

CRITICAL THINKING

Problem Set #9: Natural Deduction

Although I strongly suggest that you write out answers to all these problems, you do not have to turn in any written responses. You do, however, need to be prepared to do these types of problems, for questions on the weekly quizzes and exams will primarily be drawn from the problem sets. The solutions to these problems will be provided, so you can check your own work and seek help from me as necessary.

We will devote considerable time to these types of problems during the next in-class workshop. In order to make that workshop productive, please make a solid start on them. That way you can use the workshop to address the difficulties you are facing.

I have included an advanced logic puzzle for your own entertainment.

Part A Instructions

Each of the following problems presents a valid argument. Use natural deduction to construct that argument's formal proof of validity. Each proof can be completed in just *two* steps. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

Part A Problems

1.
$$\begin{array}{l} 1. \quad A. \\ 2. \quad B. \\ \hline \therefore (A \vee C) \& B. \end{array}$$
2.
$$\begin{array}{l} 1. \quad D \rightarrow E. \\ 2. \quad D \& F. \\ \hline \therefore E. \end{array}$$
3.
$$\begin{array}{l} 1. \quad G. \\ 2. \quad H. \\ \hline \therefore (G \& H) \vee I. \end{array}$$
4.
$$\begin{array}{l} 1. \quad J \rightarrow K. \\ 2. \quad J. \\ \hline \therefore K \vee L. \end{array}$$
5.
$$\begin{array}{l} 1. \quad M \vee N. \\ 2. \quad \sim M \& \sim O. \\ \hline \therefore N. \end{array}$$
6.
$$\begin{array}{l} 1. \quad P \& Q. \\ 2. \quad R. \\ \hline \therefore P \& R. \end{array}$$
7.
$$\begin{array}{l} 1. \quad S \rightarrow T. \\ 2. \quad \sim T \& \sim U. \\ \hline \therefore \sim S. \end{array}$$
8.
$$\begin{array}{l} 1. \quad V \vee W. \\ 2. \quad \sim V. \\ \hline \therefore W \vee X. \end{array}$$
9.
$$\begin{array}{l} 1. \quad Y \rightarrow Z. \\ 2. \quad Y. \\ \hline \therefore Y \& Z. \end{array}$$
10.
$$\begin{array}{l} 1. \quad A \rightarrow B. \\ 2. \quad (A \& B) \rightarrow C. \\ \hline \therefore A \rightarrow C. \end{array}$$
11.
$$\begin{array}{l} 1. \quad D \rightarrow E. \\ 2. \quad (E \rightarrow F) \& (F \rightarrow D). \\ \hline \therefore D \rightarrow F. \end{array}$$
12.
$$\begin{array}{l} 1. \quad (G \rightarrow H) \& (I \rightarrow J). \\ 2. \quad G. \\ \hline \therefore H \vee J. \end{array}$$
13.
$$\begin{array}{l} 1. \quad \sim(K \& L). \\ 2. \quad K \rightarrow L. \\ \hline \therefore \sim K. \end{array}$$
14.
$$\begin{array}{l} 1. \quad (M \rightarrow N) \& (M \rightarrow O). \\ 2. \quad N \rightarrow O. \\ \hline \therefore M \rightarrow O. \end{array}$$
15.
$$\begin{array}{l} 1. \quad (P \rightarrow Q) \& (R \rightarrow S). \\ 2. \quad (P \vee R) \& (Q \vee R). \\ \hline \therefore Q \vee S. \end{array}$$
16.
$$\begin{array}{l} 1. \quad (T \rightarrow U) \& (T \rightarrow V). \\ 2. \quad T. \\ \hline \therefore U \vee V. \end{array}$$
17.
$$\begin{array}{l} 1. \quad (W \vee X) \rightarrow Y. \\ 2. \quad W. \\ \hline \therefore Y. \end{array}$$
18.
$$\begin{array}{l} 1. \quad (Z \& A) \rightarrow (B \& C). \\ 2. \quad Z \rightarrow A. \\ \hline \therefore Z \rightarrow (B \& C). \end{array}$$
19.
$$\begin{array}{l} 1. \quad D \rightarrow E. \\ 2. \quad [D \rightarrow (D \& E)] \rightarrow (F \rightarrow \sim G). \\ \hline \therefore F \rightarrow \sim G. \end{array}$$
20.
$$\begin{array}{l} 1. \quad (\sim H \vee I) \vee J. \\ 2. \quad \sim(\sim H \vee I). \\ \hline \therefore J \vee \sim H. \end{array}$$
21.
$$\begin{array}{l} 1. \quad (K \rightarrow L) \rightarrow M. \\ 2. \quad \sim M \& \sim(L \rightarrow K). \\ \hline \therefore \sim(K \rightarrow L). \end{array}$$
22.
$$\begin{array}{l} 1. \quad (N \rightarrow O) \rightarrow (P \rightarrow Q). \\ 2. \quad [P \rightarrow (N \rightarrow O)] \& [N \rightarrow (P \rightarrow Q)]. \\ \hline \therefore P \rightarrow (P \rightarrow Q). \end{array}$$
23.
$$\begin{array}{l} 1. \quad R \rightarrow S. \\ 2. \quad S \rightarrow (S \& R). \\ \hline \therefore [R \rightarrow (R \& S)] \& [S \rightarrow (S \& R)]. \end{array}$$
24.
$$\begin{array}{l} 1. \quad [T \rightarrow (U \vee V)] \& [U \rightarrow (T \vee V)]. \\ 2. \quad (T \vee U) \& (U \vee V). \\ \hline \therefore (U \vee V) \vee (T \vee V). \end{array}$$
25.
$$\begin{array}{l} 1. \quad (W \& X) \rightarrow (Y \& Z). \\ 2. \quad \sim[(W \& X) \& (Y \& Z)]. \\ \hline \therefore \sim(W \& X). \end{array}$$
26.
$$\begin{array}{l} 1. \quad A \rightarrow B. \\ 2. \quad A \vee C. \\ 3. \quad C \rightarrow D \\ \hline \therefore B \vee D. \end{array}$$
27.
$$\begin{array}{l} 1. \quad (E \& F) \vee (G \rightarrow H). \\ 2. \quad I \rightarrow G. \\ 3. \quad \sim(E \& F) \\ \hline \therefore I \rightarrow H. \end{array}$$

