CRITICAL THINKING

Review Session #4

Critical Thinking

Professor David Emmanuel Gray

Final Exam: Structure

Part 1: All symbolic, nothing requiring translation from English (75 minutes, 40% of the final exam grade).

Testing for logical equivalence,
Advanced natural deduction (with 17 rules of inference), and
Assessing traditional categorical syllogisms.

15-Minute Break.

Part 2: All arguments, all in English requiring translation (90 minutes, 60% of the final exam grade).

Diagramming arguments,
Assessing arguments with truth tables, and
Assessing traditional categorical syllogisms and other types of categorical arguments.

Final Exam: Structure (There is New Material!)

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Assessing traditional categorical syllogisms and **other types of categorical arguments**.

New Compound Statement: *Material Equivalence*

Biconditional statement: A compound statement claiming that its statements have the *exact same* truth value.

Logic is fun **if and only if** logic is easy.

Logic being fun is **a necessary and sufficient condition** for logic being easy.

Logic being fun is **necessary and sufficient** for logic being easy.

Such a statement is false if *one of its statements is false while the other statement is true.* We call the statements contained within a biconditional statement the **components**.

Material Equivalence: *Translation*

So a **biconditional statement** has the form of "…if and only if…", asserting that the statements connected together have the *exact same truth* value. It is symbolized using ↔ (called "double-headed arrow").

So the biconditional statement $p \leftrightarrow q$ asserts that p and q have the same truth value: they are both true or they are both false. In this example, p and q are the **components**.

Note: As you may recall, the use of the lower-case, italic letters p and q means that *any* two generic statements can be connected together as components within a biconditional statement.

Material Equivalence: *Example*

Consider the following biconditional statement:

Logic is fun **if and only if** it is easy.

Both antecedent and consequent are simple positive statements, which are symbolized:

F: Logic is **fun**.
E: Logic is **easy**.

The entire biconditional statement is then symbolized as F ↔ E.

Note: Recall that we using those upper-case, upright letters F and E to represent the specific, simple positive statements involved.

Material Equivalence: *Truth Table*

The **biconditional statement** $p \leftrightarrow q$ asserts that p and q have the *same* truth value. So it is false just when the components have *different* truth values (that is, one component is true and the other is false). Otherwise it is always true. Here is its truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

New Concept: *Logical Equivalence*

Two statements p and q are **logically equivalent** just when the statement of their associated biconditional (that is, $p \leftrightarrow q$) is a tautology.

This means that it is *absolutely impossible* for p and q to have different truth values. In other words, they *always* have the same truth value, no matter what. Thus, p and q have the *same* logical meaning and so they may be substituted for one another while remaining logically consistent.

The claim that p and q are logically equivalent is denoted symbolically as $p \overset{\top}{\equiv} q$.

New Skill: *Testing for Logical Equivalence*

Determining if a pair of statements p and q are logically equivalent is done with a truth table according to the followings steps:

1. Construct the associated biconditional for the two statements (that is, $p \leftrightarrow q$),
2. Construct a truth for that biconditional statement,
3. Use that truth table to see if that biconditional statement is a tautology, and
4. If the biconditional statement *is* a tautology, then the two statements *are* logically equivalent (that is, $p \stackrel{\top}{\equiv} q$). If it is not a tautology, then those two statements *are not* logically equivalent.

Testing for Logical Equivalence: *Example 1*

Are p and $\sim\sim p$ logically equivalent?

Testing for Logical Equivalence: *Example 1*

Step 1: Construct the associated biconditional for the two statements.

The statements are p and $\sim\sim p$, so the associated biconditional is $p \leftrightarrow \sim\sim p$.

Testing for Logical Equivalence: *Example 1*

Step 2: Construct a truth for that biconditional statement.

p	$\sim p$	$\sim\sim p$	$p \leftrightarrow \sim\sim p$
T	F	T	T
F	T	F	T

Testing for Logical Equivalence: *Example 1*

Step 3: Use that truth table to see if that biconditional statement is a tautology.

p	$\sim p$	$\sim\sim p$	$p \leftrightarrow \sim\sim p$
T	F	T	T
F	T	F	T

This biconditional *is* a tautology.

