CRITICAL THINKING

Lecture #21

Further Categorical Inferences

Professor David Emmanuel Gray

Four Standard Forms of Categorical Statements (Generalized)

Universal Positive

A: All *X* is *Y*.

Shade in all of *X* not shared with *Y*.

Universal Negative

E: No *X* is *Y*.

Shade in all of *X* shared with *Y*.

Particular Positive

I: Some *X* is *Y*.

Dot-*x* in *X* shared with *Y*.

Particular Negative

O: Some *X* is not *Y*.

Dot-*x* in *X* not shared with *Y*.

Note: A complement like non-*S* or non-*P* can substitute *X* or *Y*.

Inferences with Categorical Statements: *Instructions*

Given that a categorical statement is true or false, draw a Venn diagram representing that statement, being sure to label its subject term (*S*) and predicate term (*P*). (Be sure to put the subject term (*S*) on the left and the predicate term (*P*) on the right.)

Now given that Venn diagram, what can you infer about other categorical statements? That is, are these other statements true, false, or unknown? Use a Venn diagram to justify each of your answers (being sure to keep each statement's subject term on the left and predicate term on the right). You may assume that neither *S* nor *P* is empty.

Exercise #1

Assume that the following categorical statement is *true*:

No students are lazy people.

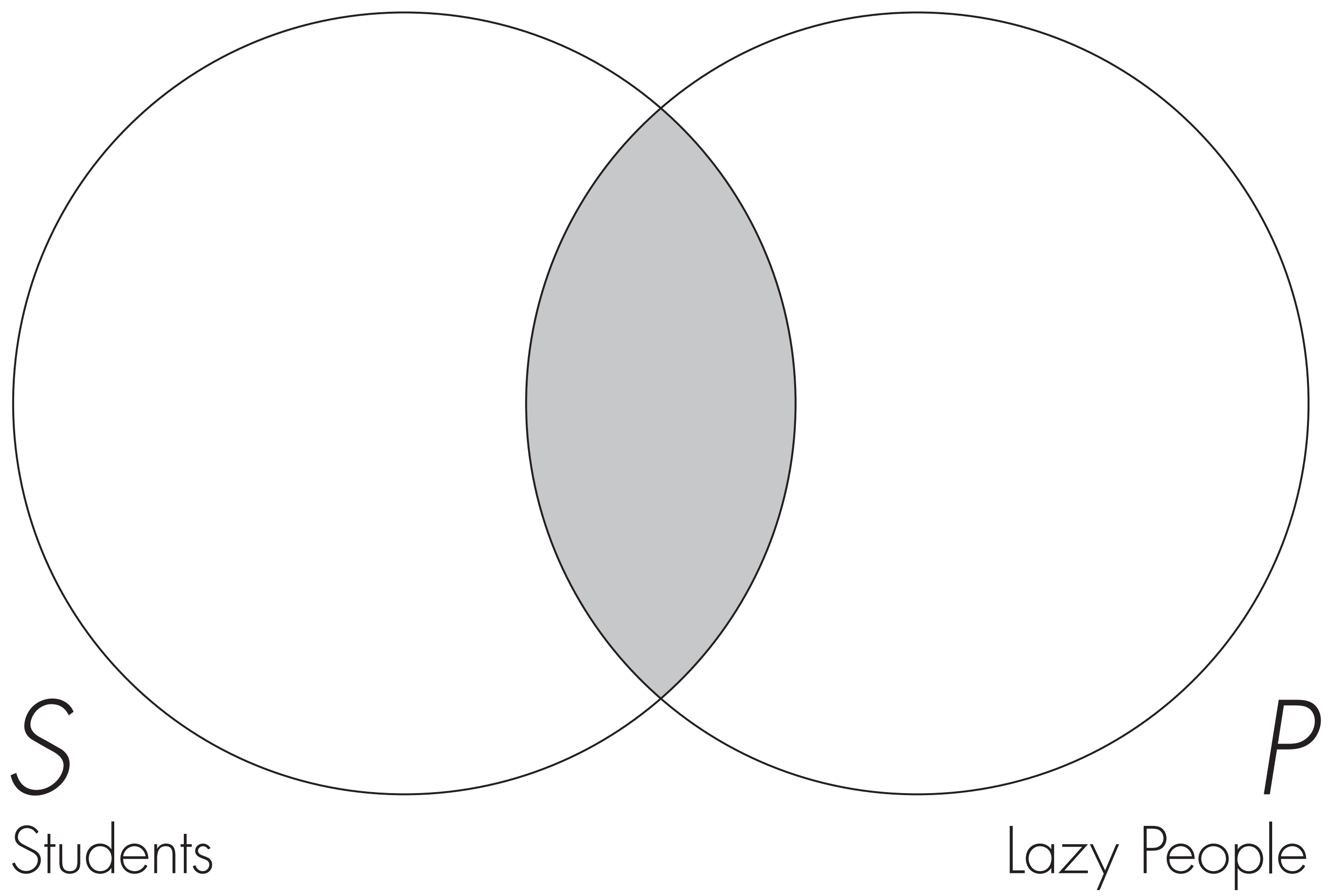
Given the truth of this statement, what can you infer about the following categorical statement?

No lazy people are students.

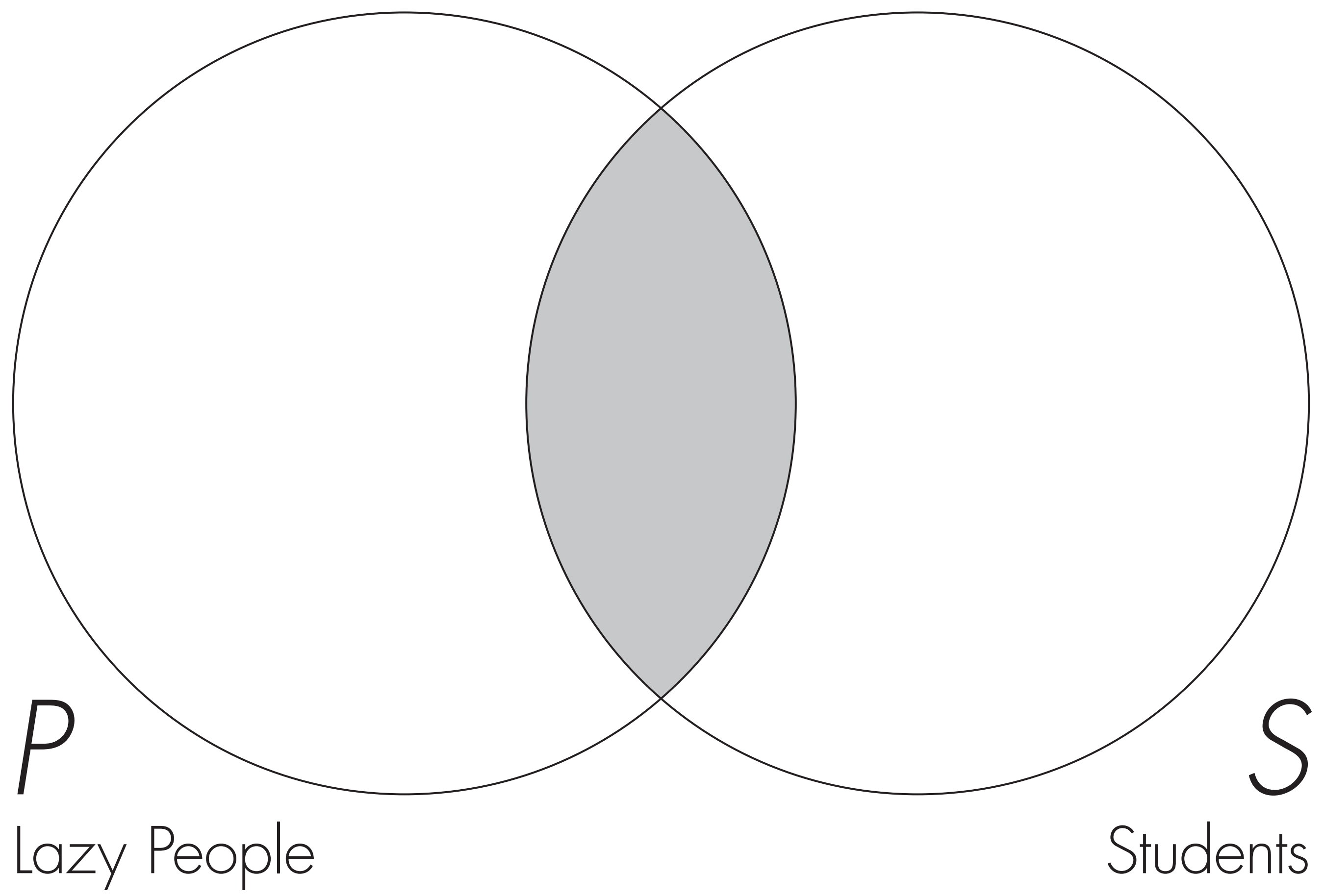
That is, is this second statement true, false, or unknown?

Exercise #1: *Venn Diagrams*

No students are lazy people.



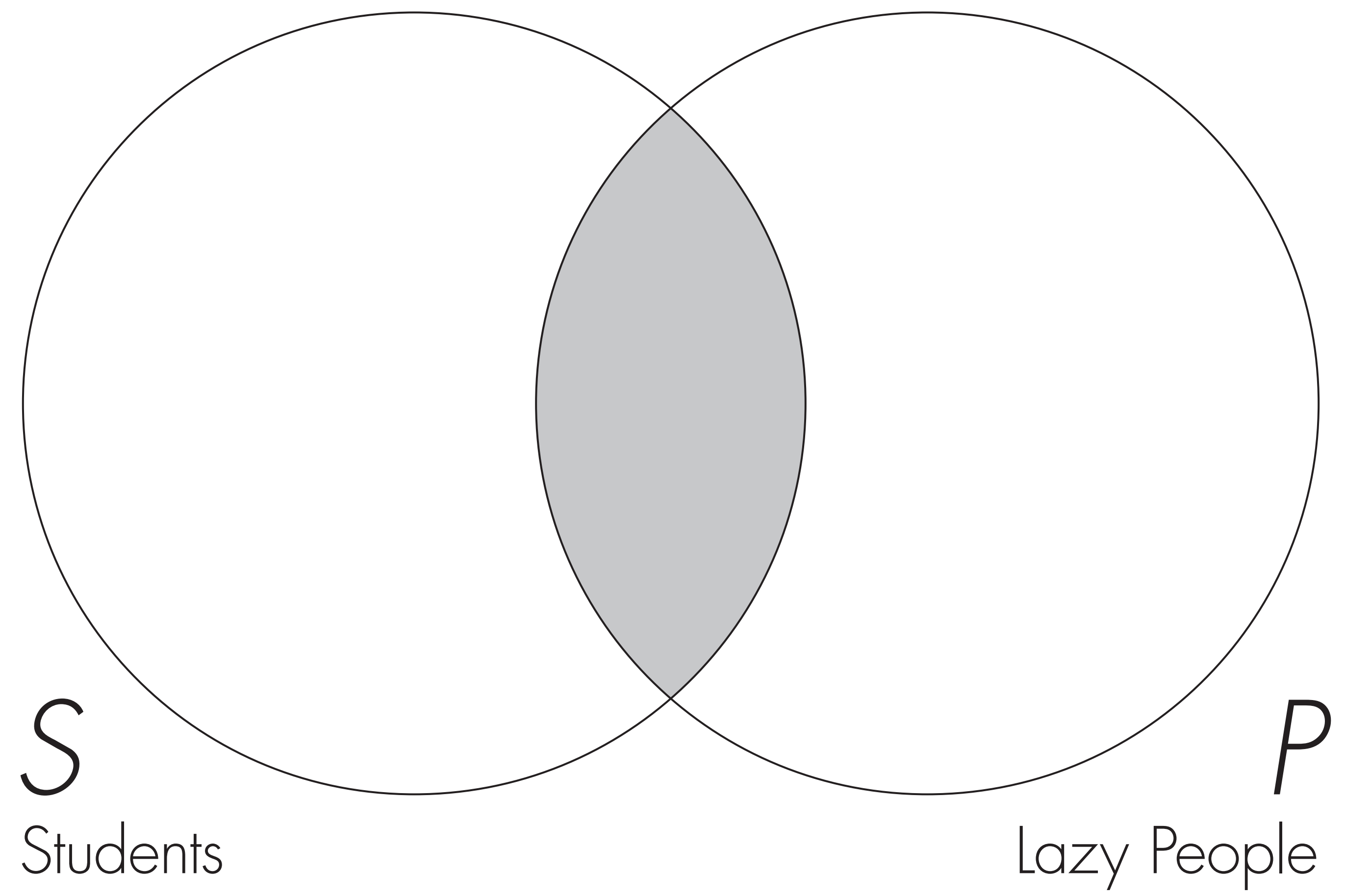
No lazy people are students.