Problem Set #9: Natural Deduction (Continued)

28. 1. $J \vee \sim K$.
2. $K \vee (L \rightarrow J)$.
3. $\sim J$
∴ $L \rightarrow J$.
29. 1. $(M \rightarrow N) \& (O \rightarrow P)$.
2. $N \rightarrow P$.
3. $(N \rightarrow P) \rightarrow (M \vee O)$.
∴ $N \vee P$.
30. 1. $Q \rightarrow (R \vee S)$.
2. $(T \& U) \rightarrow R$.
3. $(R \vee S) \rightarrow (T \& U)$.
∴ $Q \rightarrow R$.

Part B Instructions

Each of the following problems presents a valid argument. Use natural deduction to construct that argument's formal proof of validity. Each proof can be completed in just *three* steps. Keep in mind that the final line in the proof is always the conclusion of the argument being proved.

Part B Problems

1. 1. $A \vee (B \rightarrow A)$.
2. $\sim A \& C$.
∴ $\sim B$.
2. 1. $(D \vee E) \rightarrow (F \& G)$.
2. D .
∴ F .
3. 1. $(H \rightarrow I) \& (H \rightarrow J)$.
2. $H \& (I \vee J)$.
∴ $I \vee J$.
4. 1. $(K \& L) \rightarrow M$.
2. $K \rightarrow L$.
∴ $K \rightarrow [(K \& L) \& M]$.
5. 1. $N \rightarrow [(N \& O) \rightarrow P]$.
2. $N \& O$.
∴ P .
6. 1. $Q \rightarrow R$.
2. $R \rightarrow S$.
3. $\sim S$.
∴ $\sim Q \& \sim R$.
7. 1. $T \rightarrow U$.
2. $V \vee \sim U$.
3. $\sim V \& \sim W$.
∴ $\sim T$.
8. 1. $\sim X \rightarrow Y$.
2. $Z \rightarrow X$.
3. $\sim X$.
∴ $Y \& \sim Z$.

9. 1. $(A \vee B) \rightarrow \sim C$.
2. $C \vee D$.
3. A .
∴ D .
10. 1. $E \vee \sim F$.
2. $F \vee (E \vee G)$.
3. $\sim E$.
∴ G .
11. 1. $(H \rightarrow I) \& (J \rightarrow K)$.
2. $K \vee H$.
3. $\sim K$.
∴ I .
12. 1. $L \vee (M \rightarrow N)$.
2. $\sim L \rightarrow (N \rightarrow O)$.
3. $\sim L$.
∴ $M \rightarrow O$.
13. 1. $(P \rightarrow Q) \& (Q \rightarrow P)$.
2. $R \rightarrow S$.
3. $P \vee R$.
∴ $Q \vee S$.
14. 1. $(T \rightarrow U) \& (V \rightarrow W)$.
2. $(U \rightarrow X) \& (W \rightarrow Y)$.
3. T .
∴ $X \vee Y$.
15. 1. $(Z \& A) \rightarrow B$.
2. $B \rightarrow A$.
3. $(B \& A) \rightarrow (A \& B)$.
∴ $(Z \& A) \rightarrow (A \& B)$.

Note: There may a lot of exercises here. Do not feel obligated to do all of them. I often assign many exercises so that you have plenty of opportunities to practice the skills these exercises are trying to impart. I suggest doing just enough of them so that you are confident that you could use these skills on a quiz or an exam.

Logic Puzzle

In Washington, D.C., politicians never ever tell the truth, and all non-politicians always tell the truth. Last week I met three people in Washington, D.C. I asked them, "How many of you are politicians?" The first person replies, "We are all politicians". The second says, "No, just two of us are politicians". The third person then says, "That isn't true either".

Question: Is the third person a politician?

This question has a definitive right answer that can be fully justified without any guessing.