Testing for Logical Equivalence: *Example 1*

Step 4: Determine logical equivalence.

p	$\sim p$	$\sim\sim p$	$p \leftrightarrow \sim\sim p$
T	F	T	T
F	T	F	T

p and $\sim\sim p$ are logically equivalent. This is because the biconditional of both statements *is* a tautology (it is true in both lines of the truth table). That means that both statements *always* have the same truth value, no matter what. Thus, both statements have the same logical meaning, and so $p \overset{T}{\equiv} \sim\sim p$.

New Rules for Natural Deduction: *Double Negation*

Double Negation Introduction (D.N.I.)

1. *p*.

∴ ~~*p*.

Double Negation Elimination (D.N.E.)

1. ~~*p*.

∴ *p*.

Advanced Natural Deduction: *Example 2*

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. $\sim N \rightarrow \sim M$.
2. M.

$\therefore$ N.

3. $\sim\sim M$. 2; D.N.I.
4. $\sim\sim N$. 1, 3; M.T.
5. N. 4; D.N.E.

Testing for Logical Equivalence: *Example 2*

Are $p \to q$ and $q \to p$ logically equivalent?

Testing for Logical Equivalence: *Example 2*

Step 1: Construct the associated biconditional for the two statements.

The statements are $p \rightarrow q$ and $q \rightarrow p$, so the associated biconditional is $(p \rightarrow q) \leftrightarrow (q \rightarrow p)$.

Testing for Logical Equivalence: *Example 2*

Step 2: Construct a truth for that biconditional statement.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Testing for Logical Equivalence: *Example 2*

Step 3: Use that truth table to see if that biconditional statement is a tautology.

p	q	$p \to q$	$q \to p$	$(p \to q) \leftrightarrow (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

This biconditional *not* a tautology.

Testing for Logical Equivalence: *Example 2*

Step 4: Determine logical equivalence.

p	q	$p \to q$	$q \to p$	$(p \to q) \leftrightarrow (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$p \to q$ and $q \to p$ are *not* logically equivalent. This is because the biconditional of both statements is *not* a tautology (it is false in lines 2 and 3 of the truth table). That means that it *is* possible for the two statements to have *different* truth values. So these statements do *not* have the same logical meaning.

Common Logically Equivalent Expressions (Feel Free to Verify Them!)

De Morgan's Theorems:	$\sim(p \,\&\, q) \overset{T}{\equiv} \sim p \lor \sim q$ $\sim(p \lor q) \overset{T}{\equiv} \sim p \,\&\, \sim q$
Commutation:	$(p \lor q) \overset{T}{\equiv} (q \lor p)$ $(p \,\&\, q) \overset{T}{\equiv} (q \,\&\, p)$
Association:	$[p \lor (q \lor r)] \overset{T}{\equiv} [(p \lor q) \lor r]$ $[p \,\&\, (q \,\&\, r)] \overset{T}{\equiv} [(p \,\&\, q) \,\&\, r]$
Distribution:	$[p \,\&\, (q \lor r)] \overset{T}{\equiv} [(p \,\&\, q) \lor (p \,\&\, r)]$ $[p \lor (q \,\&\, r)] \overset{T}{\equiv} [(p \lor q) \,\&\, (p \lor r)]$
Double Negation:	$p \overset{T}{\equiv} \sim\sim p$
Transposition:	$(p \rightarrow q) \overset{T}{\equiv} (\sim q \rightarrow \sim p)$
Material Implication:	$(p \rightarrow q) \overset{T}{\equiv} (\sim p \lor q)$
Material Equivalence:	$(p \leftrightarrow q) \overset{T}{\equiv} [(p \rightarrow q) \,\&\, (q \rightarrow p)]$ $(p \leftrightarrow q) \overset{T}{\equiv} [(p \,\&\, q) \lor (\sim p \,\&\, \sim q)]$
Exportation:	$[(p \,\&\, q) \rightarrow r] \overset{T}{\equiv} [p \rightarrow (q \rightarrow r)]$
Tautology:	$p \overset{T}{\equiv} (p \lor p)$ $p \overset{T}{\equiv} (p \,\&\, p)$

More New Rules for Natural Deduction

Disjunctive Commutation (D.C.)

1. $p \,\&\, q.$

$\therefore \; q \,\&\, p.$

Conjunctive Commutation (C.C.)

1. $p \vee q.$

$\therefore \; q \vee p.$

Biconditional Introduction (B.I.)