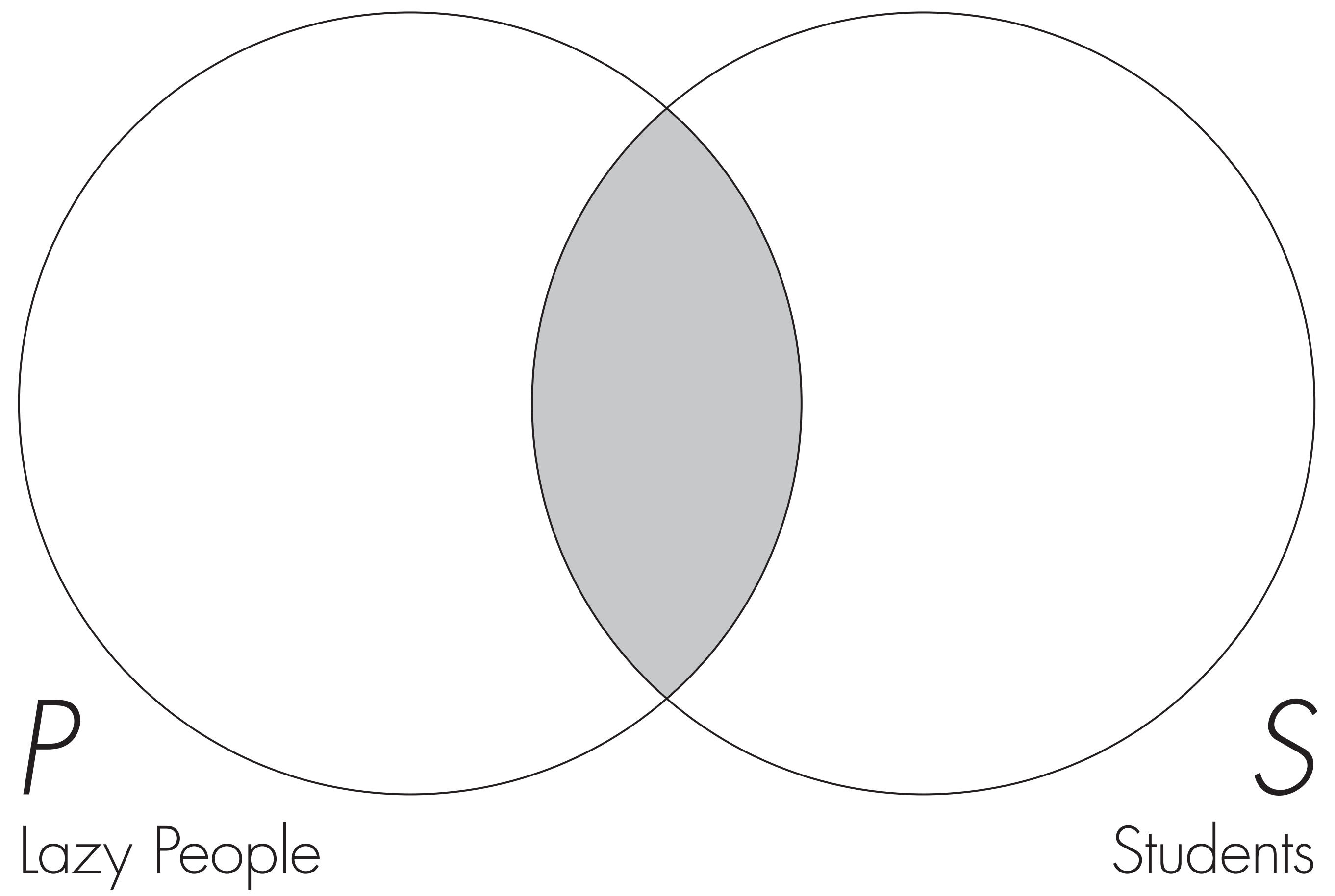
Note: To keep things constant, we fix the categories *S* and *P* using the first statement (on the left) with *S* = students and *P* = lazy people. So in the second statement (on the right), students (*S*) is now the predicate and lazy people (*P*) is now the subject.

Exercise #1: *Inference Determined*

No students are lazy people.



No lazy people are students.



The statement on the right is *true*.

The truth of of the statement on the left implies that there is nothing in the area of overlap between students and lazy people. This is also seen in the Venn diagram for the statement on the right (this diagram is just a mirror image of the left statement with students and lazy people swapped). So statement on the right must be true.

Exercise #2

Assume that the following categorical statement is *true*:

Some students are lazy people.

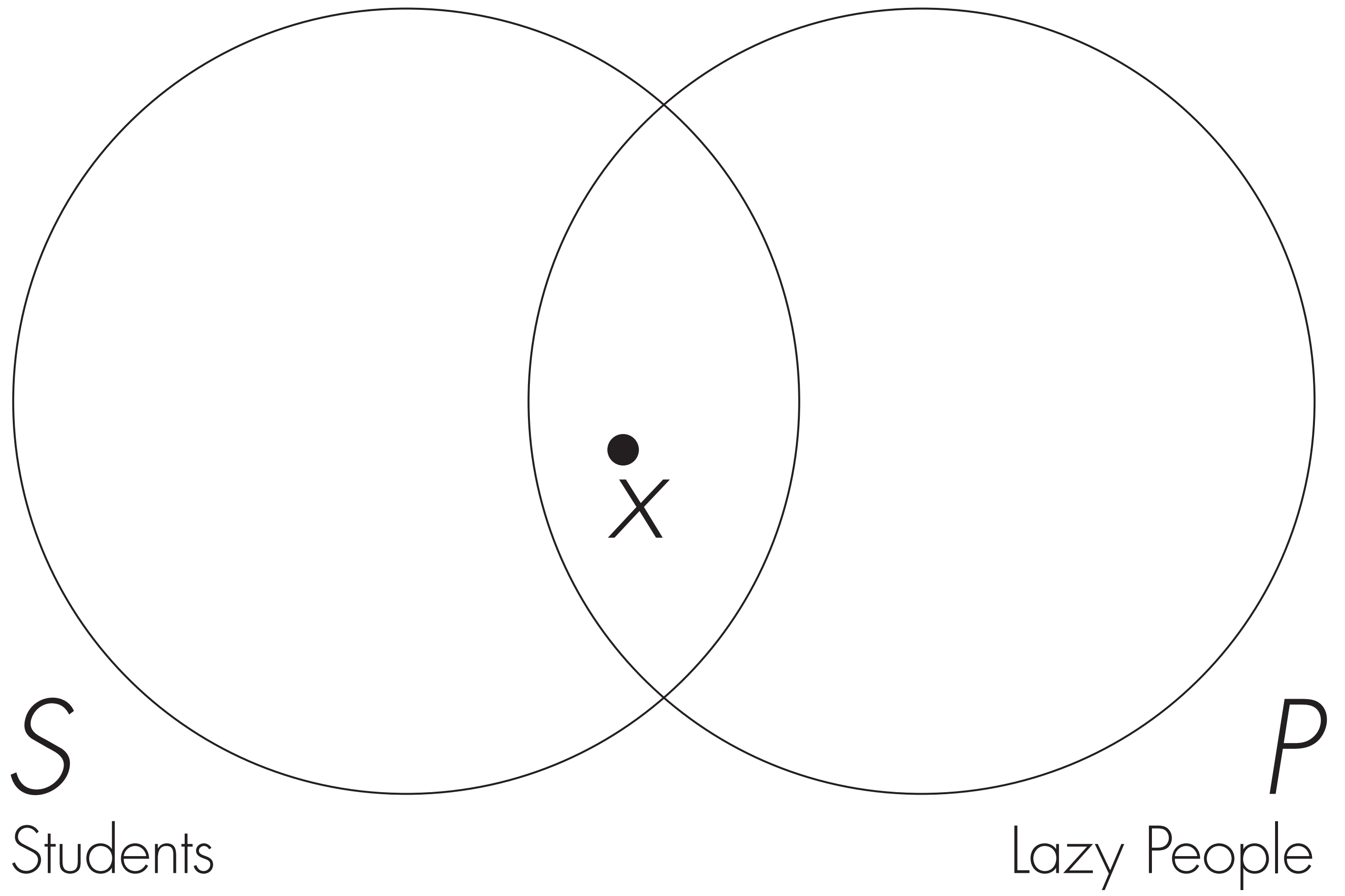
Given the truth of this statement, what can you infer about the following categorical statement?

Some lazy people are students.

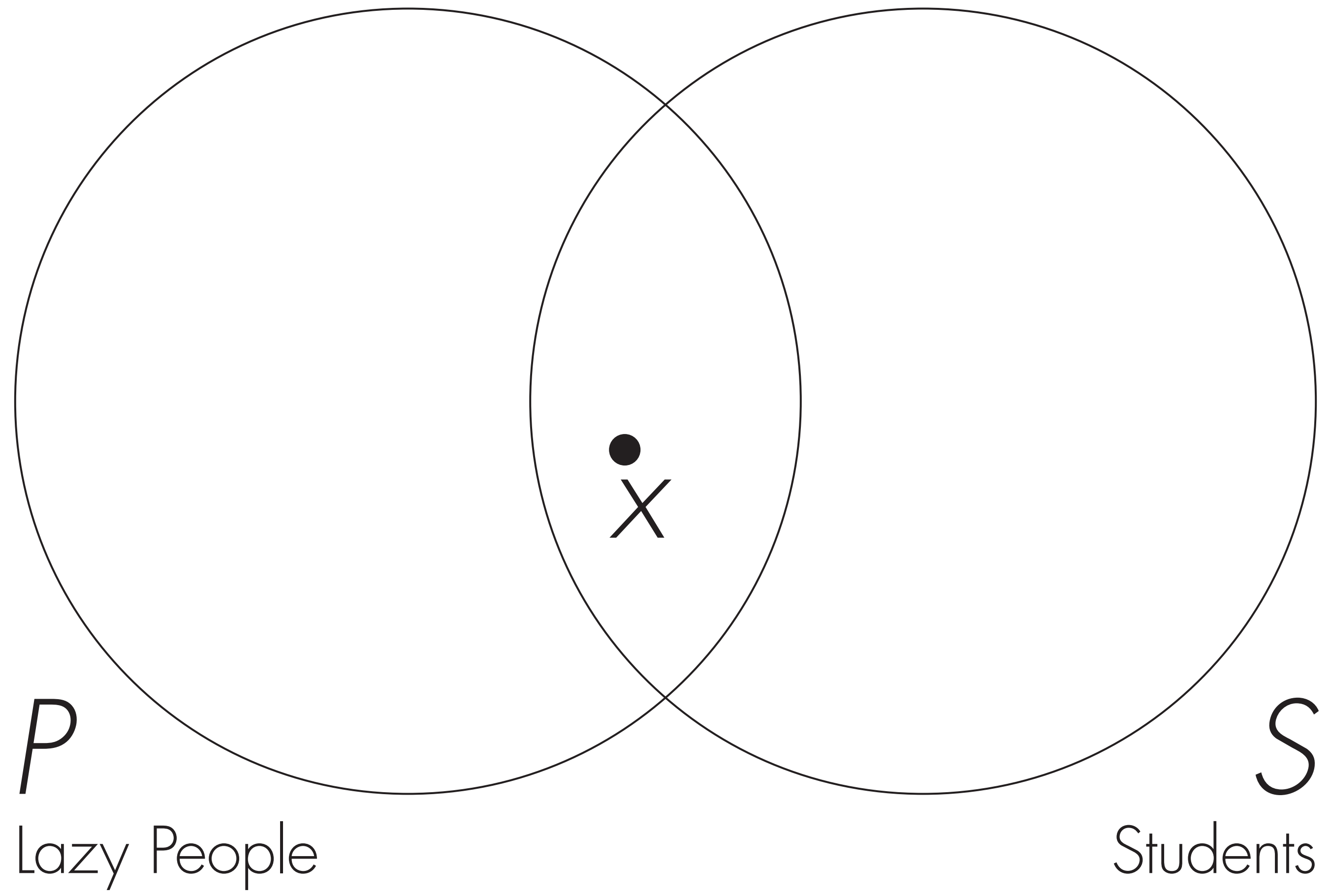
That is, is this second statement true, false, or unknown?

