Introduction to Logical Reasoning

Creating Proofs by Natural Deduction

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The Nine Rules of Inference

1. *Modus Ponens* (M.P.)
   1. $p \rightarrow q$.
   2. p.

   $\therefore$ q.

2. *Modus Tollens* (M.T.)
   1. $p \rightarrow q$.
   2. $\sim q$.

   $\therefore$ $\sim p$.

3. Hypothetical Syllogism (H.S.)
   1. $p \rightarrow q$.
   2. $q \rightarrow r$.

   $\therefore$ $p \rightarrow r$.

4. Disjunctive Syllogism (D.S.)
   1. $p \vee q$.
   2. $\sim p$.

   $\therefore$ q.

5. Constructive Dilemma (C.D.)
   1. $(p \rightarrow q)\ \&\ (\mathrm{r} \rightarrow \mathrm{s})$.
   2. $p \vee r$.

   $\therefore$ $q \vee s$.

6. Absorption (Abs.)
   1. $p \rightarrow q$.

   $\therefore$ $p \rightarrow (p\ \&\ q)$.

7. Simplification (Simp.)
   1. $p\ \&\ q$.

   $\therefore$ p.

8. Conjunction (Conj.)
   1. p.
   2. q.

   $\therefore$ $p\ \&\ q$.

9. Addition (Add.)
   1. p.

   $\therefore$ $p \vee q$.

Natural Deduction

So far we have just seen formal proofs that only required being able to recognize the rules of inference (the "patterns") being applied. Now we can begin to begin to construct proofs where the steps are not given to us. To introduce you to this process, today we will look at arguments whose proofs can be done in just two steps.

Argument 1

1. A.
2. B.

$\therefore$ (A $\vee$ C) & B.

Argument 1

1. A.
2. B.

∴ (A ∨ C) & B.

3. A ∨ C. 1; Add.
4. (A ∨ C) & B. 3, 2; Conj.

Argument 2

1. $A \rightarrow B$.
2. $A \lor C$.
3. $C \rightarrow D$.

$\therefore B \lor D$.

Argument 2

1. $A \rightarrow B$.
2. $A \vee C$.
3. $C \rightarrow D$.

∴ $B \vee D$.

4. $(A \rightarrow B) \& (C \rightarrow D)$. 1, 3; Conj.
5. $B \vee D$. 4, 2; C.D.

Natural Deduction

Of course, the real goal with natural deduction to be able to take arguments in English and verify their validity. So the following argument must first be translated into the language of logic, and then verified with natural deduction. The proof can be done in just two steps.

Argument 3

If Layli is present then Majnun is happy. If Layli is present and Majnun is happy, then Cala is pleased. Therefore, if Layli is present then Cala is pleased.

Argument 3

1. $L \rightarrow M$.
2. $(L \& M) \rightarrow C$.

$\therefore L \rightarrow C$.

Argument 3

1. $L \rightarrow M$.
2. $(L \& M) \rightarrow C$.

$\therefore L \rightarrow C$.

3. $L \rightarrow (L \& M)$. 1; Abs.
4. $L \rightarrow C$. 3, 2; H.S.

Learning Natural Deduction

There are only three ways to learn natural deduction:

1. Practice,
2. Practice, and
3. Practice.

If you do not practice this, then you will not be able to do it. I trust you now understand *modus ponens* and *modus tollens*, so you can follow the implications here.

Next Class…

We will look at arguments that require even longer proofs of validity. Don't panic.