1. $(p \rightarrow q) \,\&\, (q \rightarrow p).$

$\therefore \; p \leftrightarrow q.$

Biconditional Elimination (B.E.)

1. $p \leftrightarrow q.$

$\therefore \; (p \rightarrow q) \,\&\, (q \rightarrow p).$

Material Implication 1 (M.I.1)

1. $p \rightarrow q.$

$\therefore \; {\sim}p \vee q.$

Material Implication 2 (M.I.2)

1. ${\sim}p \vee q.$

$\therefore \; p \rightarrow q.$

The Seventeen Rules of Inference

1. *Modus Ponens* (M.P.)

1.	$p \rightarrow q.$
2.	$p.$
∴	$q.$

2. *Modus Tollens* (M.T.)

1.	$p \rightarrow q.$
2.	$\sim q.$
∴	$\sim p.$

3. Hypothetical Syllogism (H.S.)

1.	$p \rightarrow q.$
2.	$q \rightarrow r.$
∴	$p \rightarrow r.$

4. Disjunctive Syllogism (D.S.)

1.	$p \vee q.$
2.	$\sim p.$
∴	$q.$

5. Constructive Dilemma (C.D.)

1.	$(p \rightarrow q) \,\&\, (r \rightarrow s).$
2.	$p \vee r.$
∴	$q \vee s.$

6. Absorption (Abs.)

1.	$p \rightarrow q.$
∴	$p \rightarrow (p \,\&\, q).$

7. Simplification (Simp.)

1.	$p \,\&\, q.$
∴	$p.$

8. Conjunction (Conj.)

1.	$p.$
2.	$q.$
∴	$p \,\&\, q.$

9. Addition (Add.)

1.	$p.$
∴	$p \vee q.$

10. Double Negation Introduction (D.N.I.)

1.	$p.$
∴	$\sim\sim p.$

11. Double Negation Elimination (D.N.E.)

1.	$\sim\sim p.$
∴	$p.$

12. Disjunctive Commutation (D.C.)

1.	$p \vee q.$
∴	$q \vee p.$

12. Conjunctive Commutation (C.C.)

1.	$p \,\&\, q.$
∴	$q \,\&\, p.$

14. Biconditional Introduction (B.I.)

1.	$(p \rightarrow q) \,\&\, (q \rightarrow p).$
∴	$p \leftrightarrow q.$

15. Biconditional Elimination (B.E.)

1.	$p \leftrightarrow q.$
∴	$(p \rightarrow q) \,\&\, (q \rightarrow p).$

16. Material Implication 1 (M.I.1)

1.	$p \rightarrow q.$
∴	$\sim p \vee q.$

17. Material Implication 2 (M.I.2)

1.	$\sim p \vee q.$
∴	$p \rightarrow q.$

Advanced Natural Deduction: *Example 3*

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1. Y.

∴ X → Y.

2. Y v ~X. 1; Add.
3. ~X v Y. 2; D.C.
4. X → Y. 3; M.I.2.

Advanced Natural Deduction: *Example 4*

The following is a valid argument. Use natural deduction to construct this argument's formal proof of validity.

1.	~A v (B → C).
2.	A → B
3.	B → (C → B).
4.	A.
∴	C ↔ B.

5.	B.	2, 4; M.P.
6.	C → B.	3, 5; M.P.
7.	~~A.	4; D.N.I.
8.	B → C.	7, 1; D.S.
9.	(C → B) & (B → C).	6, 8; Conj.
10.	C ↔ B.	9; B.I.

5.	B.	2, 4; M.P.
6.	C → B.	3, 5; M.P.
7.	A → (B → C).	1; M.I.2.
8.	B → C.	7, 4; M.P.
9.	(C → B) & (B → C).	6, 8; Conj.
10.	C ↔ B.	9; B.I.

Categorical Arguments

So far, the only categorical arguments that you have assessed have been traditional categorical syllogism. (Two premises involving three categories.)

Now you should be able to use your Venn diagramming skills to assess categorical arguments involving one premise or even three premises. You may also see arguments involving only two categories. (I will not have you assess arguments involving more than three categories because Venn diagrams at that point get unwieldy!)

While these arguments may seem complex: do not panic. Just follow your training and you will be surprised at how straightforward these actually become with only a little practice.