Exercise #2: *Venn Diagrams*

Some students are lazy people.



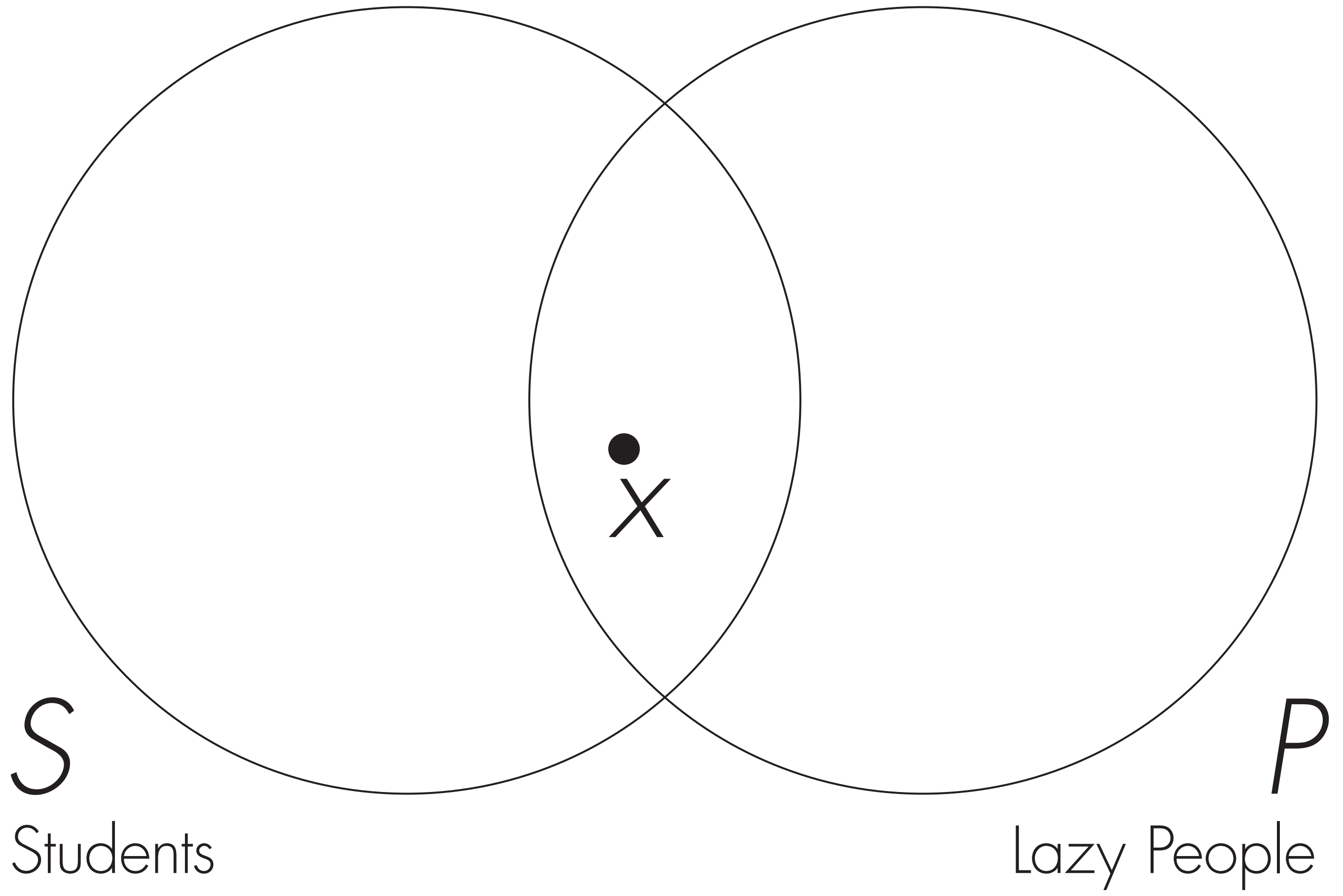
Some lazy people are students.



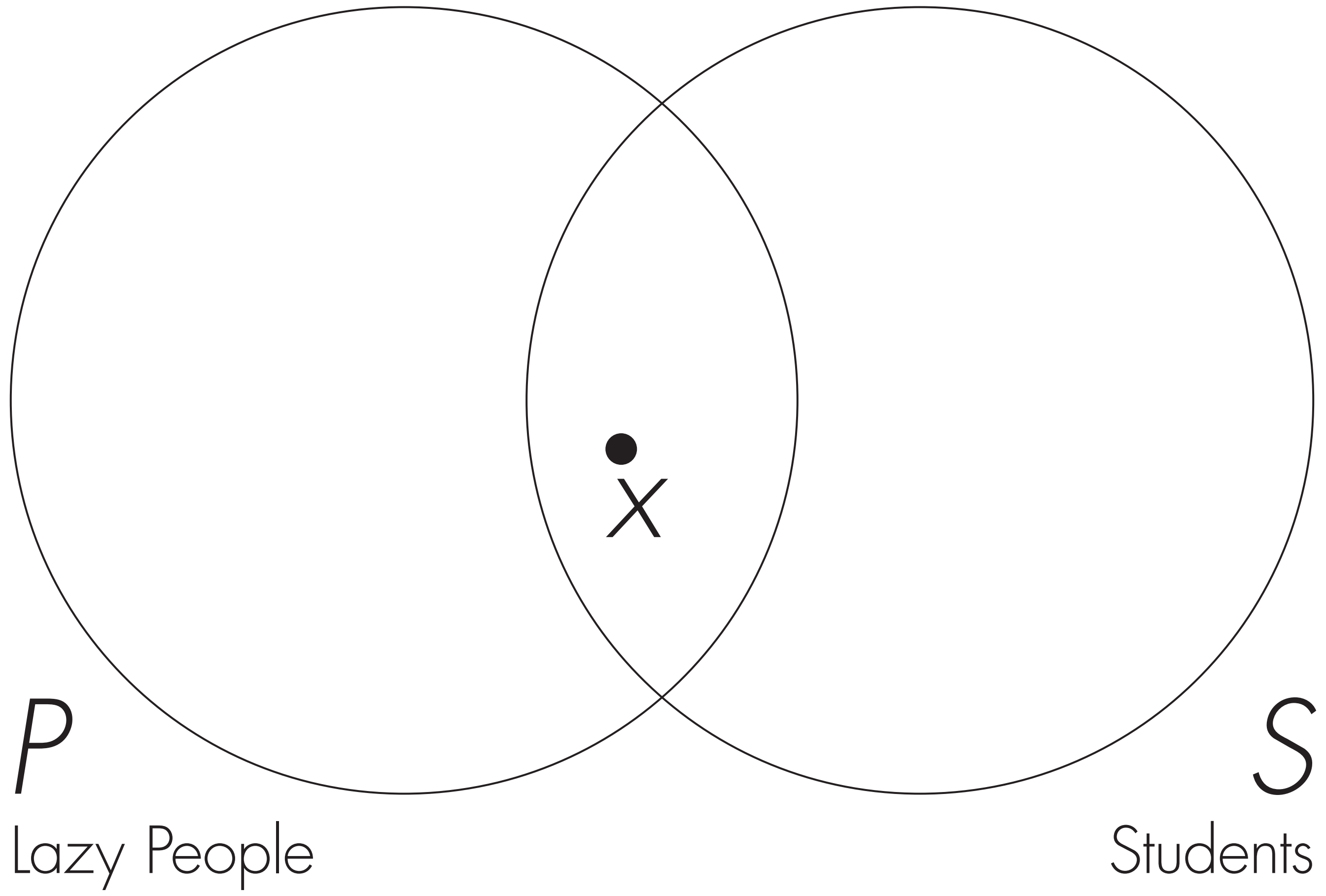
Note: As before, we fix the categories *S* and *P* using the first statement (on the left) with *S* = students and *P* = lazy people. So in the second statement (on the right), students (*S*) is now the predicate and lazy people (*P*) is now the subject.

Exercise #2: *Inference Determined*

Some students are lazy people.



Some lazy people are students.



The statement on the right is *true*.

The truth of of the statement on the left implies that there is something in the area of overlap between students and lazy people. This is also seen in the Venn diagram for the statement on the right (this diagram is just a mirror image of the left statement with students and lazy people swapped). So statement on the right must be true.

Categorical Statements: *Conversion*

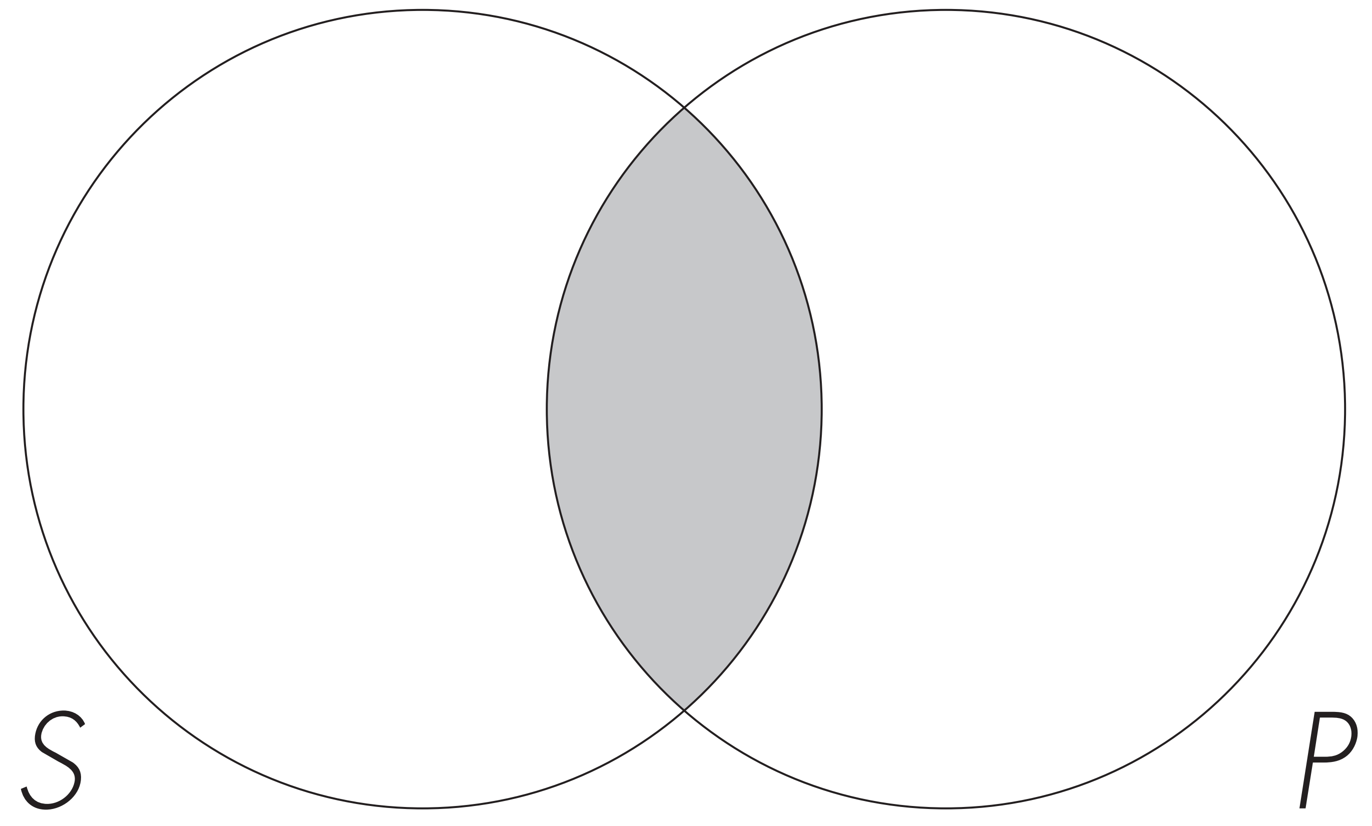
The **conversion** of a categorical statement swaps its subject (*S*) and predicate (*P*) terms to create a new categorical statement.

In some instances, the new statement will be logically equivalent to the original one. For example, the statement "No students are lazy people" (**E**: No *S* is *P*) is logically the same as "No lazy people are students" (**E**: No *P* is *S*).

