Introduction to Logical Reasoning

The Square of Opposition

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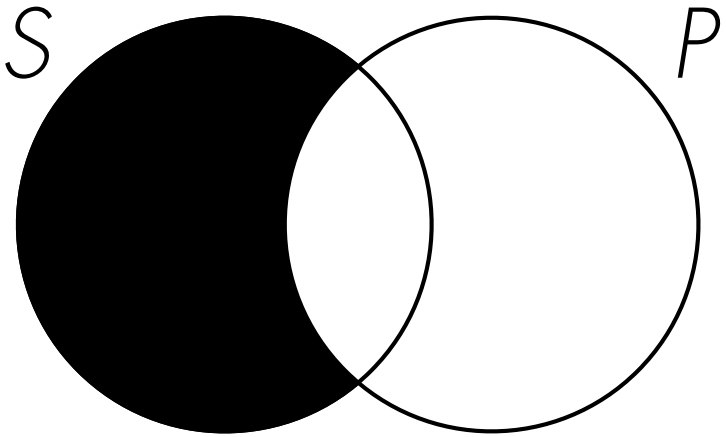
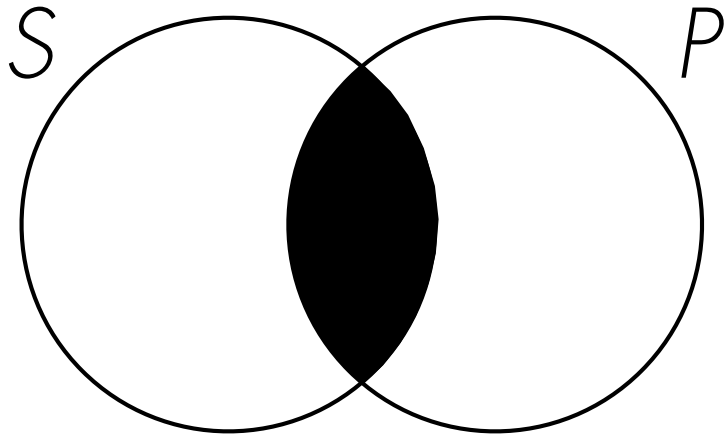
Categorical Statements

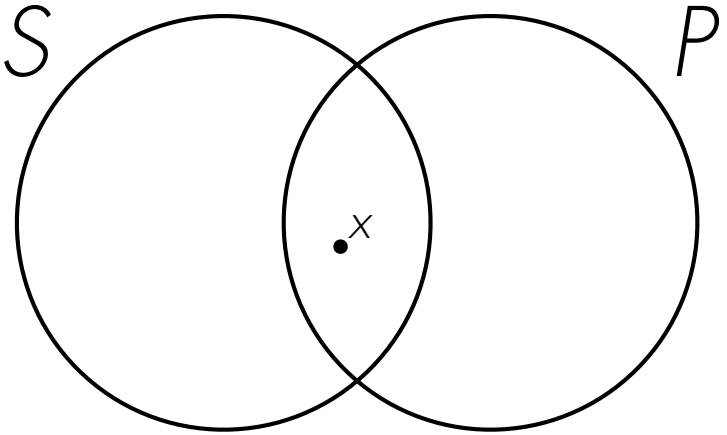
Recall the four standard forms of categorical statements:



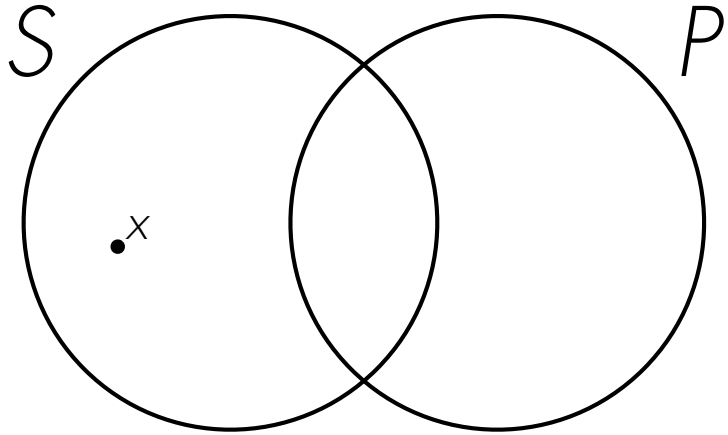
A: All *S* is *P*.



E: No *S* is *P*.



I: Some *S* is *P*.



O: Some *S* is not *P*.

Statement 1

Consider the following categorical statement:

All students are hard workers.

Question 1

Suppose statement 1 is true.

What can we then infer about the claim that "Some students are not hard workers"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 1 tells us nothing about the truth or falsity of the above claim).

Question 2

Suppose statement 1 is false.

