Introduction to Logical Reasoning

Problem Set #11

Although I strongly suggest that you write out answers to all these problems, you do *not* have to turn in any written responses. You do, however, need to be prepared to do these types of problems, for questions on the weekly quizzes and exams will primarily be drawn from the problem sets. The solutions to these problems will be provided, so you can check your own work and seek help from me as necessary.

We will devote considerable time to these types of problems during the next in-class workshop. In order to make that workshop productive, please make a solid start on them. That way you can use the workshop to address the difficulties you are facing.

If you do the extra credit logic puzzle, you must turn in a computer-typewritten solution at the beginning of class on Monday, November 21st.

Part A Instructions

Let A, B, and C be any sets with universal set U. For each set given below, draw a *neat* Venn diagram, labeling all the diagram's parts and indicating the set being diagramed by shading it in. Do not forget the universal set (the box).

Part A Problems

1. A.
2. The complement of A ($\overline{A}$).
3. The intersection of A and B. This is set $A \cap B$.
4. The union of A and B. This is set $A \cup B$.
5. The union of A and B-complement. This is set $A \cup \overline{B}$.
6. The intersection of B and A-complement. This is set $\overline{A} \cap B$.
7. The complement of the union of A and B. This is set $\overline{A \cup B}$.
8. The complement of the intersection of A and B. This is set $\overline{A \cap B}$.
9. The union of A, B, and C. This is set $(A \cup B) \cup C$.
10. The intersection of A, B, and C. This is set $(A \cap B) \cap C$.
11. The intersection of A, B-complement, and C. This is set $(A \cap \overline{B}) \cap C$.
12. Play around with other combinations of A, B, C, their complements, unions, intersections, and so forth.

Part B Instructions

Consider the following sets:

$A = \{a, b, c\}$,
$B = \{a, i\}$,
$C = \{g, s\}$,
$D = \{1, 2, 3\}$,
$E = \{1, 5\}$,
$F = \{4, 10\}$,
$G = \{c, d, e, 1, 2\}$,
$H = \{l, m, 7, 8\}$,
$I = \{c, 2, 3, 5\}$,
$J = \{m, n, o, p, 8\}$,
$K = \{1, 3, 5, 7, 9\}$,
$L = \{2, 4, 6, 8, 10\}$, and
$M = \{a, e, i, o, u\}$.

Specify the sets given below. Use the roster method, unless told otherwise.

Part B Problems

1. All the subsets of A, labeling them $A1$, $A2$, etc.
2. All the subsets of B, labeling them $B1$, $B2$, etc.
3. All the subsets of C, labeling them $C1$, $C2$, etc.
4. All the subsets of D, labeling them $D1$, $D2$, etc.
5. All the subsets of E, labeling them $E1$, $E2$, etc.
6. All the subsets of F, labeling them $F1$, $F2$, etc.
7. All the subsets of H, labeling them $H1$, $H2$, etc. (Beware: there are several of them!).
8. $A \cup B$.
9. $A \cap B$.
10. $B \cup C$.
11. $B \cap C$.
12. $D \cup E$.
13. $D \cap E$.
14. $E \cup F$.
15. $E \cap F$.
16. $G \cup I$.
17. $G \cap I$.
18. $H \cup J$.
19. $H \cap J$.
20. K by the rule method.
21. L by the rule method.
22. M by the rule method.
23. $K \cup L$ by the rule method.
24. Let $U = \{x \mid x \text{ is a letter of the English alpabet}\}$. Specify the complement of M by the rule method.
25. Let $U = \{a, b, c, d, e\}$ be the universal set for A. Specify the complement of A by the roster method.
26. Let $U = \{1, 2, 3, 4, 5\}$ be the universal set for D. Specify the complement of D by the roster method.

Note: There may a lot of exercises here. Do not feel obligated to do all of them. I often assign many exercises so that you have plenty of opportunities to practice the skills these exercises are trying to impart. I suggest doing just enough of them so that you are confident that you could use these skills on a quiz or an exam.

Extra Credit Logic Puzzle

In Washington, D.C., politicians never ever tell the truth, and all non-politicians always tell the truth. Suppose you are working at a hospital in Washington, D.C. You are assisting Dr. Torres, a non-politician, in an amputation, but she has forgotten which leg to cut off! Torres tells you that the patient's doctors, Grey and Yang, know which leg is being amputated, so she sends you to find one of them and solve this problem. However, Torres warns you that one of Grey and Yang is a politician and one of them is a non-politician, but Torres forgets who is who. Leaving the operating room, you immediately run into Dr. Grey.

Question: What one question can you ask Dr. Grey to find out which leg to amputate?

To receive full credit you must justify your answer with a logical argument showing why you are 100% right. That is to say, this question has a definitive answer that can be justified without *any* guessing on your part.