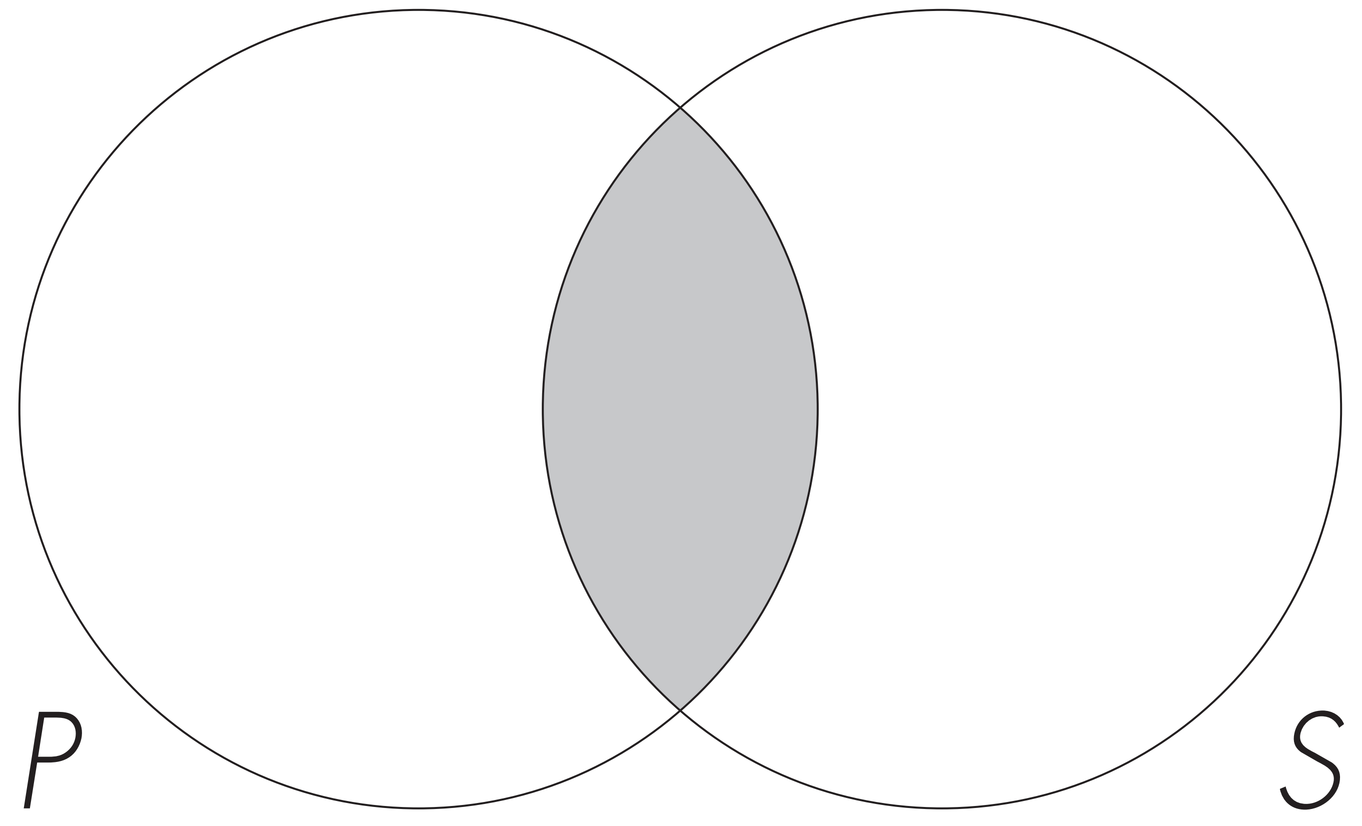
Note: Again, to keep things constant, we fix the categories *S* and *P* using the first statement (with *S* = students and *P* = lazy people). So in the second statement, students (*S*) is now the predicate and lazy people (*P*) is now the subject.

Conversion: **E**

In general, any **E** statement and its conversion are logically the same.



E Statement: No *S* is *P*.

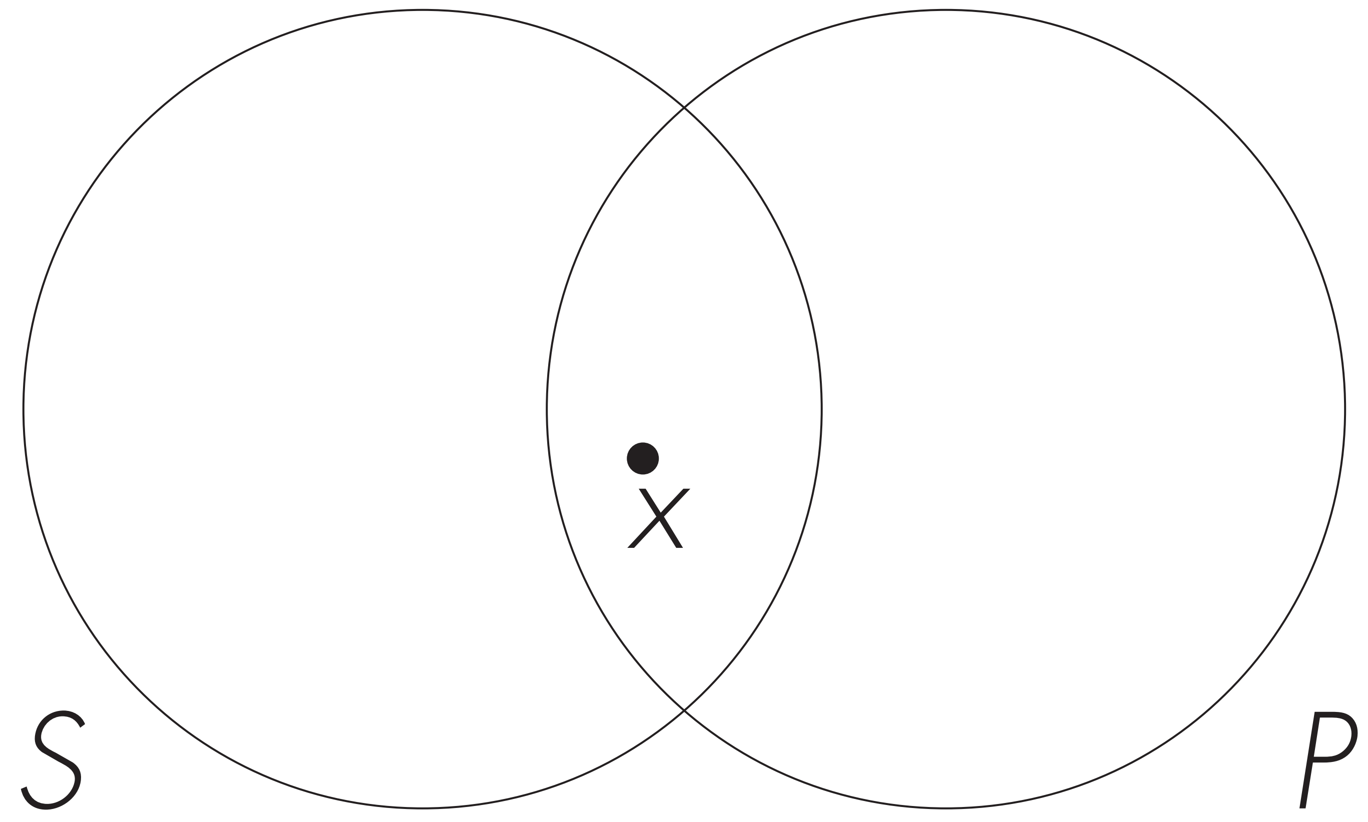


E's Conversion: No *P* is *S*.

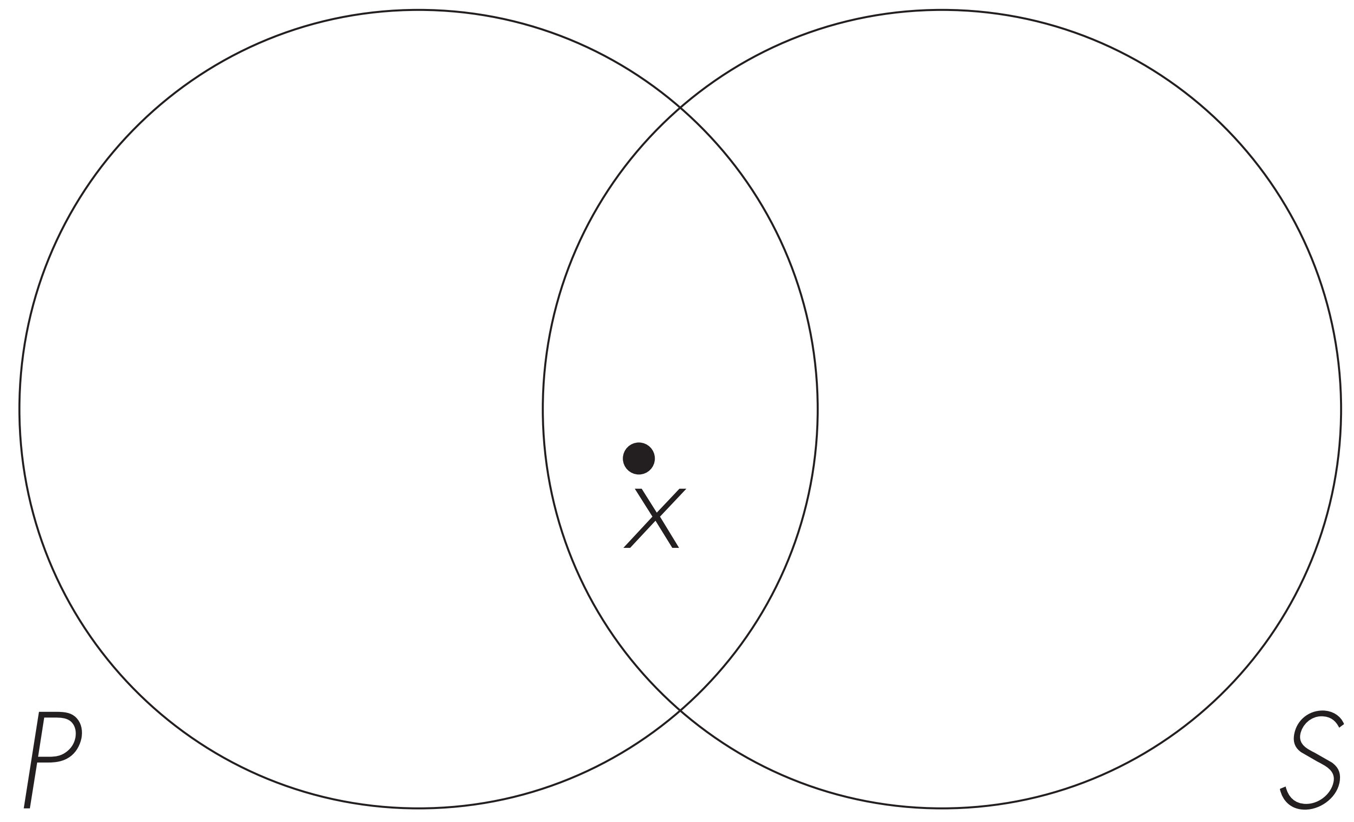
Note: Even though *S* and *P* are the same in both statements, the Venn diagram of the second statement (like all diagrams) has its left circle represent the statement's subject (now *P*) and its right circle represent the statement's predicate (now *S*).

Conversion: **I**

Similarly, any **I** statement and its conversion are logically the same.



I Statement: Some *S* is *P*.



I's Conversion: Some *P* is *S*.

Exercise #3

Assume that the following categorical statement is *true*:

Some students are not lazy people.