What can we infer about the claim that "Some students are not hard workers"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 1 tells us nothing about the truth or falsity of the above claim).

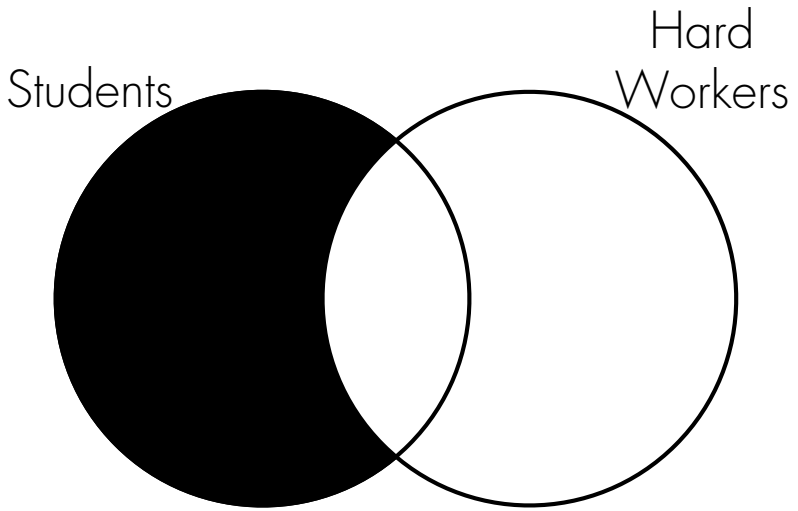
Contradictories

Two statements are **contradictories** if they both cannot be true and both cannot be false.

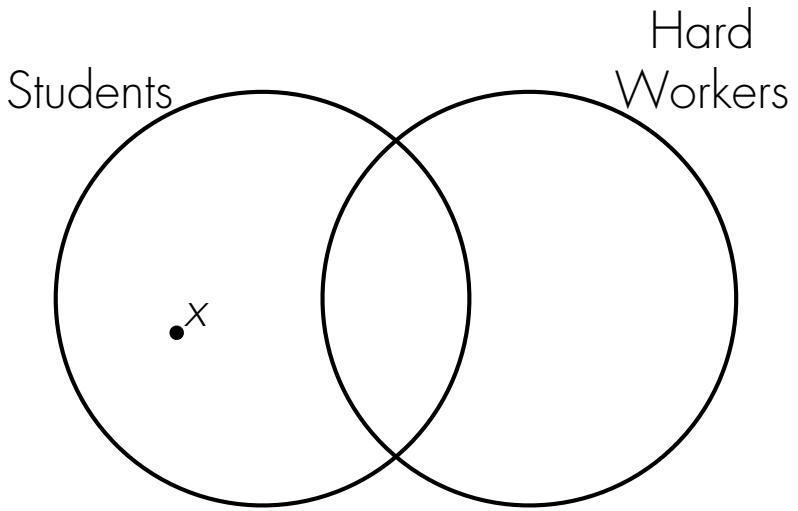
For instance, the statements "All students are hard workers" (**A**) and "Some students are not hard workers" (**O**) are contradictories. Both cannot be true and both cannot be false. So if you know one is true, the other must be false, and vice versa.

Contradictories

The Venn diagrams for **A** and **O** statements confirm that they are contradictories.



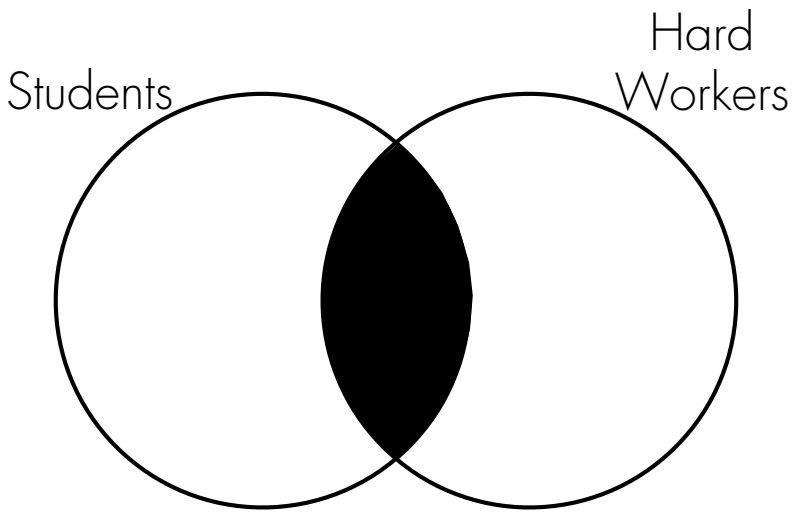
A Statement
(All *S* is *P*)



