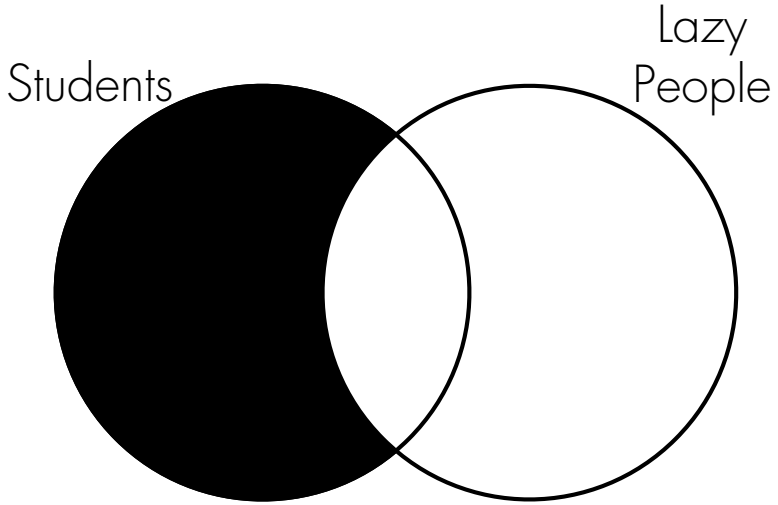
Contraposition

According to **contraposition**, a categorical statement is changed by (1) replacing its subject with that subject's complement, (2) replacing its predicate with that predicate's complement, and (3) swapping this new subject and new predicate.

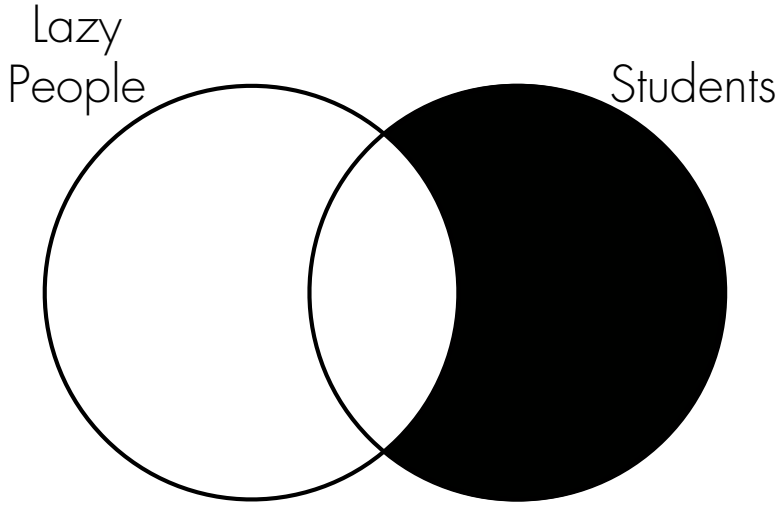
In some instances, the new statement will be logically equivalent to the original one. For example, the proposition "All students are lazy" (**A**) is logically the same as "All non-lazy people are non-students" (**A**).

Contraposition

The Venn diagrams for **A** and its contrapositive confirm that they are logically the same.



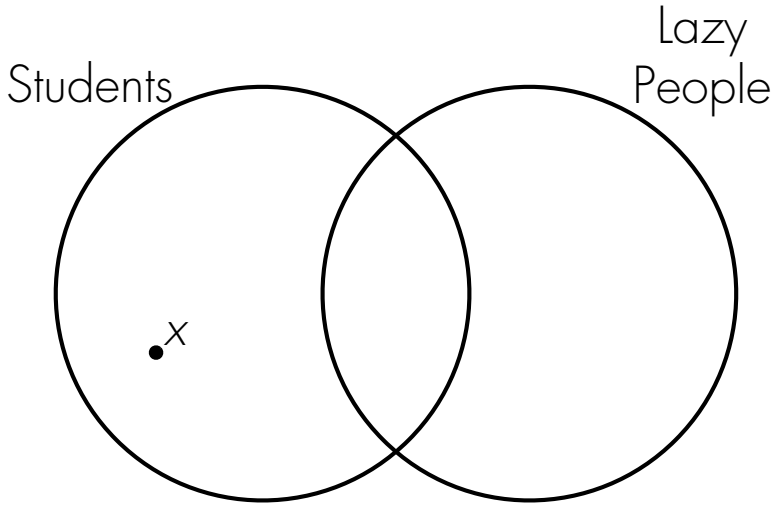
A Statement
(All *S* is *P*)