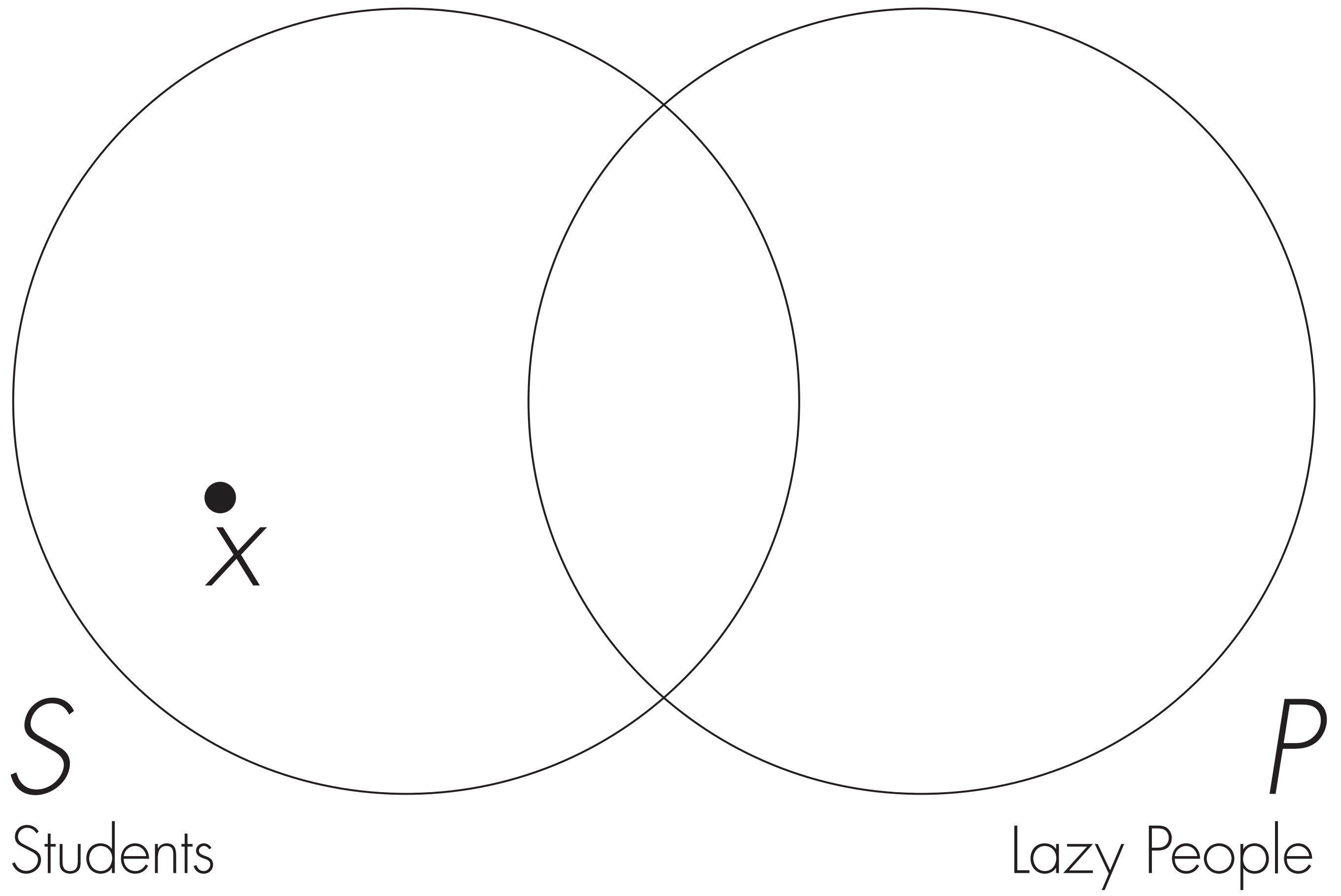
Given the truth of this statement, what can you infer about the following categorical statement?

Some lazy people are not students.

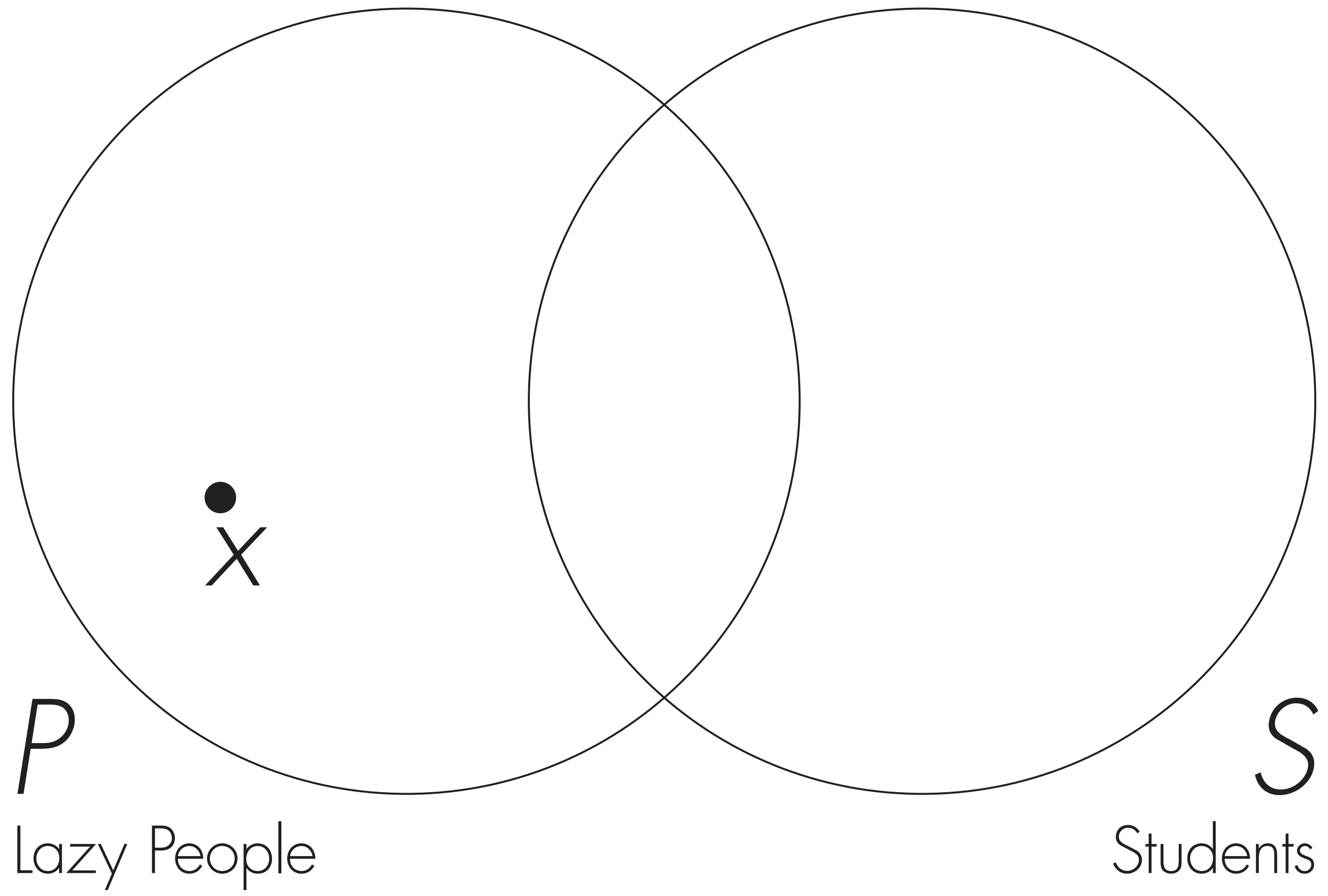
That is, is this second statement true, false, or unknown?

Exercise #3: *Venn Diagrams*

Some students are not lazy people.



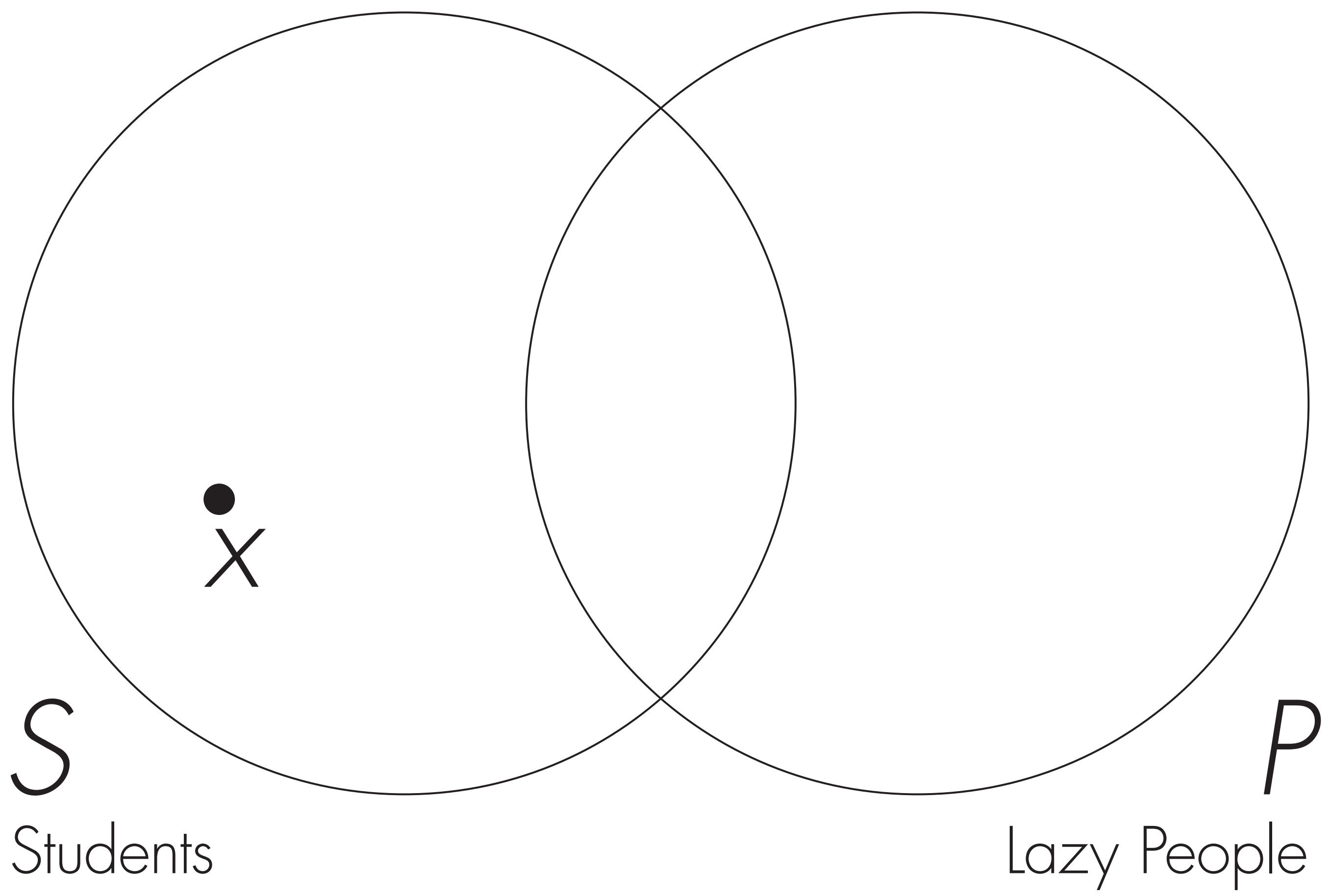
Some lazy people are not students.