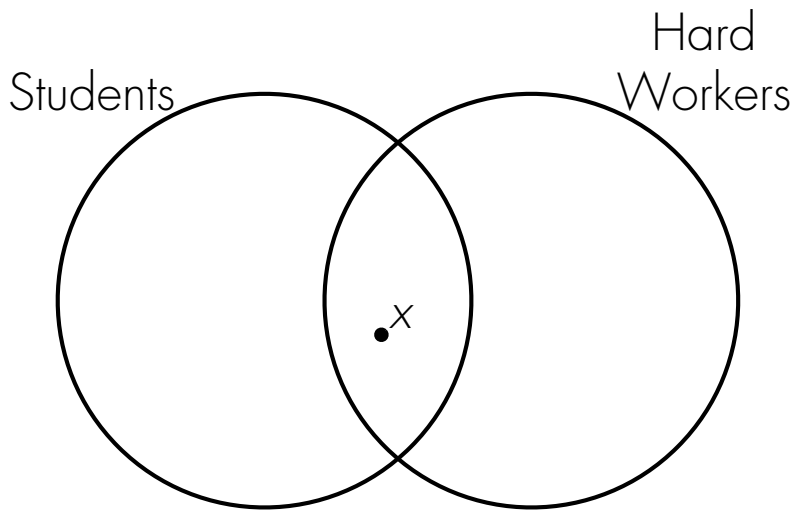
O Statement
(Some *S* is not *P*)

Contradictories

Similarly, the Venn diagrams for **E** and **I** statements confirm that they are also contradictories.



E Statement
(No *S* is *P*)



I Statement
(Some *S* is *P*)

Question 3

Suppose statement 1 is true.

What can we infer about the claim that "No students are hard workers"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 1 tells us nothing about the truth or falsity of the above claim).

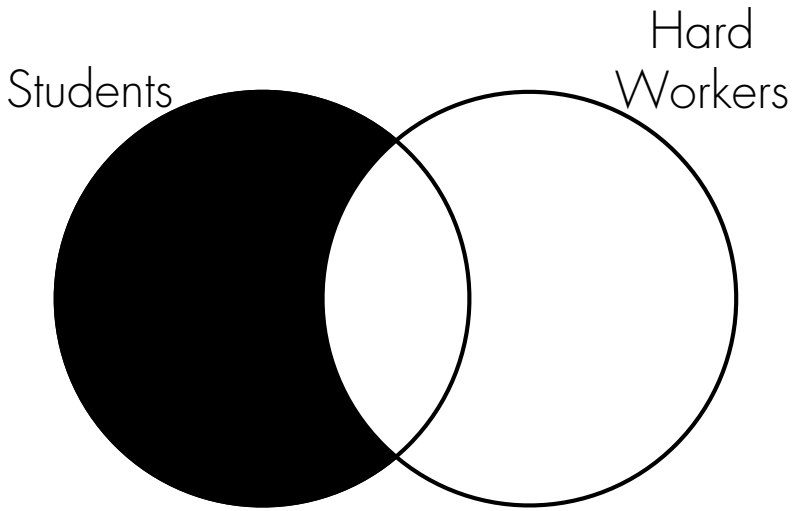
Contraries

Two statements are **contraries** if they both cannot be true, though both may be false.

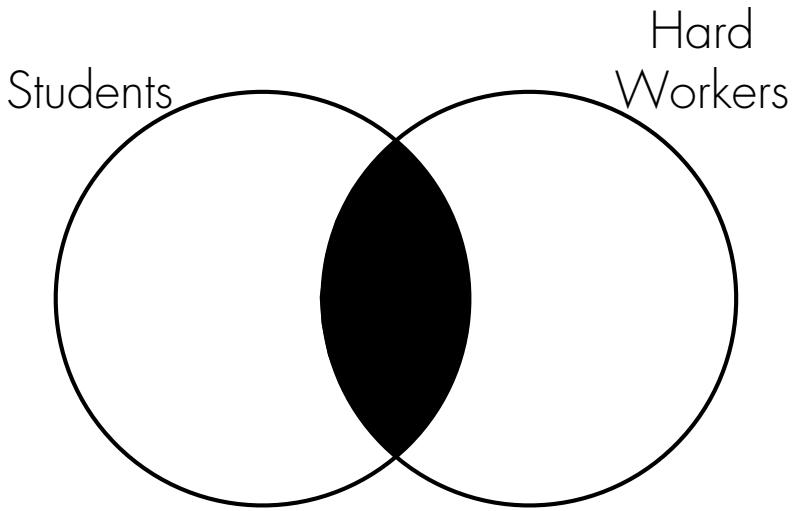
For instance, the statements "All students are hard workers" (**A**) and "No students are hard workers" (**E**) are contraries. Both cannot be true: if one is true, the other must be false. However, both positions could, in fact, be false. There might be some students who are hard workers and some others who are not.

