CRITICAL THINKING

Problem Set #14: Advanced Logic

Although I strongly suggest that you write out answers to all these problems, you do not have to turn in any written responses. You do, however, need to be prepared to do these types of problems, for some questions on the final exam will be drawn from this problem set. The solutions to these problems will be provided, so you can check your own work and seek help from me as necessary.

Part A Instructions

Each of the following problems presents a pair of statements in logical form. For each pair, use the truth table method from class to determine whether these statements are logically equivalent or not. Be sure to briefly explain how the truth table supports your answer.

Part A Problems

1. p and $\sim\sim p$.
2. $p \rightarrow q$ and $\sim q \rightarrow \sim p$
3. $p \rightarrow q$ and $\sim p \rightarrow \sim q$.
4. $p \,\&\, q$ and $q \,\&\, p$.
5. $p \vee q$ and $q \vee p$.
6. $p \rightarrow q$ and $q \rightarrow p$.
7. $p \rightarrow q$ and $\sim p \vee q$.
8. $(p \rightarrow q) \rightarrow r$ and $(q \rightarrow p) \rightarrow r$.
9. $p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \rightarrow r)$.
10. p and $p \,\&\, (p \vee q)$.
11. p and $p \vee (p \,\&\, q)$.
12. p and $p \,\&\, (p \rightarrow q)$.
13. p and $p \,\&\, (q \rightarrow p)$.
14. $\sim(p \,\&\, q)$ and $\sim p \,\&\, \sim q$.
15. $\sim(p \,\&\, q)$ and $\sim p \vee \sim q$.
16. $\sim(p \vee q)$ and $\sim p \vee \sim q$.
17. $\sim(p \vee q)$ and $\sim p \,\&\, \sim q$.

Part B Instructions

Each of the following problems presents a valid argument. Use natural deduction (with the 17 rules of inference) to construct that argument's formal proof of validity. The number of steps in these proofs will vary. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

Part B Problems

1. 1. $\sim N \rightarrow \sim M$.
   2. M.
   $\therefore$ N.
2. 1. Y.
   $\therefore$ $X \rightarrow Y$.
3. 1. $B \,\&\, (C \,\&\, D)$.
   $\therefore$ $C \,\&\, (D \,\&\, B)$.
4. 1. $H \,\&\, (I \,\&\, J)$.
   $\therefore$ $J \,\&\, (I \,\&\, H)$.
5. 1. $\sim A \vee (B \rightarrow C)$.
   2. $A \rightarrow B$.
   3. $B \rightarrow (C \rightarrow B)$.
   4. A.
   $\therefore$ $C \leftrightarrow B$.
6. 1. $A \rightarrow B$.
   2. $C \rightarrow \sim B$.
   $\therefore$ $A \rightarrow \sim C$.
7. 1. $A \vee B$.
   2. $\sim B$.
   $\therefore$ A.
8. 1. $U \rightarrow \sim V$.
   2. V.
   $\therefore$ $\sim U$.
9. 1. $(\sim Q \rightarrow R) \,\&\, (\sim S \rightarrow T)$.
   2. $\sim\sim(\sim S \vee \sim Q)$.
   $\therefore$ $R \vee T$.
10. 1. $[(E \vee F) \,\&\, (G \vee H)] \rightarrow (F \,\&\, I)$.
    2. $(G \vee H) \,\&\, (E \vee F)$.
    $\therefore$ I.
11. 1. $C \rightarrow \sim D$.
    2. $\sim E \rightarrow D$.
    $\therefore$ $C \rightarrow \sim\sim E$.
12. 1. $(E \,\&\, F) \rightarrow (G \,\&\, H)$.
    2. $F \,\&\, E$.
    $\therefore$ $H \,\&\, G$.
13. 1. $(G \rightarrow \sim H) \rightarrow I$.
    2. $\sim H \vee \sim G$.
    $\therefore$ $\sim H \vee I$.
14. 1. $G \leftrightarrow H$.
    $\therefore$ $H \leftrightarrow G$.
15. 1. $G \leftrightarrow H$.
    $\therefore$ $(H \vee \sim G) \,\&\, (\sim H \vee G)$.
16. 1. $A \rightarrow B$.
    $\therefore$ $(A \,\&\, B) \vee \sim A$.
17. 1. A.
    $\therefore$ $\sim A \rightarrow B$.

Part C Instructions

Each of the following problems presents a categorical argument. For each, use Venn diagrams to determine whether it is a valid or invalid argument.

In particular, for each of the following arguments, (1) identify the major term (*P*) and the minor term (*S*), (2) identify any other term (*M*) if there is one; (3) put the argument into standard symbolic form; (4) create a Venn diagram of the premises, (5) create a Venn diagram of the conclusion; and (6) use those two Venn diagrams to explain whether the syllogism is valid or invalid. It is now possible that *P*, *S*, and *M* are empty.

Part C Problems

1. All non-philosophers are students, and so no non-students are non-philosophers.

Please continue ▶

Problem Set #14: Advanced Logic (Continued)

2. Some non-students are philosophers because there is a non-musician who is a philosopher.
3. Some students are philosophers for three reasons. First, all non-musicians are philosophers. Second, no philosopher is a musician. Third, some students are not musicians.
4. All philosophers are students and some musicians are not students. Therefore, there is a non-student who is a non-philosopher.
5. All non-students are philosophers because all non-students are musicians.
6. No students are philosophers because all students are musicians.
7. All non-students are philosophers because all non-students are musicians and all musicians are philosophers.
8. No students are philosophers because all students are musicians and all musicians are students.
9. There is a non-philosopher who is a non-musician. Therefore, Some non-student is a non-philosopher.
10. There is a non-philosopher who is a non-musician and all students are philosophers. Therefore, some non-student is a non-philosopher.

Note: There may a lot of exercises here. Do not feel obligated to do all of them. I often assign many exercises so that you have plenty of opportunities to practice the skills these exercises are trying to impart. I suggest doing just enough of them so that you are confident that you could use these skills on the final exam.