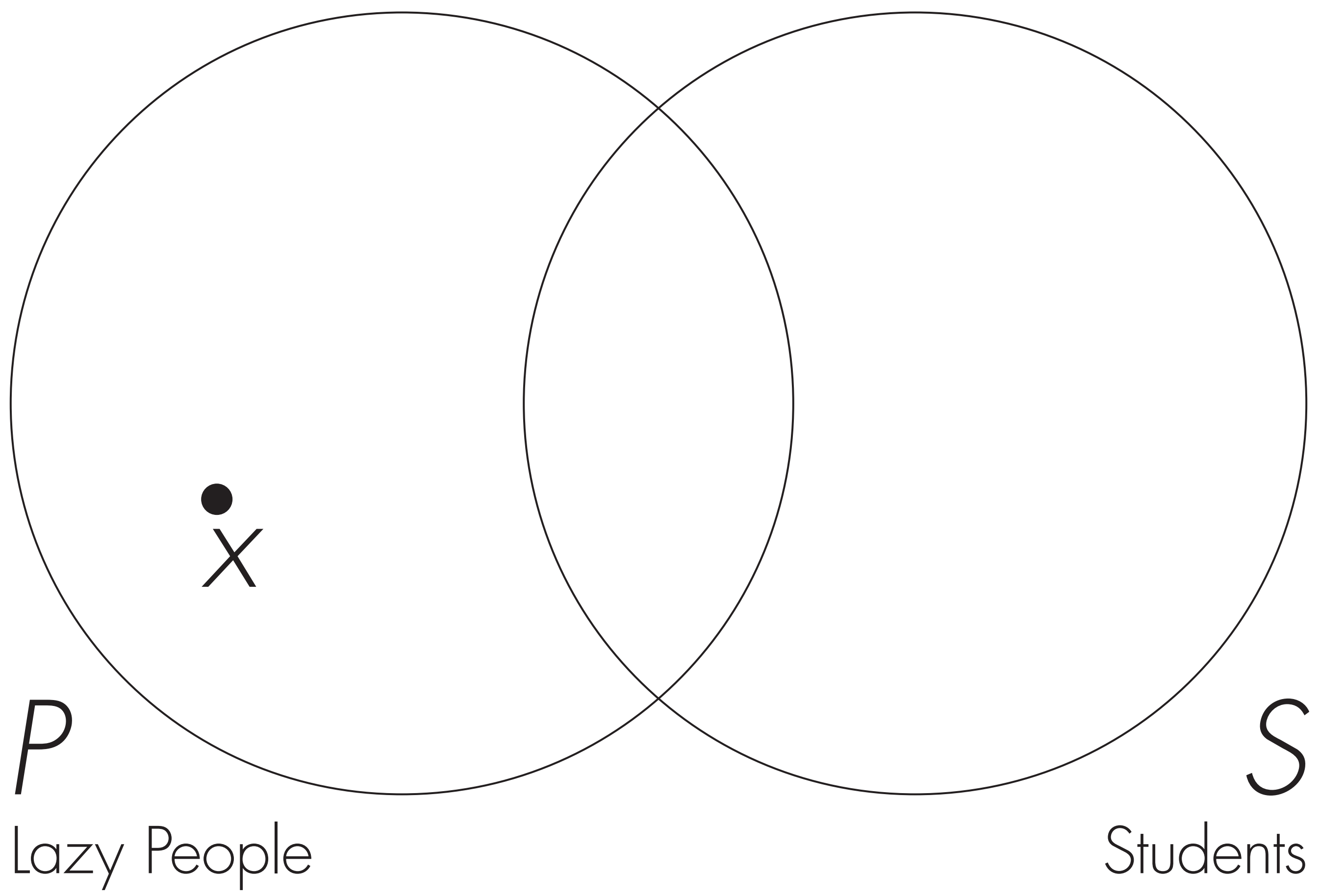
Note: Once again, we fix the categories *S* and *P* using the first statement (on the left) with *S* = students and *P* = lazy people. So in the second statement (on the right), students (*S*) is now the predicate and lazy people (*P*) is now the subject.

Exercise #3: *Inference Determined*

Some students are not lazy people.



Some lazy people are not students.

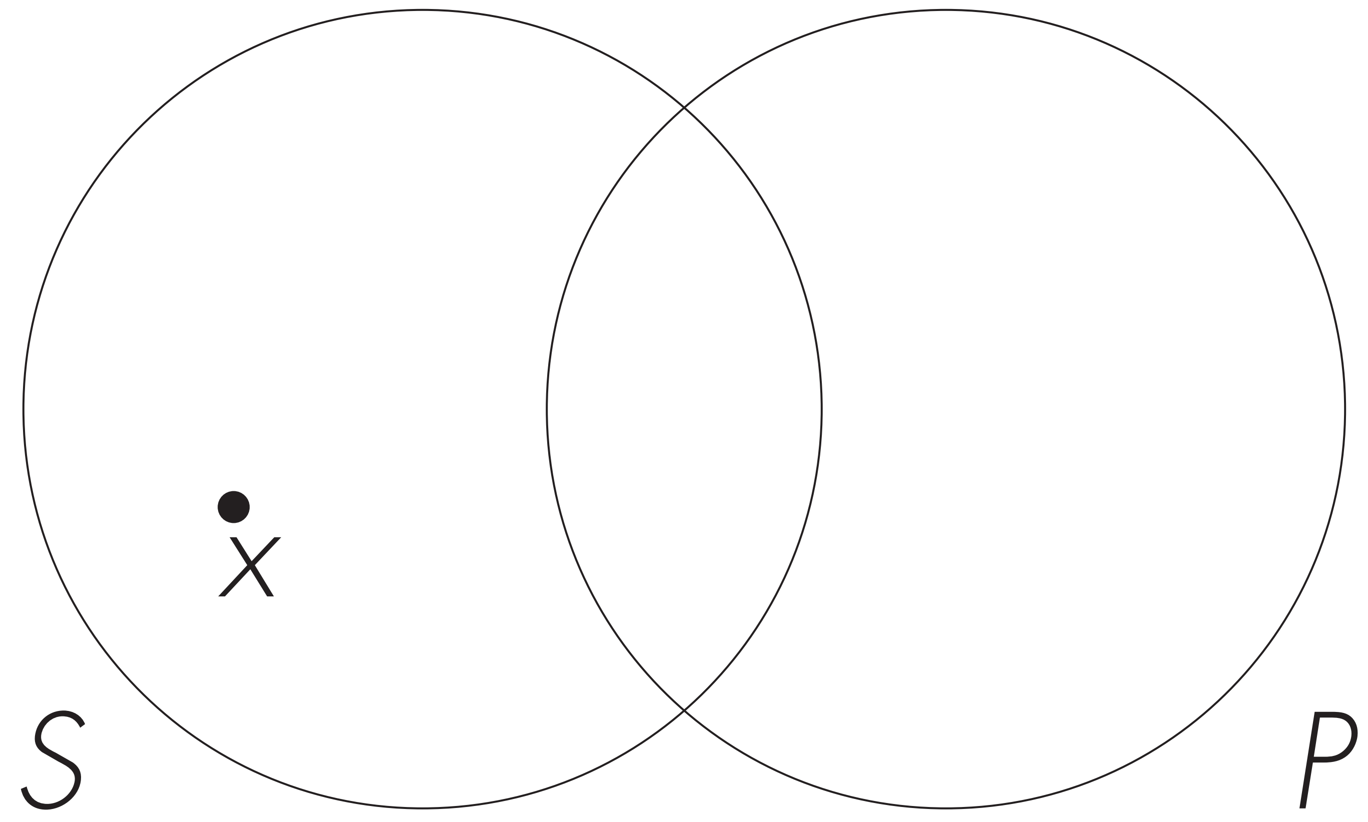


The statement on the right is *unknown*.

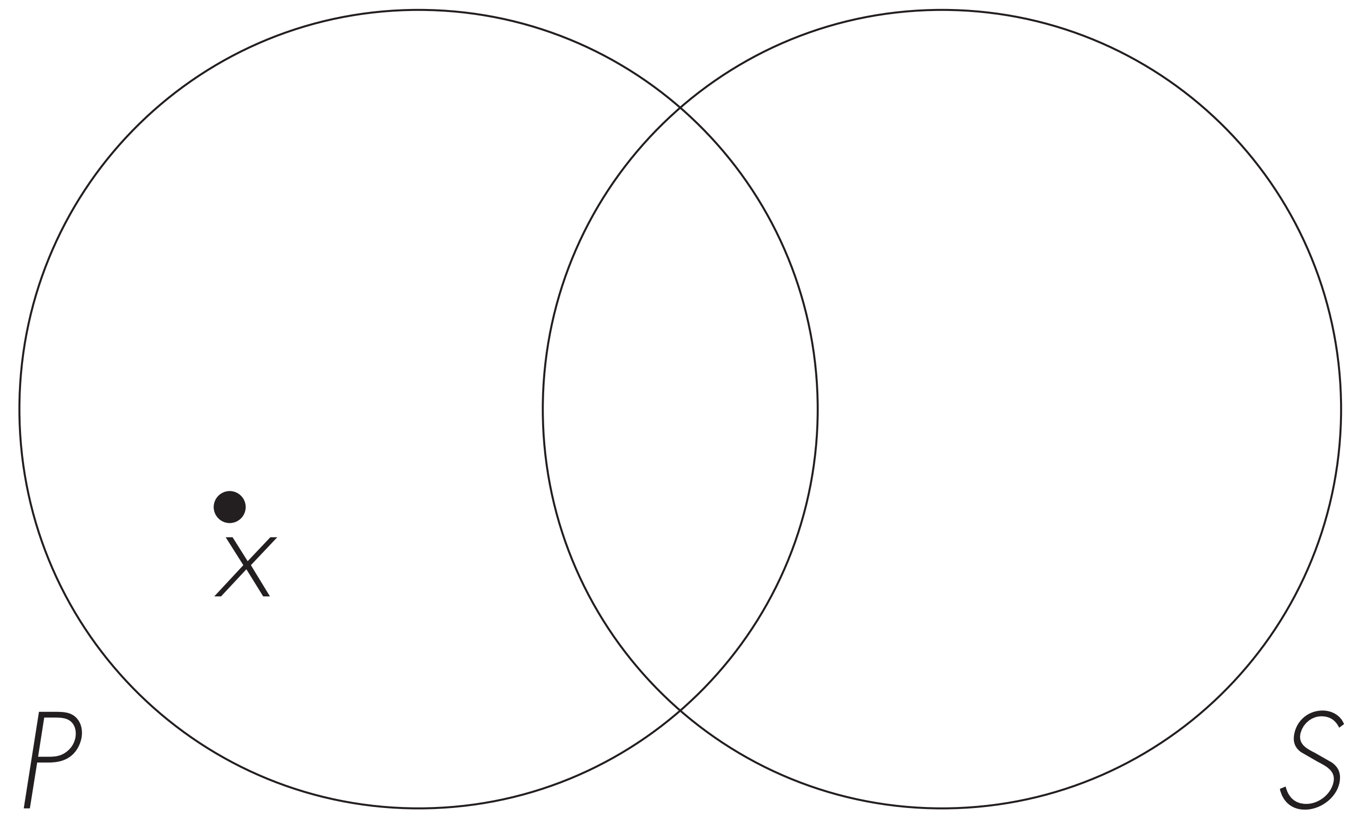
The truth of the statement on the left only tells us about the area of students outside of lazy people while telling us nothing about what is going on inside the area of lazy people. Since the statement on the right only tells us about what is going on inside that area of lazy people, it is therefore impossible to know whether the statement on the right is true or false.

Conversion: O

In general, any **O** statement and its conversion are *not* logically the same.



O Statement: Some *S* is *P*.

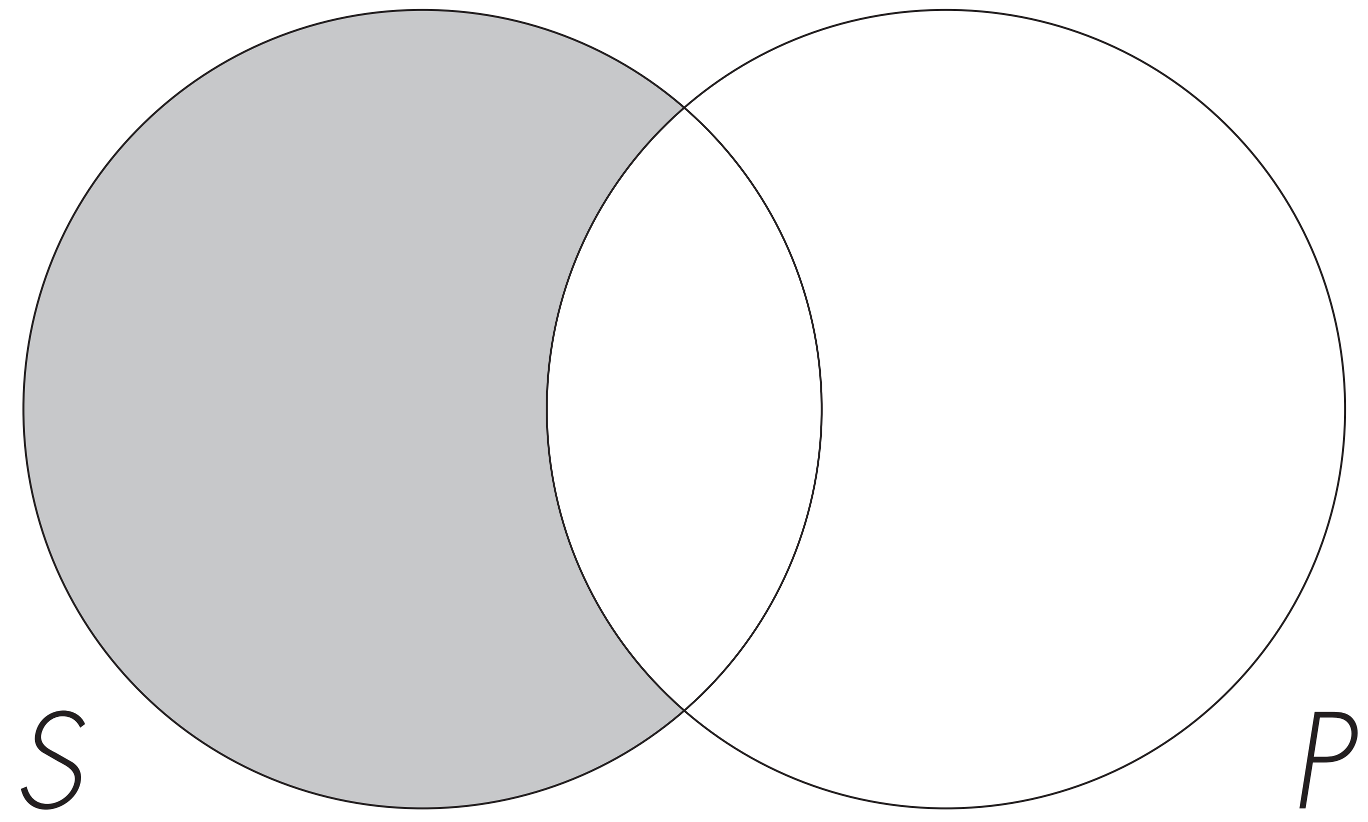


O's Conversion: Some *P* is *S*.