Contraries

Venn diagrams confirm that corresponding **A** and **E** statements cannot both be true.*



A Statement
(All *S* is *P*)

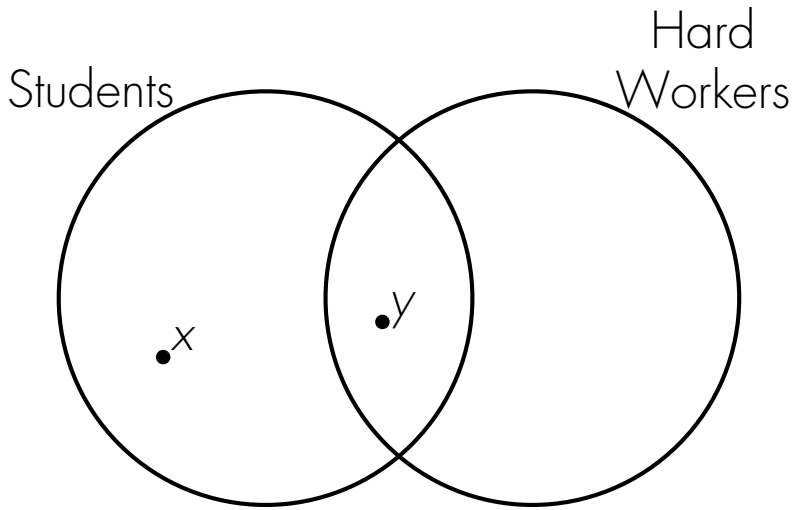


E Statement
(No *S* is *P*)

*This only works as long at the subject category is not empty! For this course, we will always assume each category is non-empty.

Contraries

But this Venn diagram shows a situation in which the corresponding **A** and **E** statements are *both* false.



Statement 2

Consider the following categorical statement:

Some students are hard workers.

Question 4

Suppose statement 2 is true.

What can we infer about the claim that "Some students are not hard workers"?

(**A**) It is true,

(**B**) It is false, or

(**C**) Nothing (statement 2 tells us nothing about the truth or falsity of the above claim).

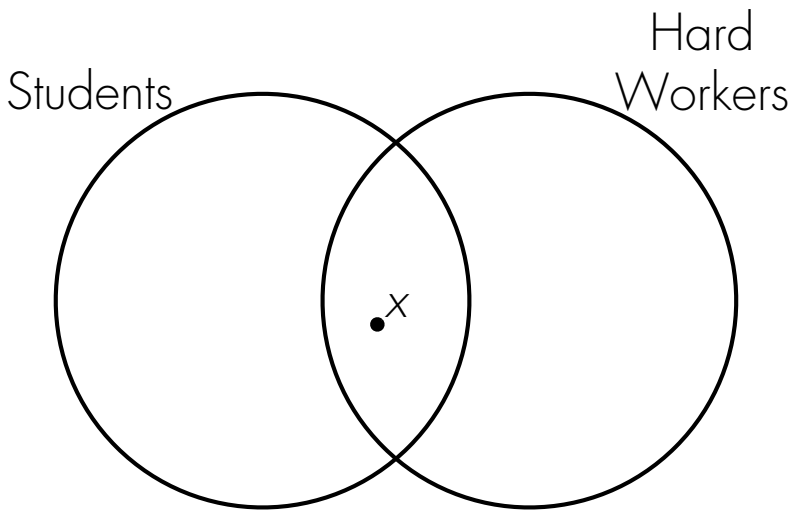
Subcontraries

Two statements are subcontraries if they both cannot be false, though they both may be true.

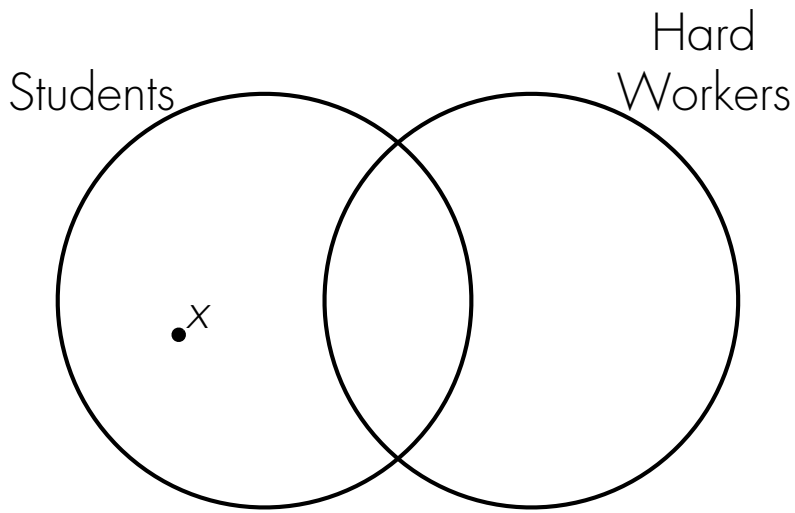
For instance, the statements "Some students are hard workers" (**I**) and "Some students are not hard workers" (**O**) are subcontraries. Both cannot be false: if one is false, the other must be true. However, both statements could be true. As already noted, there might be some students who are hard workers and some others who are not.