Categorical Arguments: *Example 4*

All non-philosophers are students, and so no non-students are non-philosophers.

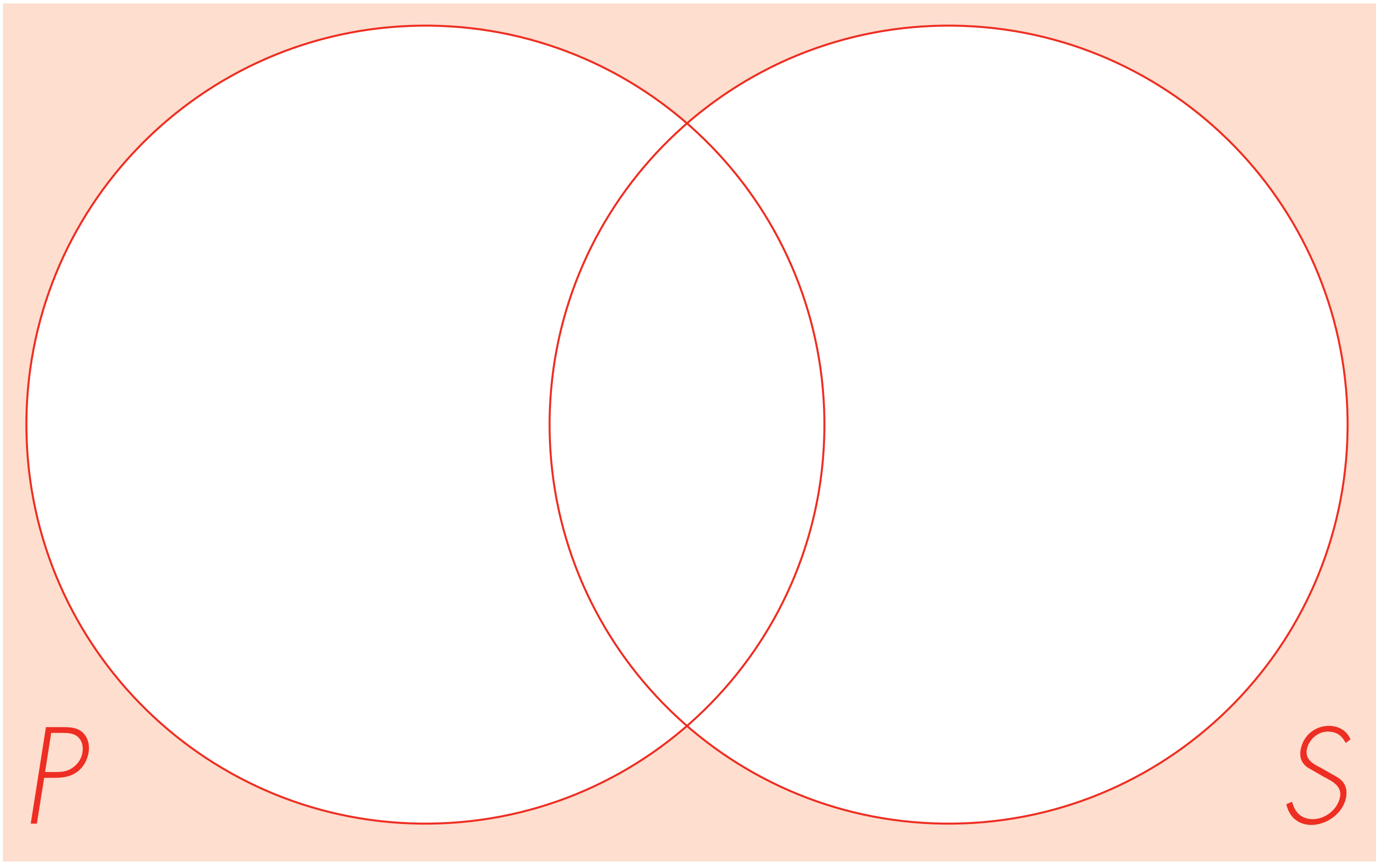
Major term (*P*): Philosophers.
Minor term (*S*): Students.