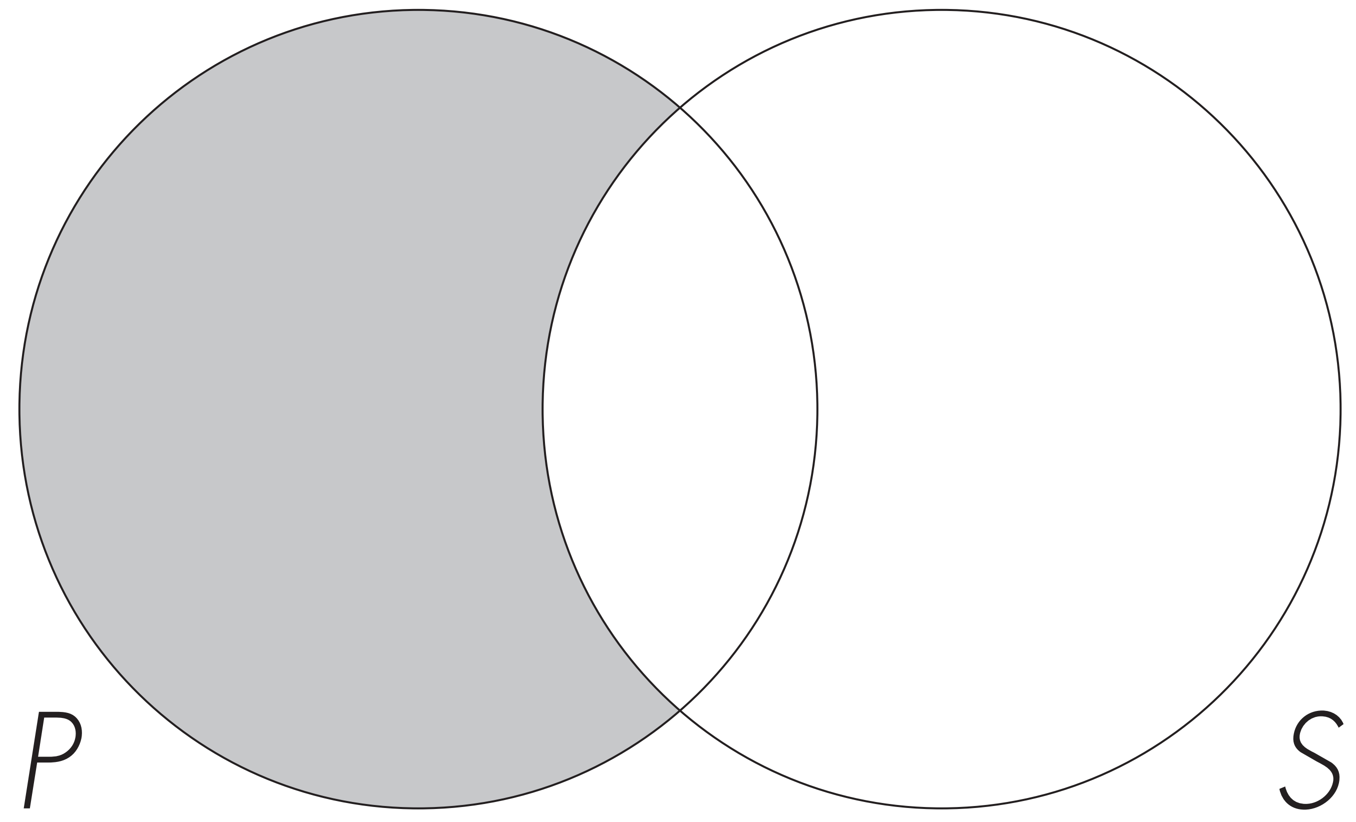
Look closely and you will see that the dot-*x* is actually *not* in the same place in both diagrams.

Conversion: **A**

Similarly, any **A** statement and its conversion are *not* logically the same.



A Statement: All *S* is *P*.



A's Conversion: All *P* is *S*.

Look closely and you will see that the shaded area is actually *not* the same in both diagrams.

Categorical Statements: *Complements*

Recall that for any subject (*S*) or predicate (*P*) term in a categorical statement, we may consider its complement. The **complement** of a category consists of everything *not* in that category. The complement of the subject term *S* is denoted as non-*S*; the complement of the predicate term *P* is denoted by non-*P*.

Exercise #4

Assume that the following categorical statement is *true*:

All students are lazy people.

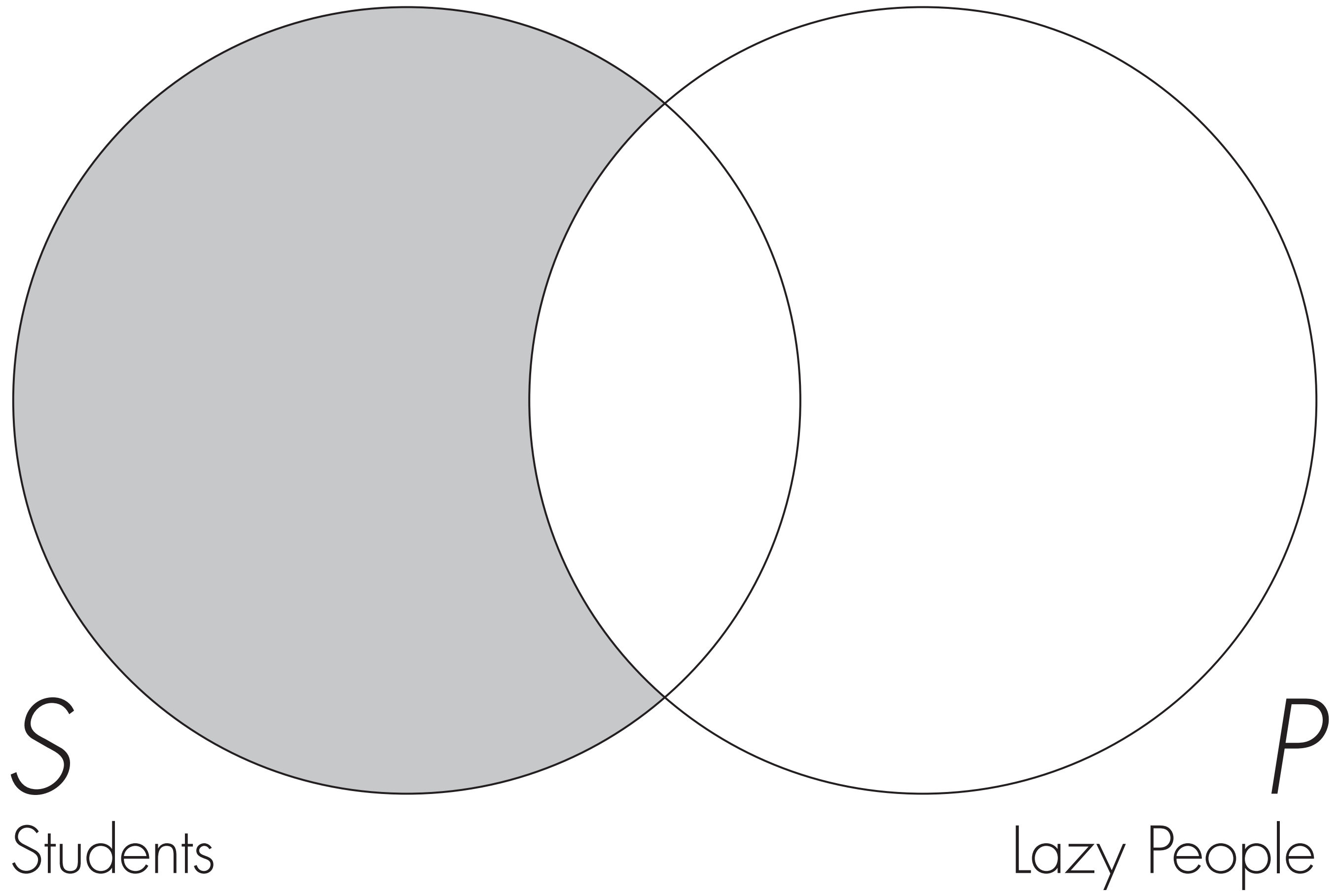
Given the truth of this statement, what can you infer about the following categorical statement?

No students are non-lazy people.

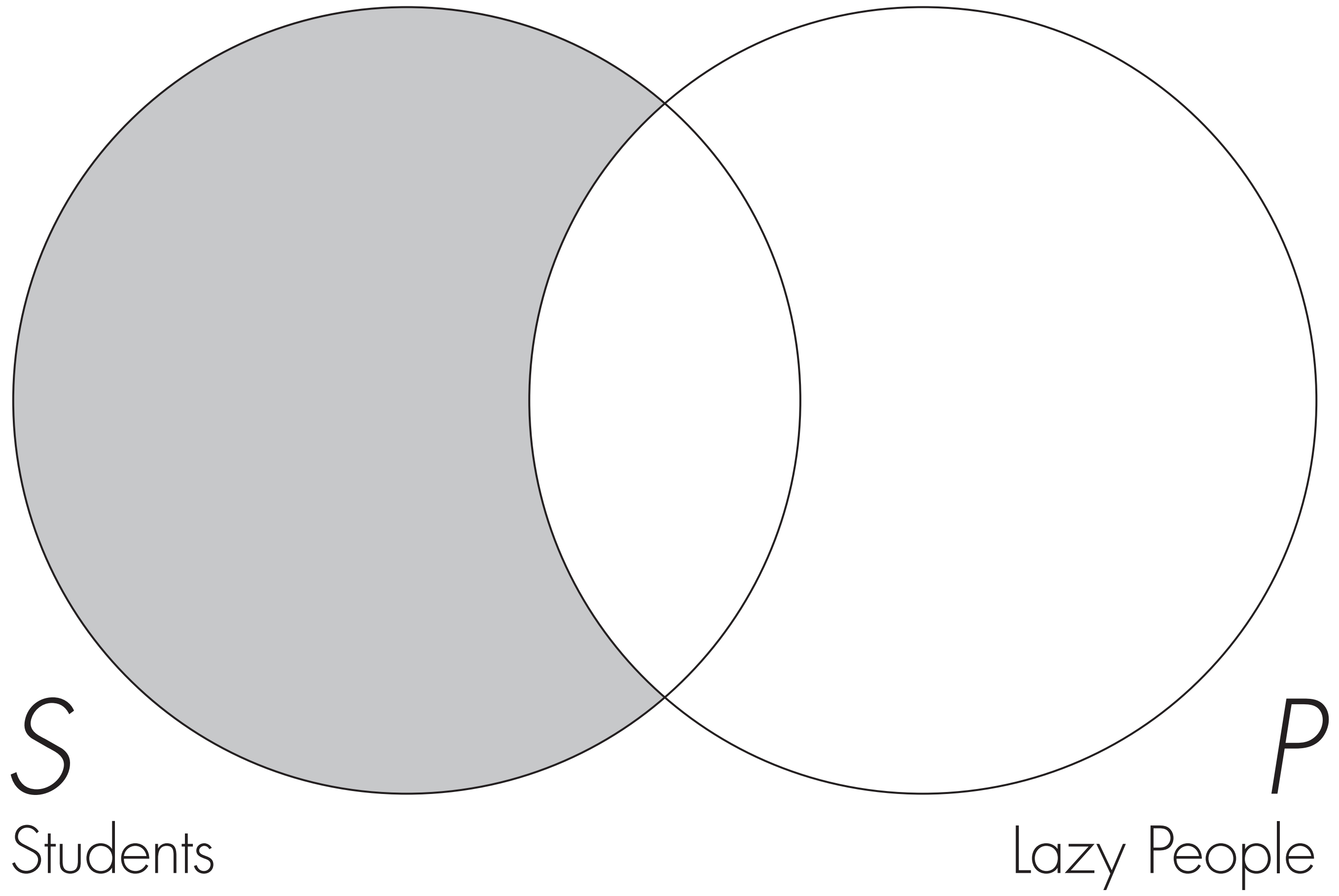
That is, is this second statement true, false, or unknown?