Subcontraries

These Venn diagrams confirm that corresponding **I** and **O** statements cannot both be false.*



I Statement
(Some *S* is *P*)

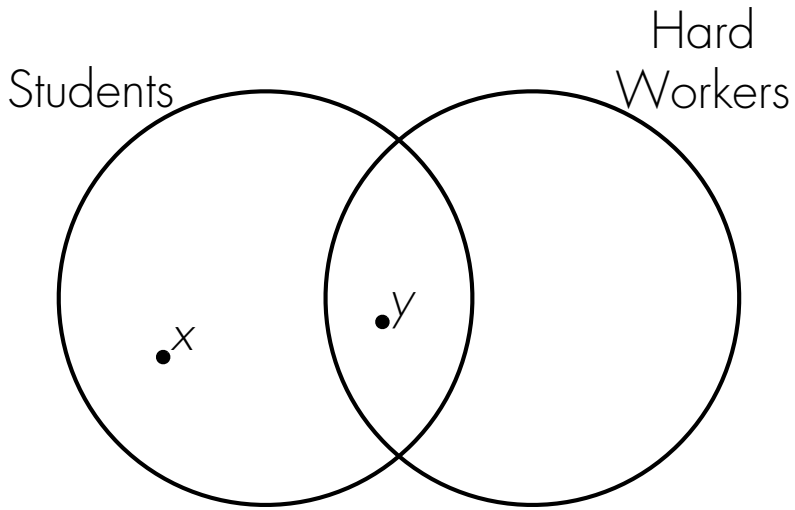


O Statement
(Some *S* is not *P*)

*This also only works as long at the subject category is not empty!

Subcontraries

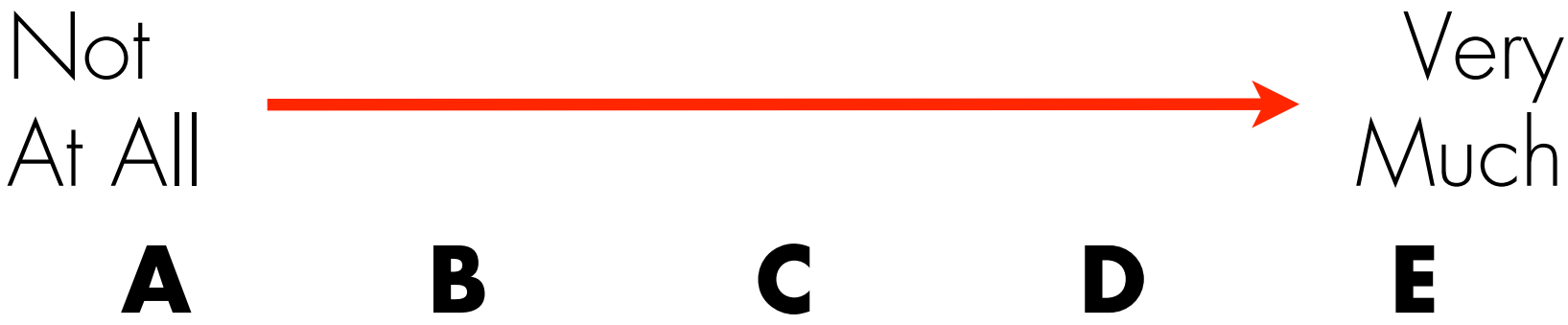
But this Venn diagram now shows a situation where the corresponding **I** and **O** statements are *both* true.



Question 5

Indicate how you feel about solving these types of problems:

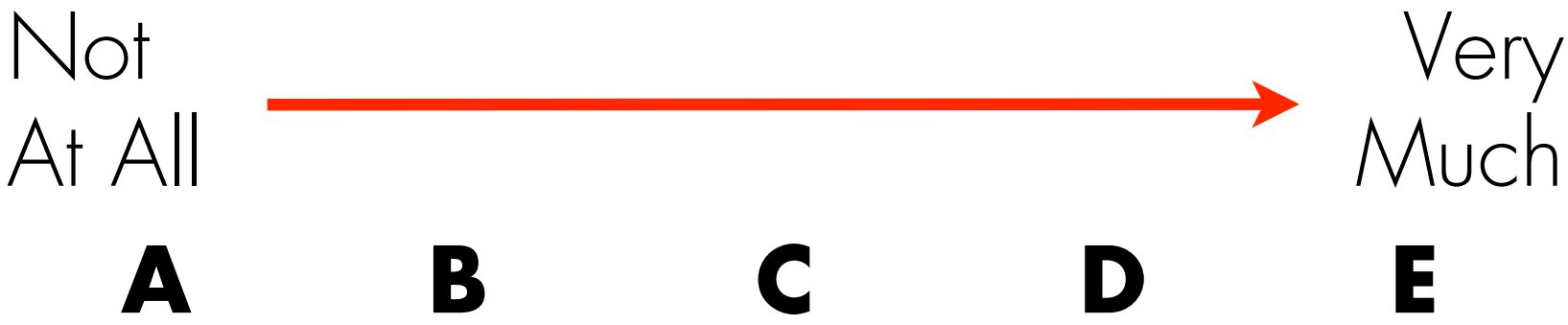
Do you understand what you are supposed to do to solve these types of problems?