1. All non-*P* is *S*.

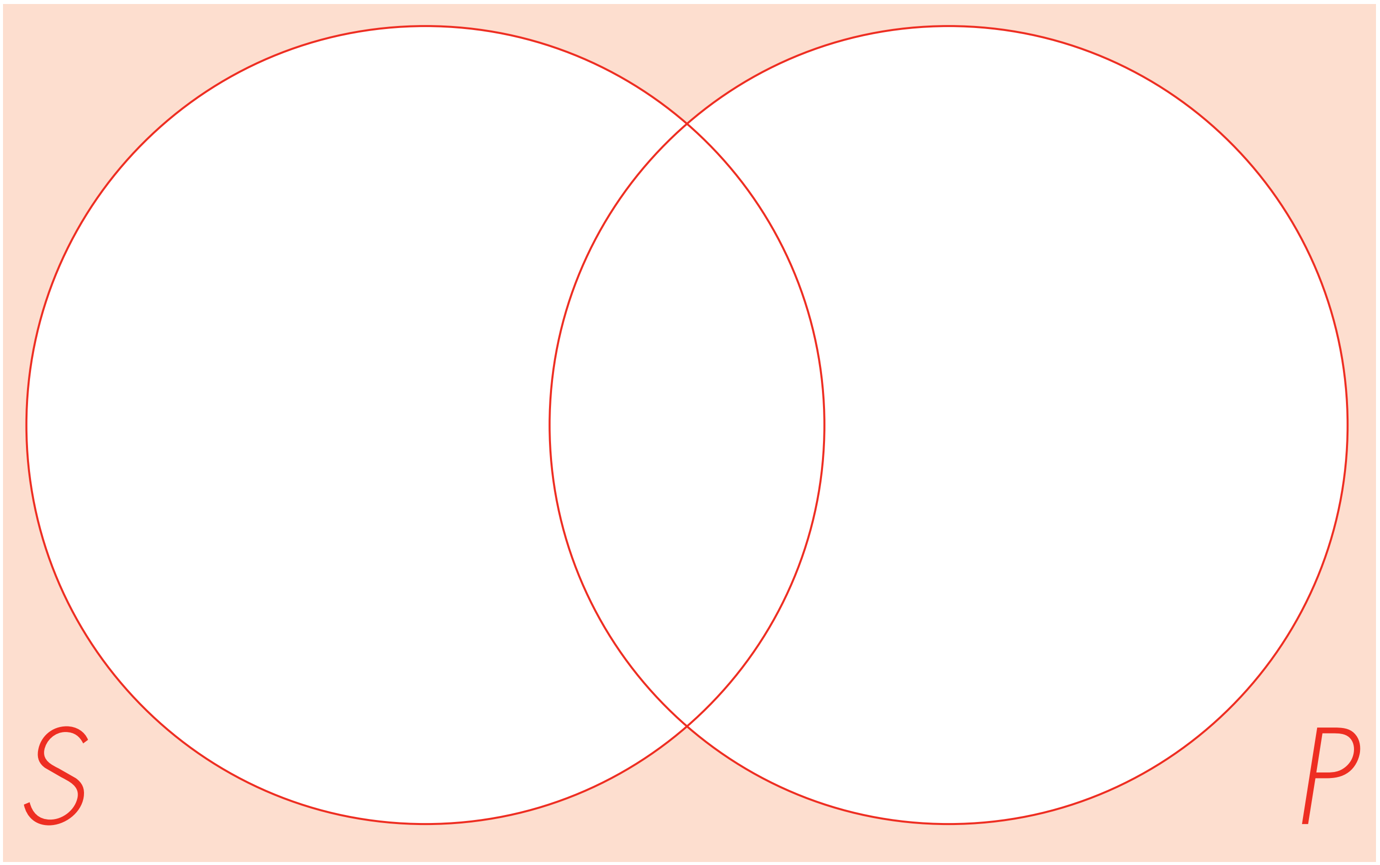
∴ No non-*S* is non-*P*.

The argument is *valid*. The conclusion claims that the area outside of both students and philosophers is completely empty, and the premise confirms this. So assuming the truth of the premise means that the conclusion is true as well, making this argument valid.

Premise:



Conclusion:



Categorical Arguments: *Example 5*

Some non-students are philosophers because there is a non-musician who is a philosopher.