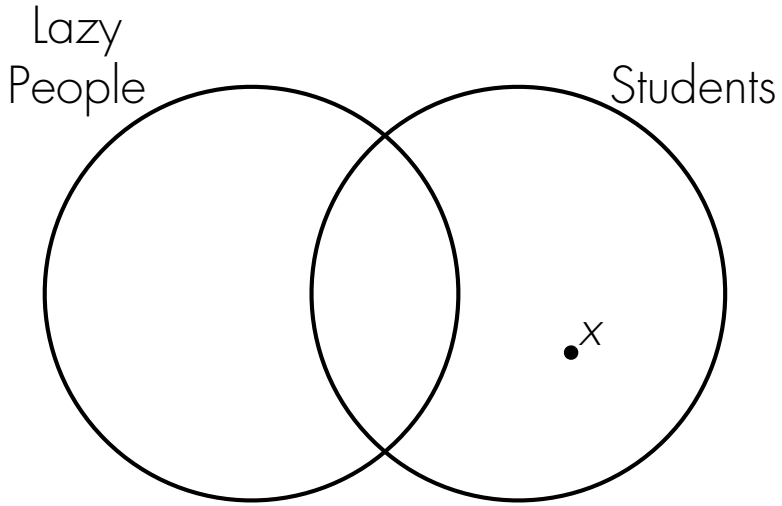
A's Contrapositive
(All non-*P* is non-*S*)

Contraposition

Similarly, the diagrams for **O** and its contrapositive confirm that they are also logically the same.



O Statement
(Some *S* is not *P*)



O's Contrapositive
(Some non-*P* is not non-*S*)

Statement 4

Consider the following categorical statement:

Some students are lazy people.

Question 9

Suppose statement 4 is *true*.

What can we then infer about the claim that "Some non-lazy people are non-students"?