Question 6

Indicate how you feel about solving these types of problems:

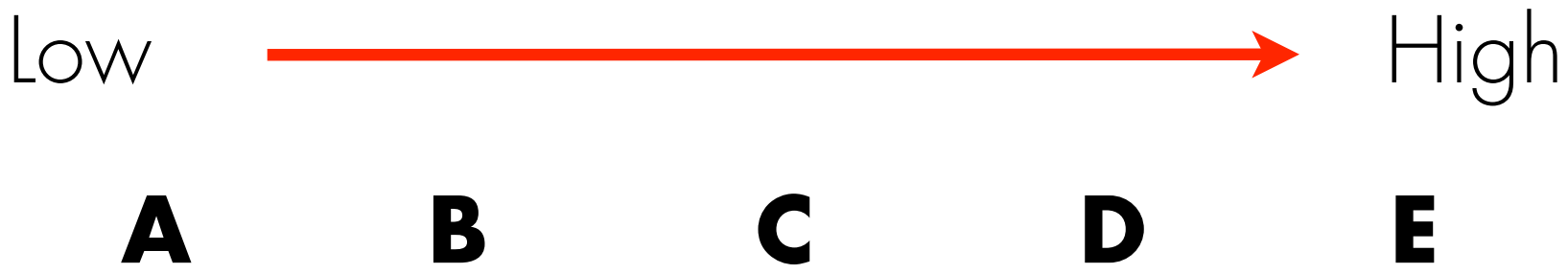
Did you receive adequate feedback on how to solve these types of problems?



Question 7

Indicate how you feel about solving these types of problems:

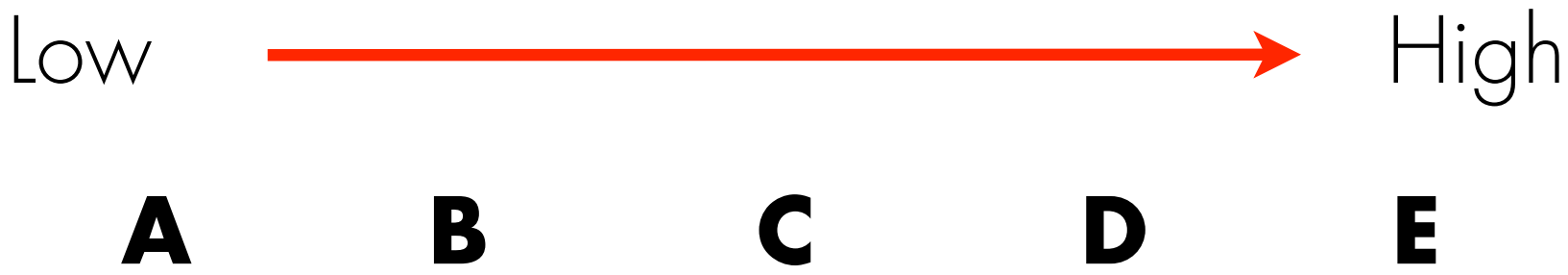
The challenges of solving these types of problems.



Question 8

Indicate how you feel about solving these types of problems:

Your skills in solving these types of problems.



