Introduction to Logical Reasoning

Advanced Natural Deduction

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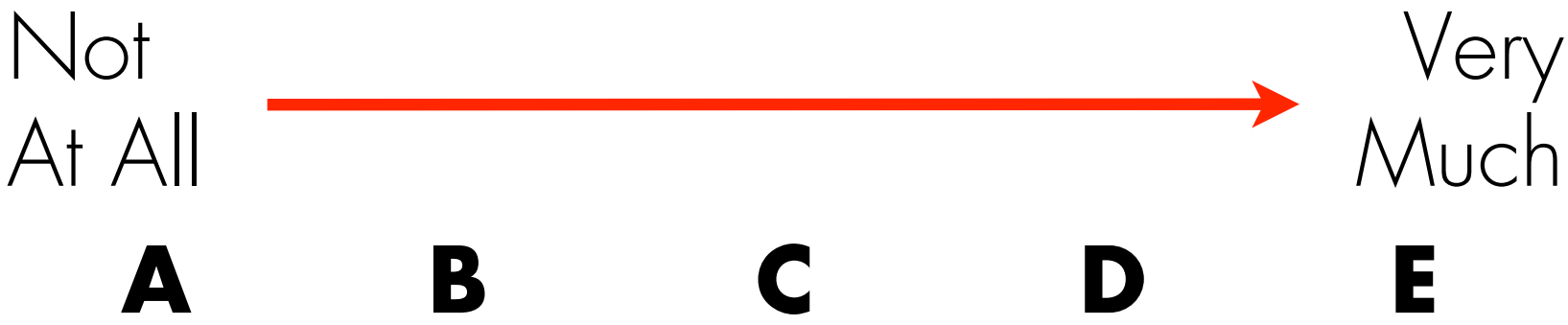
Question 1

Indicate how you felt about solving the problems on today's quiz:

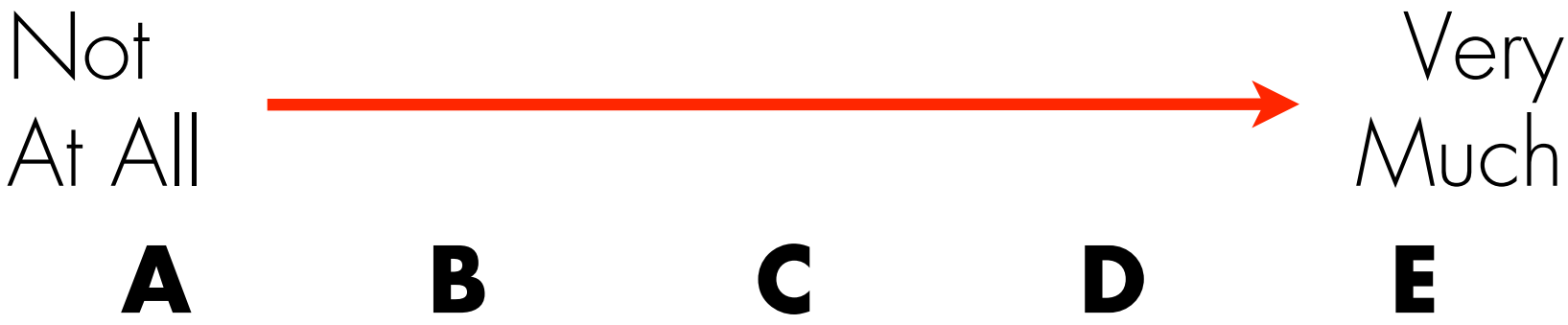
Did you understand what you were supposed to do to solve these problems?



Question 2

Indicate how you felt about solving the problems on today's quiz:

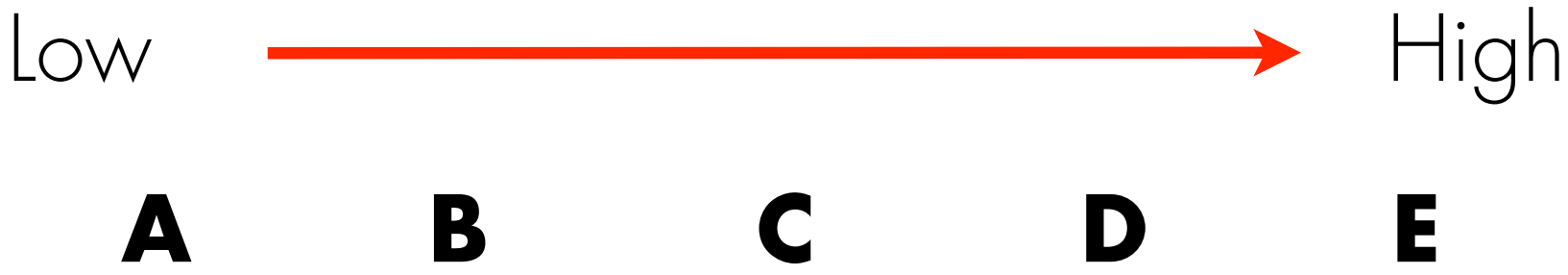
Did you receive adequate feedback for solving these problems?