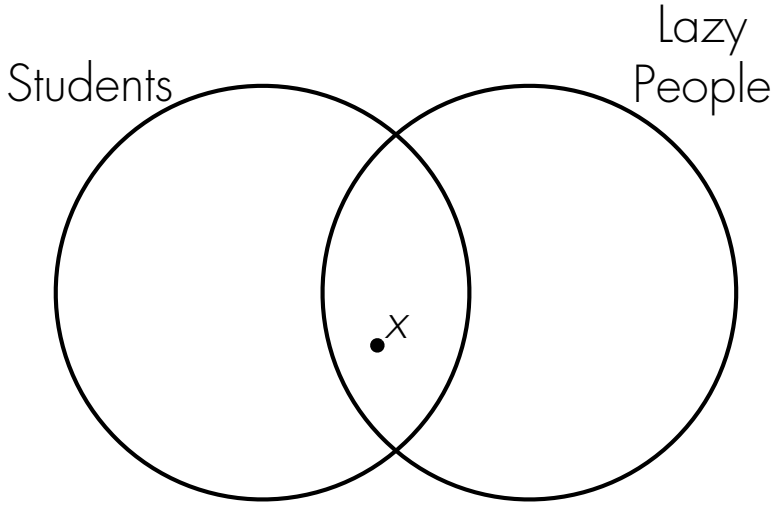
(**A**) It is true,

(**B**) It is false, or

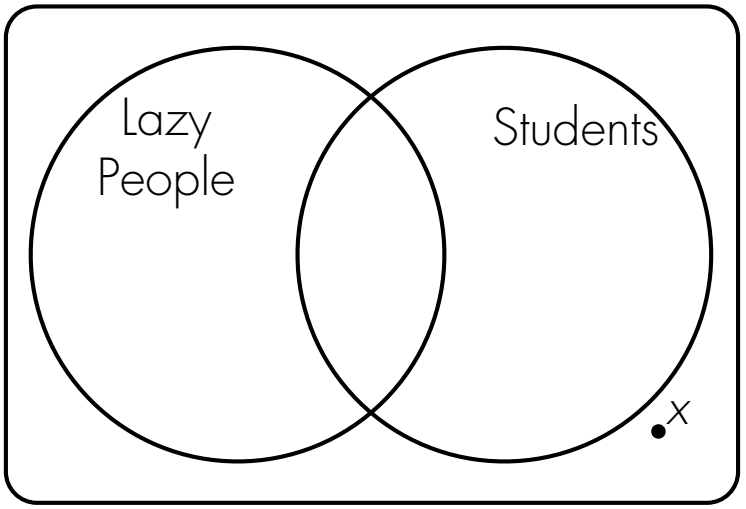
(**C**) Nothing (statement 3 tells us nothing about the truth or falsity of the above claim).

Contraposition

However, the Venn diagrams for **I** and its contraposition are *not* logically the same.



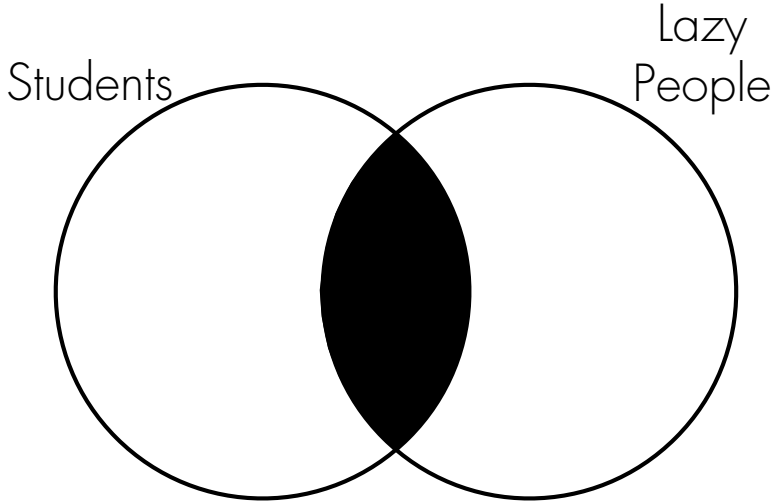
I Statement
(Some *S* is *P*)



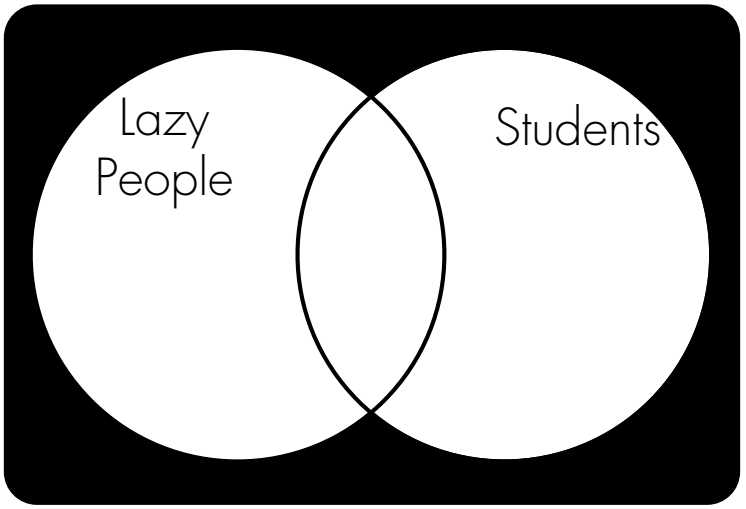
I's Contrapositive
(Some non-*P* is non-*S*)

Contraposition

Similarly, the diagrams for **E** and its contraposition reveal that they are not logically the same either.



E Statement
(No *S* is *P*)



E's Contrapositive
(No non-*P* is non-*S*)

Categorical Inferences

Don't let this table overwhelm you. Never forget, if you get lost, make a Venn diagram. From this simple picture, you should be able verify any of these inferences.

Next Class...

We will look at how to assess the validity of arguments known as categorical syllogisms.