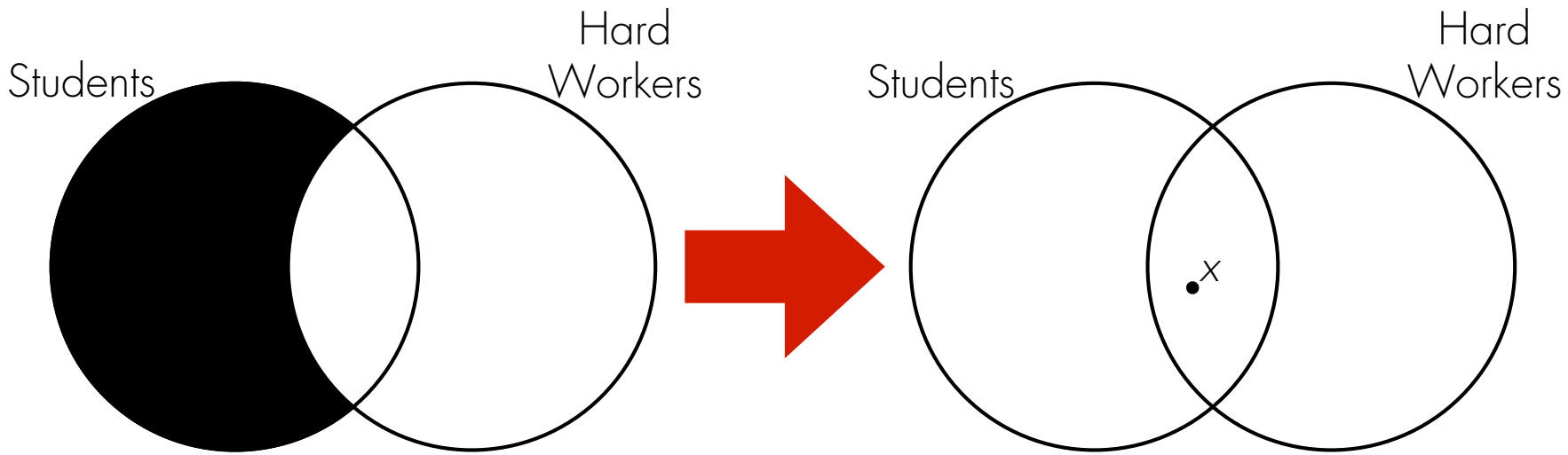
Subalternation

According to **subalternation**, any true *universal* categorical statement may be transformed into a true *particular* one. Going the other direction, subalternation says that any false particular categorical statement may be transformed into a false universal one.

So, for instance, if the statement "All students are hard workers" (**A**) is true, then "Some students are hard workers" (**I**) is trivially true as well.

Subalternation

Venn diagrams confirm that a true **A** statement may be transformed into a true **I** statement.*



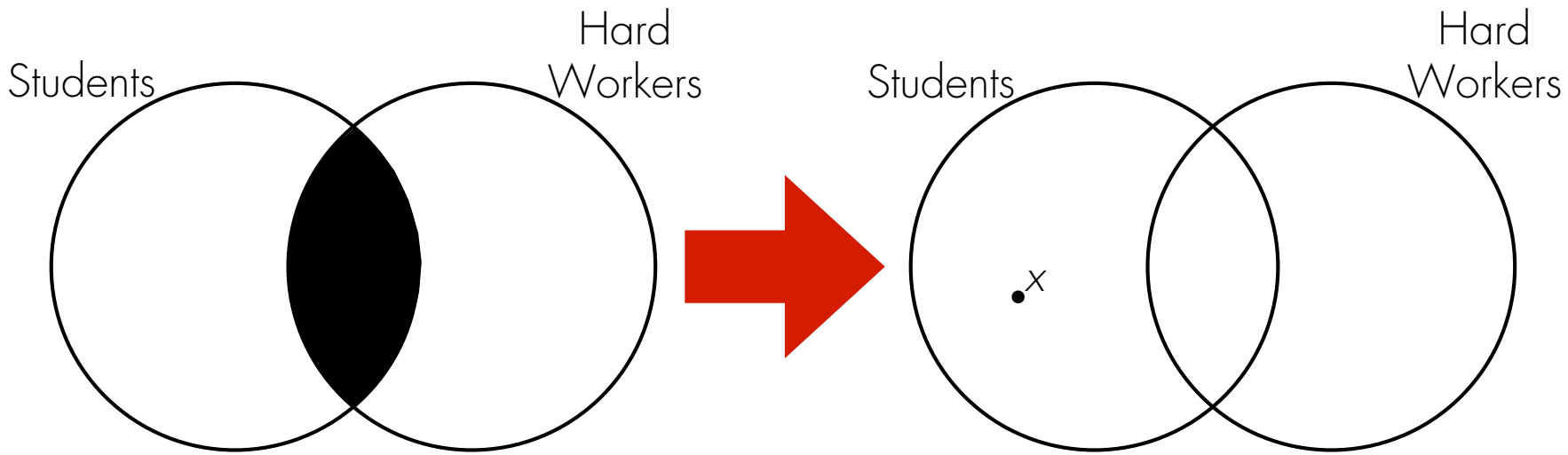
A Statement
(All *S* is *P*)

I Statement
(Some *S* is *P*)

*This also only works as long at the subject category is not empty!

