Introduction to Logical Reasoning

Argument Patterns

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Modus Ponens

Consider the argument:

1. If I study hard, then I pass the class.
2. I study hard.

∴ I pass the class.

This can be formalized as follows:

1. $S \rightarrow P$.
2. S.

∴ P.

Modus Ponens

A truth table shows that this argument is valid:

S	P	S → P
T	T	T
T	F	F
F	T	T
F	F	T
Premise 2	Conclusion	Premise 1

Modus Ponens

This argument has the following general form, which is known as ***modus ponens*** (M.P.):

1. $p \rightarrow q$.
2. p.

$\therefore q$.

So any inference that has this form—i.e., affirming (1) a hypothetical and (2) its antecedent to imply (∴) affirming its consequent—is logically valid.

Identifying Patterns

This same pattern may appear in arguments that appear to be more complicated:

1. If I study hard and I attend every class, then I either pass the class or die trying.
2. I study hard and I attend every class.

∴ I either pass the class or die trying.

Notice this is still just a (1) a hypothetical and (2) its antecedent implying (∴) affirming its consequent.

Identifying Patterns

This can be seen more clearly when formalizing the argument:

1. $(S \,\&\, A) \rightarrow (P \vee D)$.
2. $S \,\&\, A$.

∴ $P \vee D$.

You can then see the pattern of M.P. emerge:

1. $\overbrace{(S \,\&\, A)}^{p} \rightarrow \overbrace{(P \vee D)}^{q}$.
2. $\overbrace{S \,\&\, A}^{p}$.

∴ $\overbrace{P \vee D}^{q}$.

Argument Patterns

Knowing commonly used argument patterns is extremely useful. Once you know that a particular pattern is logically valid, if you see that same pattern appear in another argument, you then know right away that this new argument is also logically valid.

Argument Patterns

So, for instance, any argument that has the pattern of *modus ponens*—no matter what content statements *p* and *q* may have, and no matter whether they positive, negative, or compound—is logically valid.

Argument 1

Consider the following argument:

> If you are eighteen, then you can vote. You are eighteen. Therefore you can vote.

Argument 2

Consider the following argument:

> If you are eighteen, then you can vote. You not eighteen. Therefore you cannot vote.

Modus Tollens

Another common argument pattern is known as ***modus tollens*** (M.T.):

1. $p \rightarrow q$.
2. $\sim q$.

$\therefore \sim p$.

In this case (1) affirming a hypothetical statement but(2) denying its consequent is said to imply ($\therefore$) denying its antecedent.

Modus Tollens

And a truth table shows that this form is also valid:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T
		Conclusion	Premise 2	Premise 1

Identifying Patterns

So anytime you see an inference where (1) a hypothetical is affirmed, (2) its consequent is denied, it is valid to conclude by (∴) denying its antecedent. Once again, this is valid even when these three things are more complex:

1. $(A \rightarrow B) \rightarrow \sim(C \vee D)$.
2. $\sim\sim(C \vee D)$.

∴ $\sim(A \rightarrow B)$.

1. $\overbrace{(A \rightarrow B)}^{p} \rightarrow \overbrace{\sim(C \vee D)}^{q}$.
2. $\sim\overbrace{\sim(C \vee D)}^{q}$.

∴ $\sim\overbrace{(A \rightarrow B)}^{p}$.

Argument 3

Consider the following argument:

> If you are eighteen, then you can vote. You cannot vote. Therefore you are not eighteen.

Fallacy of Affirming the Consequent

Now all argument patterns are good, however. Consider the following common argument pattern:

1. $p \rightarrow q$.
2. q.

$\therefore p$.

The pattern here is affirming both (1) a hypothetical and (2) its consequent in order to conclude (∴) by affirming its antecedent.

Fallacy of Affirming the Consequent

A truth table shows that this form is invalid:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T
Conclusion	Premise 2	Premise 1

Fallacy of Affirming the Consequent

This is an extremely common fallacy known as **the fallacy of affirming the consequent**. For instance:

> If I have good business skills, then I will earn a lot of money. I earn a lot of money. Therefore, I have good business skills.

On a quick read this (rather common) argument may seem logically valid. But on closer inspection, it has the same pattern as this fallacy. So it is invalid!

Fallacy of Denying the Antecedent

Here is another bad argument pattern:

1. $p \rightarrow q$.
2. $\sim p$.

$\therefore \sim q$.

The pattern here is (1) affirming a hypothetical but (2) denying its antecedent in order to conclude (∴) by denying its consequent.

Fallacy of Denying the Antecedent

And a truth table shows that this form is also invalid:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T
		Premise 2	Conclusion	Premise 1

Fallacy of Denying the Antecedent

This is another extremely common fallacy known as **the fallacy of denying the antecedent**. For instance:

> If I have good business skills, then I will earn a lot of money. I do not have good business skills. Therefore, I will not earn a lot of money.

On a quick read this may seem logically valid. But it has the same pattern as this fallacy. So it is invalid!

Valid vs. Invalid Patterns

It is sometimes easy to confuse a valid argument with a fallacy, so you need to be on guard!

– Do not confuse M.P. (affirming the *antecedent*) with the fallacy of affirming the *consequent*, and

– Do not confuse M.T. (denying the *consequent*) with the fallacy of denying the *antecedent*.

Next Class…

We will do a workshop on using truth tables to assess the validity of arguments.

We will work more on identifying argument patterns in the next unit on natural deduction.