CRITICAL THINKING

Problem Set #10: Advanced Natural Deduction

Although I strongly suggest that you write out answers to all these problems, you do not have to turn in any written responses. You do, however, need to be prepared to do these types of problems, for questions on the weekly quizzes and exams will primarily be drawn from the problem sets. The solutions to these problems will be provided, so you can check your own work and seek help from me as necessary.

We will devote considerable time to these types of problems during the next in-class workshop. In order to make that workshop productive, please make a solid start on them. That way you can use the workshop to address the difficulties you are facing.

I have included an advanced logic puzzle for your own entertainment.

Part A Instructions

Each of the following problems presents a valid argument. Use natural deduction to construct that argument's formal proof of validity. The number of steps in these proofs will vary, but some might require up to *four* or *five* steps to complete. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

Part A Problems

1. 1. $A \rightarrow B$.
   2. $A \vee (C \,\&\, D)$.
   3. $\sim B \,\&\, \sim E$.
   ∴ C.

2. 1. $(F \rightarrow G) \,\&\, (H \rightarrow I)$.
   2. $J \rightarrow K$.
   3 $(F \vee J) \,\&\, (H \vee L)$.
   ∴ $G \vee K$.

3. 1. $(\sim M \,\&\, \sim N) \rightarrow (O \rightarrow N)$.
   2. $N \rightarrow M$.
   3. $\sim M$.
   ∴ $\sim O$.

4. 1. $(K \vee L) \rightarrow (M \vee N)$.
   2. $(M \vee N) \rightarrow (O \,\&\, P)$.
   3. K.
   ∴ O.

5. 1. $(Q \rightarrow R) \,\&\, (S \rightarrow T)$.
   2. $(U \rightarrow V) \,\&\, (W \rightarrow X)$.
   3. $Q \vee U$.
   ∴ $R \vee V$.

6. 1. $W \rightarrow X$.
   2. $(W \,\&\, X) \rightarrow Y$.
   3. $(W \,\&\, Y) \rightarrow Z$.
   ∴ $W \rightarrow Z$.

7. 1. $A \rightarrow B$.
   2. $C \rightarrow D$.
   3. $A \vee C$.
   ∴ $(A \,\&\, B) \vee (C \,\&\, D)$.

8. 1. $(E \vee F) \rightarrow (G \,\&\, H)$.
   2. $(G \vee H) \rightarrow I$.
   3. E.
   ∴ I.

9. 1. $J \rightarrow K$.
   2. $K \vee L$.
   3. $(L \,\&\, \sim J) \rightarrow (M \,\&\, \sim J)$.
   4. $\sim K$.
   ∴ M.

10. 1. $(N \vee O) \rightarrow P$.
    2. $(P \vee Q) \rightarrow R$.
    3. $Q \vee N$.
    4. $\sim Q$.
    ∴ R.

Part B Instructions

Each of the following problems presents a valid argument in English. For each, (1) translate it into the language of symbolic logic, using the indicated capital letters to label each simple positive statement involved, (2) put it into its argumentative form, and (3) use natural deduction to construct that argument's formal proof of validity. The number of steps in these proofs will vary, but some might require up to *six* steps to complete. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

Part B Problems

1. If either Gertrude or Herbert wins, then both Jens and Kenneth lose. Gertrude wins. Therefore Jens loses. (G, H, J, K)

2. If Adriana joins, then the club's social prestige will rise; and if Boris joins, then the club's financial position will be more secure. Either Adriana or Boris will join. If the club's social prestige rises, then Boris will join; and if the club's financial position becomes more secure, then Wilson will join. Therefore either Boris or Wilson will join. (A, S, B, F, W.)