Subalternation

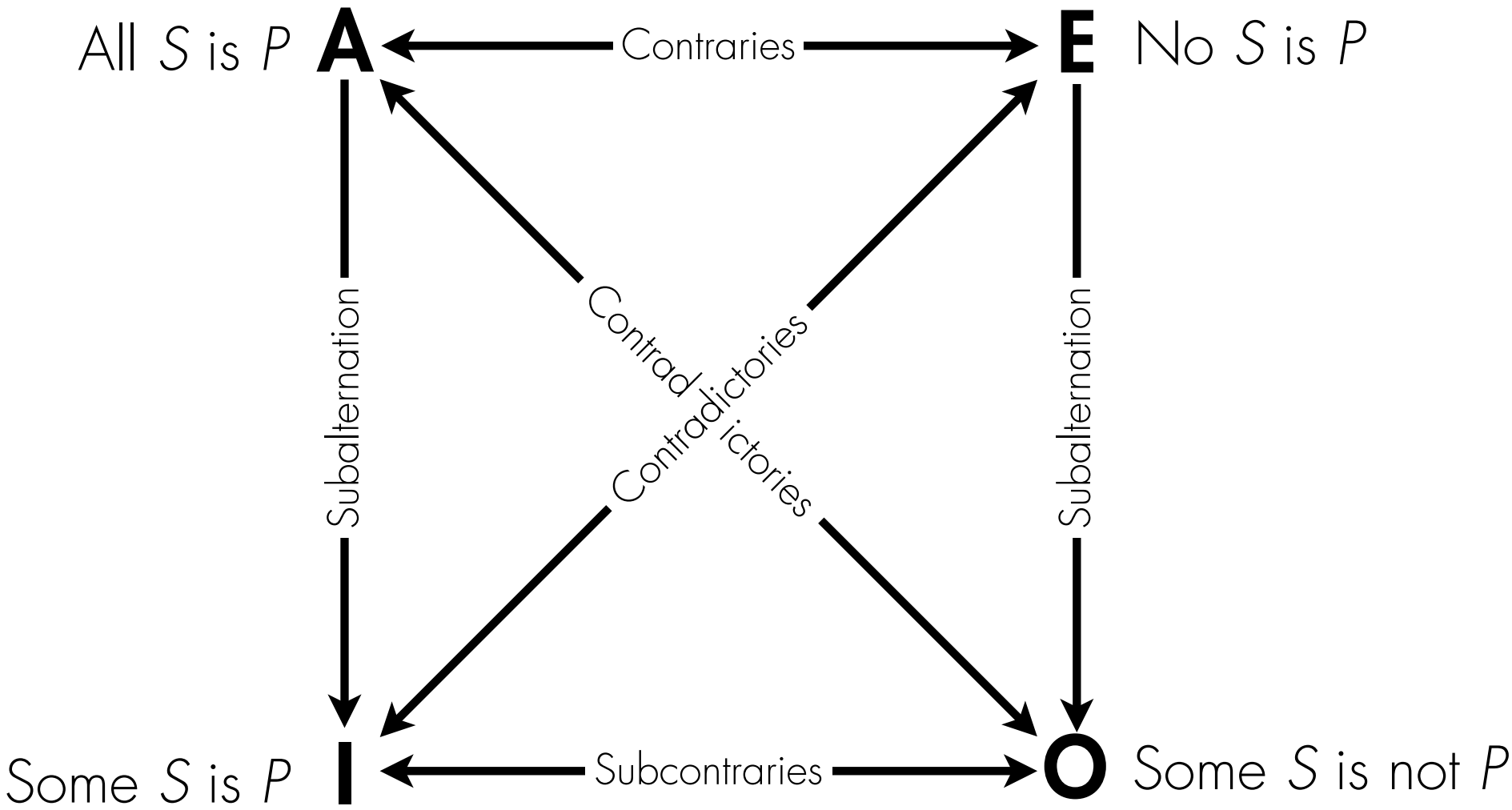
Venn diagrams also confirm that a true **E** statement may be transformed into a true **O** statement.



E Statement
(No *S* is *P*)

O Statement
(Some *S* is not *P*)

The Square of Opposition



Inferences from the Square

Fix the subject (S) and the predicate (P). Then the square of oppositions reveals these inferences:

– If **A** is true: **E** is false; **I** is true; **O** is false.
– If **A** is false: **O** is true; **E** and **I** are undetermined.

– If **E** is true: **A** is false; **I** is false; **O** is true.
– If **E** is false: **I** is true; **A** and **O** are undetermined.

– If **I** is true: **E** is false; **A** and **O** are undetermined.
– If **I** is false: **A** is false; **E** is true; **O** is true.

– If **O** is true: **A** is false; **E** and **I** are undetermined.
– If **O** is false: **A** is true; **E** is false; **I** is true.

Inferences from the Square

The square of opposition contains a lot of useful information concerning what you can infer from a single categorical statement, but Venn diagrams provide intuitive ways to figure out these inferences.

Next Class...

We will do a workshop on drawing venn diagrams foe categorical statements and seeing what can be inferred from them about other statements.