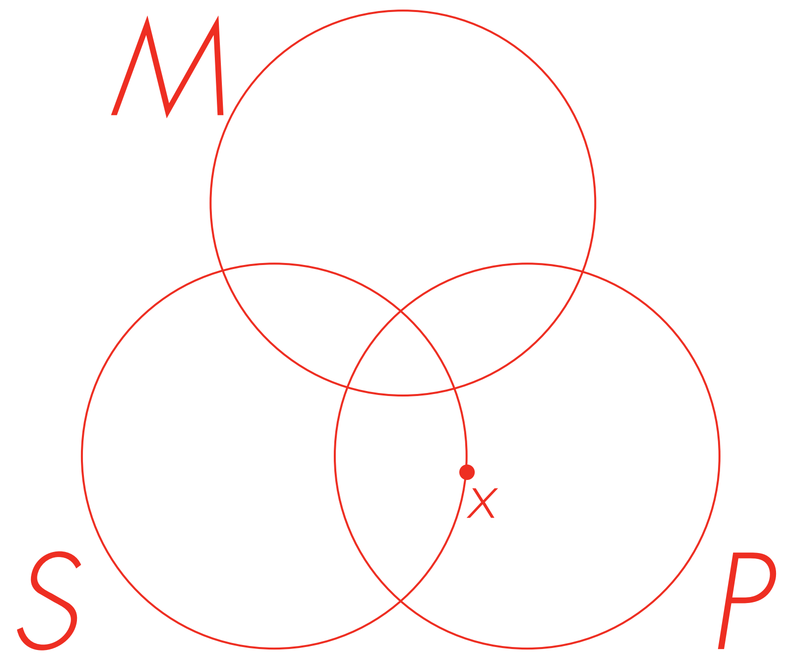
Major term (*P*): Philosophers.
Minor term (*S*): Students.
Other term (*M*): Musicians.

1. Some non-*M* is *P*.

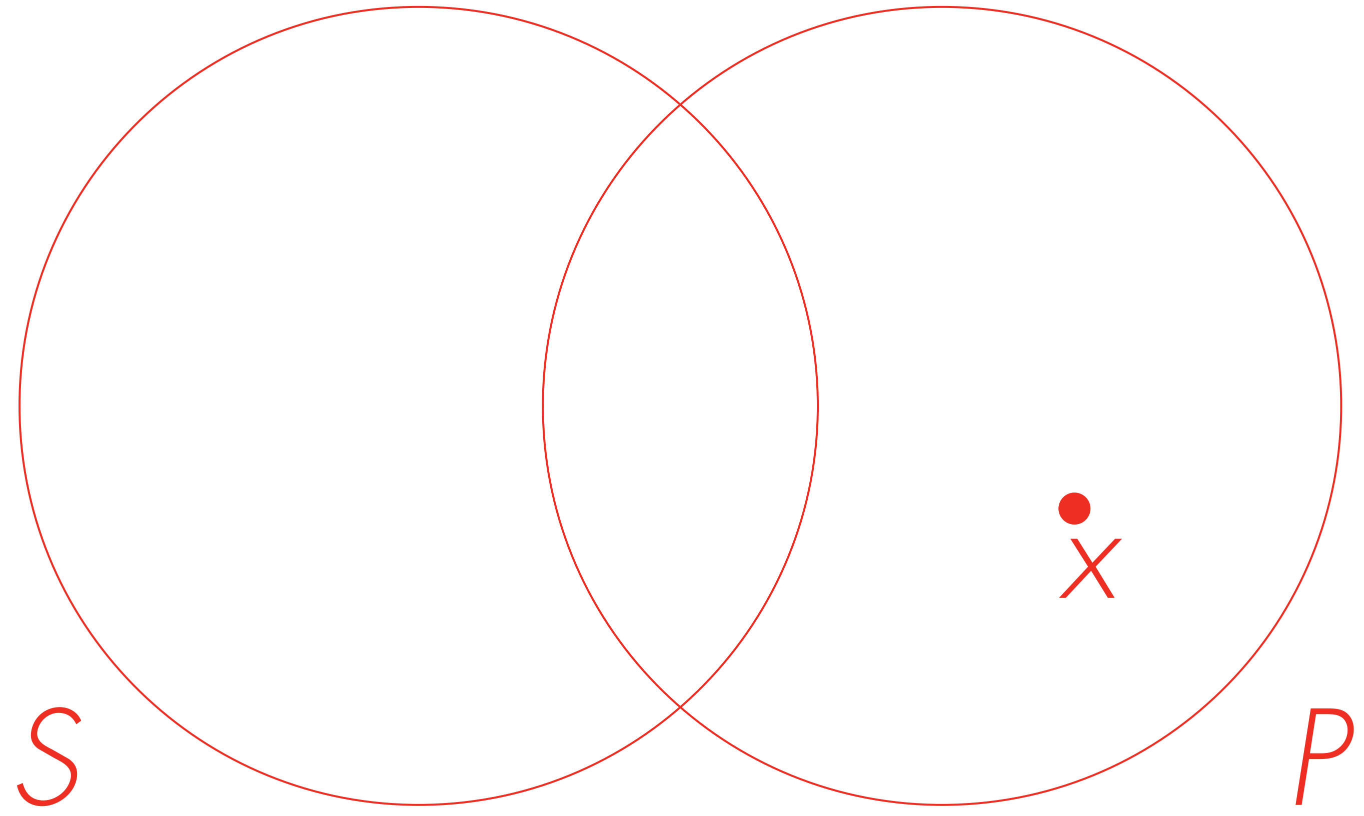
∴ Some non-*S* is *P*.