3. If Brown received the message, then she took the plane; and if she took the plane, then she will not be late for the meeting. If the message was incorrectly addressed, then Brown will be late for the meeting. Either Brown received the message or the message was incorrectly addressed. Therefore either Brown took the plane or she will be late for the meeting. (R, P, L, T)

4. If Nihar buys the lot, then an office building will be constructed; whereas if Payton buys the lot, then it will be quickly sold again. If Rivers buys the lot, then a store will be constructed; and if a store is constructed, then Thompson will offer to lease it. Either Nihar or Rivers will buy the lot. Therefore either an office building or a store will be constructed. (N, O, P, Q, R, S, T)

5. If rain continues, then the river rises. If rain continues and the river rises, then the bridge will wash out. If the continuation of rain would cause the bridge to wash out, then a single road is not sufficient for the town. Either a single road is sufficient for the town or the traffic engineers have made a mistake. Therefore the traffic engineers have made a mistake. (C, R, B, S, M)

Please continue ▶

Problem Set #10: Advanced Natural Deduction (Continued)

6. If Jonas goes to the meeting, then a complete report will be made; but if Jonas does not go to the meeting, then a special election will be required. If a complete report is made, then an investigation will be launched. If Jonas' going to the meeting implies that a complete report will be made, and the making of a complete report implies that an investigation will be launched, then either Jonas goes to the meeting and an investigation is launched or Jonas does not go to the meeting and no investigation is launched. If Jonas goes to the meeting and an investigation is launched, then some members will have to stand trial. But if Jonas does not go to the meeting and no investigation is launched, then the organization will disintegrate very rapidly. Therefore either some members will have to stand trial or the organization will disintegrate very rapidly. (J, R, E, I, T, D)

7. If Ann is present, then Bill is present. If Ann and Bill are both present, then either Charles or Doris will be elected. If either Charles or Doris is elected, then Elmer does not really dominate the club. If Ann's presence implies that Elmer does not really dominate the club, then Florence will be the new president. So Florence will be the new president. (A, B, C, D, E, F)

8. If Mr. Jones is the manager's next-door neighbor, then Mr. Jones' annual earnings are exactly divisible by 3. If Mr. Jones' annual earnings are exactly divisible by 3, then $40,000 is exactly divisible by 3. But $40,000 is not exactly divisible by 3. If Mr. Robinson is the manager 's next-door neighbor, then Mr. Robinson lives halfway between Detroit and Chicago. If Mr. Robinson lives in Detroit, then he does not live halfway between Detroit and Chicago. Mr. Robinson lives in Detroit. If Mr. Jones is not the manager 's next-door neighbor, then either Mr. Robinson or Mr. Smith is the manager's next-door neighbor. Therefore Mr. Smith is the manager's next-door neighbor. (J, E, T, R, H, D, S)

9. If Mr. Smith is the manager's next-door neighbor, then Mr. Smith lives halfway between Detroit and Chicago. If Mr. Smith lives halfway between Detroit and Chicago, then he does not live in Chicago. Mr. Smith is the manager's next-door neighbor. If Mr. Robinson lives in Detroit, then he does not live in Chicago. Mr. Robinson lives in Detroit. Mr. Smith lives in Chicago or else either Mr. Robinson or Mr. Jones lives in Chicago. If Mr. Jones lives in Chicago, then the manager is Jones. Therefore the manager is Jones. (S, W, L, D, I, C, B)

10. If Smith once beat the editor at billiards, then Smith is not the editor. Smith once beat the editor at billiards. If the manager is Jones, then Jones is not the editor. The manager is Jones. If Smith is not the editor and Jones is not the editor, then Robinson is the editor. If the manager is Jones and Robinson is the editor, then Smith is the publisher. Therefore Smith is the publisher. (O, M, B, N, F, G)

Note: There may a lot of exercises here. Do not feel obligated to do all of them. I often assign many exercises so that you have plenty of opportunities to practice the skills these exercises are trying to impart. I suggest doing just enough of them so that you are confident that you could use these skills on a quiz or an exam.

Logic Puzzle

In Washington, D.C., politicians never ever tell the truth, and all non-politicians always tell the truth. Last summer, I did a census in Washington, D.C., to see whether there was any correlation between truth-telling and smoking. I interviewed everyone in Washington, D.C., and they all said the same thing: "At least one politician in Washington, D.C., smokes".

Questions: What can be determined about Washington, D.C.? Are there any non-politicians? Any politicians? Any smokers? Any nonsmokers?

These questions have definitive right answers that can be fully justified without any guessing.