Question 3

Indicate how you felt about solving the problems on today's quiz:

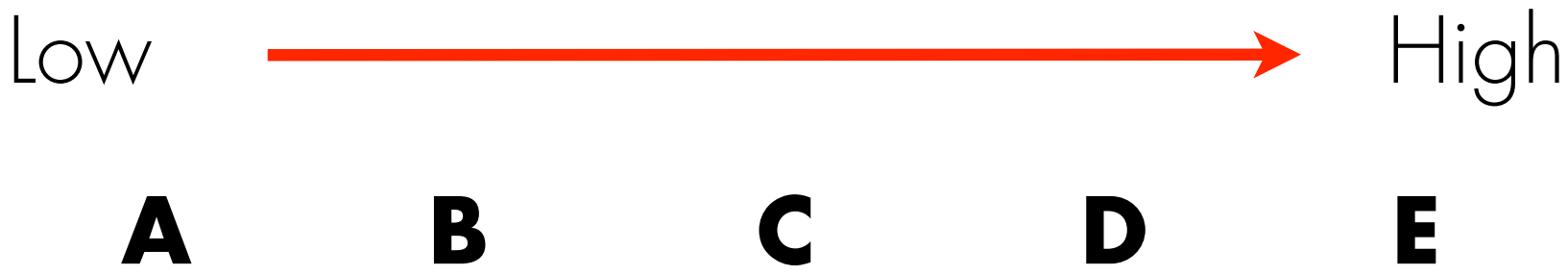
The challenges of solving these problems.



Question 4

Indicate how you felt about solving the problems on today's quiz:

Your skills in solving these problems.



The Nine Rules of Inference

1. *Modus Ponens* (M.P.)

   1. $p \rightarrow q$.
   2. p.

   $\therefore\ q$.

2. *Modus Tollens* (M.T.)

   1. $p \rightarrow q$.
   2. $\sim q$.

   $\therefore\ \sim p$.

3. Hypothetical Syllogism (H.S.)

   1. $p \rightarrow q$.
   2. $q \rightarrow r$.

   $\therefore\ p \rightarrow r$.

4. Disjunctive Syllogism (D.S.)

   1. $p \vee q$.
   2. $\sim p$.

   $\therefore\ q$.

5. Constructive Dilemma (C.D.)

   1. $(p \rightarrow q)\ \&\ (\mathrm{r} \rightarrow \mathrm{s})$.
   2. $p \vee r$.

   $\therefore\ q \vee s$.

6. Absorption (Abs.)

   1. $p \rightarrow q$.

   $\therefore\ p \rightarrow (p\ \&\ q)$.

7. Simplification (Simp.)

   1. $p\ \&\ q$.

   $\therefore\ p$.

8. Conjunction (Conj.)

   1. p.
   2. q.

   $\therefore\ p\ \&\ q$.

9. Addition (Add.)

   1. p.

   $\therefore\ p \vee q$.

Natural Deduction

Today we finally bring all of our skills in natural deduction together. We look at proofs where we do not know in advance how many steps it will take to solve. However, the process remains the same.

Argument 1

1. $A \rightarrow B$.
2. $A \vee (C \,\&\, D)$.
3. $\sim B \,\&\, \sim E$.

$\therefore$ C.

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1. $A \rightarrow B$.
2. $A \vee (C \,\&\, D)$.
3. $\sim B \,\&\, \sim E$.

$\therefore$ C.

4. $\sim B$. 3; Simp.
5. $\sim A$. 1, 4; M.T.
6. $C \,\&\, D$. 2, 5; D.S.
7. C. 6; Simp.

Argument 2

1. $(\sim M \,\&\, \sim N) \rightarrow (O \rightarrow N)$.
2. $N \rightarrow M$.
3. $\sim M$.

$\therefore\ \sim O$.

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1. $(\sim M \,\&\, \sim N) \rightarrow (O \rightarrow N)$.
2. $N \rightarrow M$.
3. $\sim M$.

$\therefore \sim O$.

4. $\sim N$.	2, 3; M.T.
5. $\sim M \,\&\, \sim N$.	3, 4; Conj.
6. $O \rightarrow N$.	1, 5; M.P.
7. $\sim O$.	6, 4; M.T.

Argument 3

If Layli is present then Majnun is happy. If Layli is present and Majnun is happy, then Cala is pleased. If Layli is present and Cala is pleased, then Dirran is pleased. Therefore, if Layli is present then Dirran is pleased.

Argument 3

1. $L \rightarrow M$.
2. $(L \,\&\, M) \rightarrow C$.
3. $(L \,\&\, C) \rightarrow D$.

$\therefore$ $L \rightarrow D$.

Argument 3

1. $L \rightarrow M$.
2. $(L \,\&\, M) \rightarrow C$.
3. $(L \,\&\, C) \rightarrow D$.

$\therefore$ $L \rightarrow D$.

4. $L \rightarrow (L \,\&\, M)$. 1; Abs.
5. $L \rightarrow C$. 4, 2; H.S.
6. $L \rightarrow (L \,\&\, C)$. 5; Abs.
7. $L \rightarrow D$. 6, 3; H.S.

Learning Natural Deduction

There are only three ways to learn natural deduction:

1. Practice,

2. Practice, and

3. Practice.

If you do not practice this, then you will not be able to do it. I trust you now understand *modus ponens* and *modus tollens*, so you can follow the implications here.

Next Class…

We will have a review for Exam #2, which is Sunday.