The argument is *invalid*. The conclusion claims that there is a philosopher who is not a student. The premise, however, fails to confirm this: according to it, there is a philosopher that is not a musician, but that premise does not say whether that philosopher is a student or not. (This is because the dot-*x* is on the students' line, leaving it unclear whether it is a student or not.) So assuming the truth of the premise is not enough to show that the conclusion must be true, making this argument invalid.

Premise:



Conclusion:



Categorical Arguments: *Example 6*

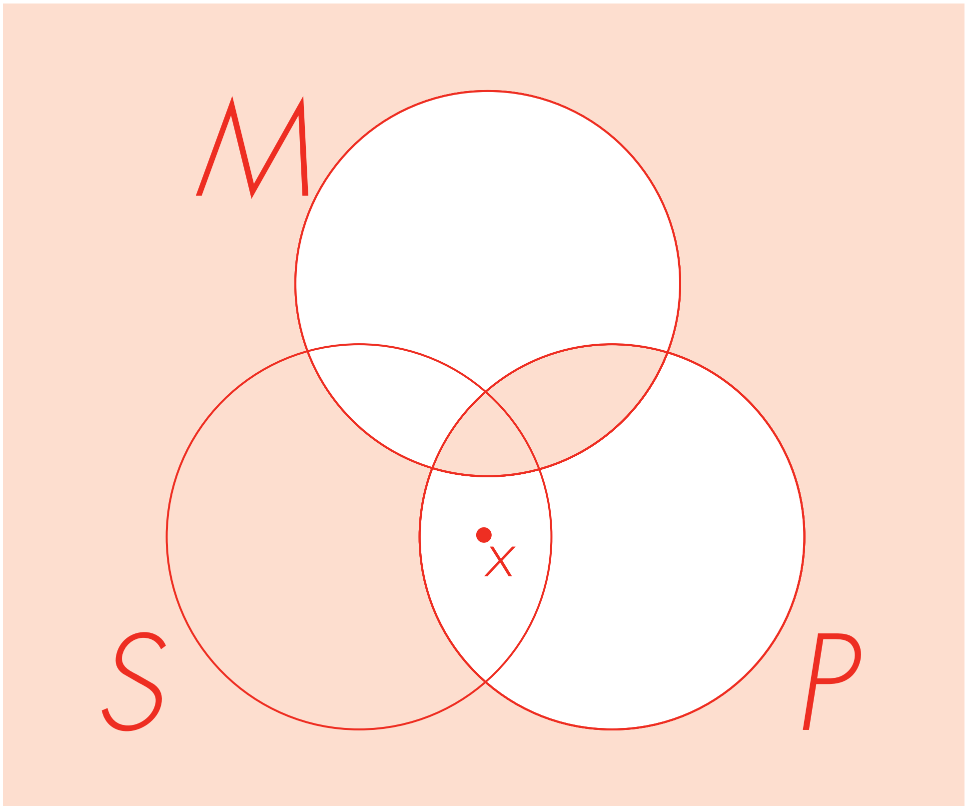
Some students are philosophers for three reasons. First, all non-musicians are philosophers. Second, no philosopher is a musician. Third, some students are not musicians.

Major term (*P*): Philosophers.
Minor term (*S*): Students.
Other term (*M*): Musicians.

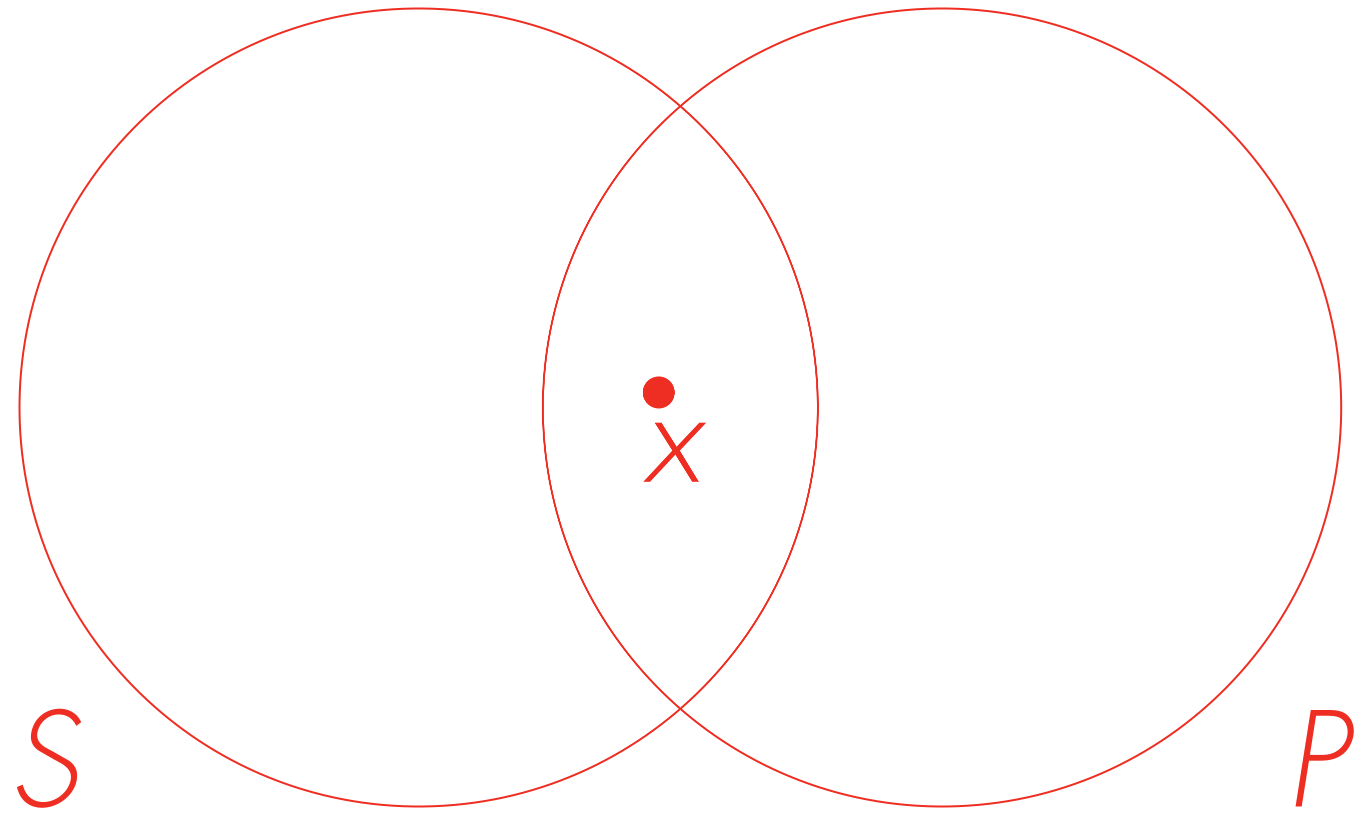
1.	All non-*M* is *P*.
2.	No *P* is *M*.
3.	Some S is not *M*.
∴	Some *S* is *P*.

The argument is *valid*. The conclusion claims that there is a student who is also a philosopher, and the premises confirm this. So assuming the truth of the premises means that the conclusion is true as well, making this argument valid.

Premises:



Conclusion:



Next Class…

We will have the **final exam**.

Keep practicing! You can do this!