Exercise #4: *Venn Diagrams*

All students are lazy people.

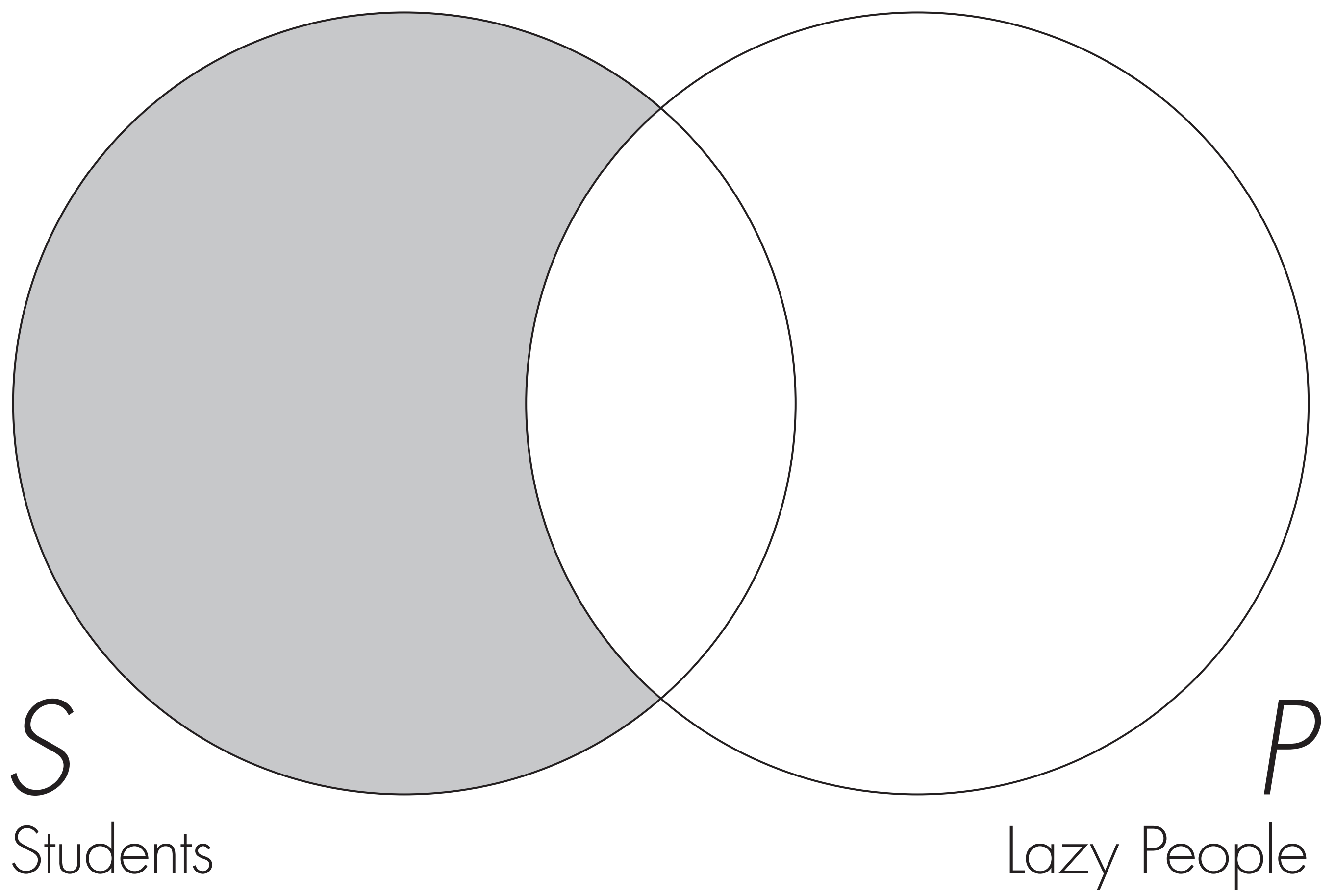


No students are non-lazy people.

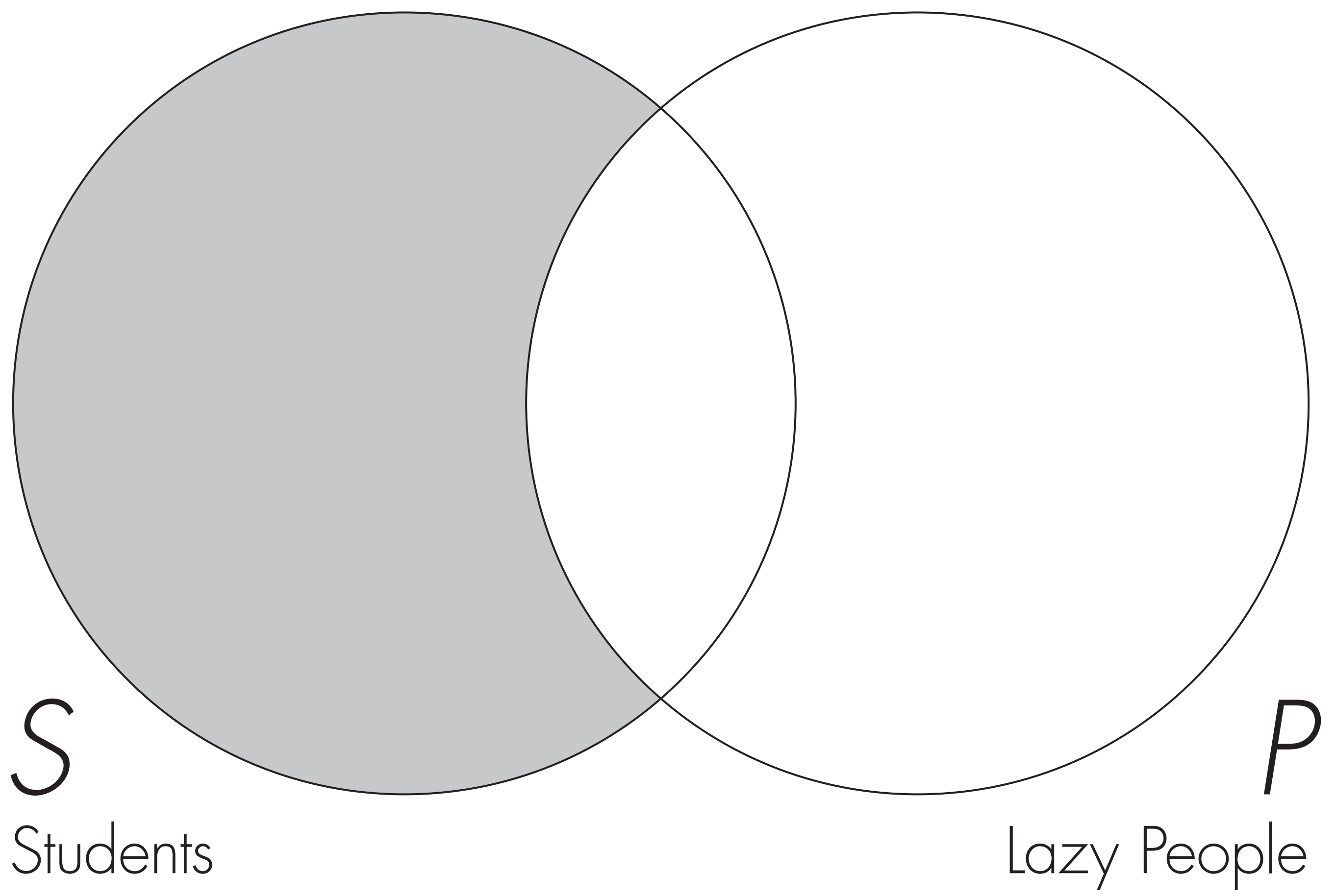


Exercise #4: *Inference Determined*

All students are lazy people.



No students are non-lazy people.



The statement on the right is *true*.

The truth of the statement on the left implies that there is nothing in the area of students outside of lazy people. Indeed the Venn diagram for the truth of the statement on the left is the same as that for the truth of the statement on the right. So the statement on the right must be true.

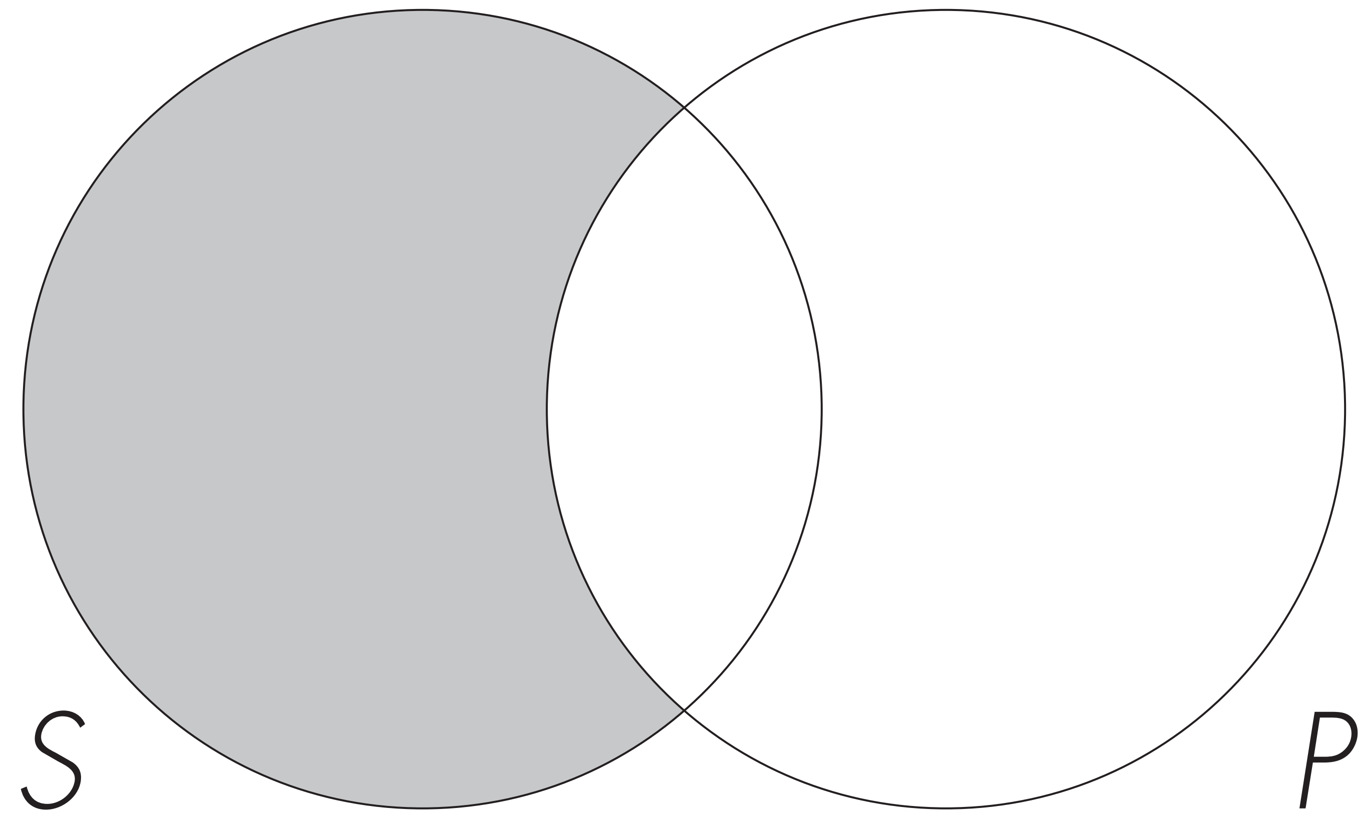
Categorical Statements: *Obversion*

The **obversion** of a categorical statement comes from flipping its quality and replacing the predicate (*P*) with that predicate's complement (non-*P*).

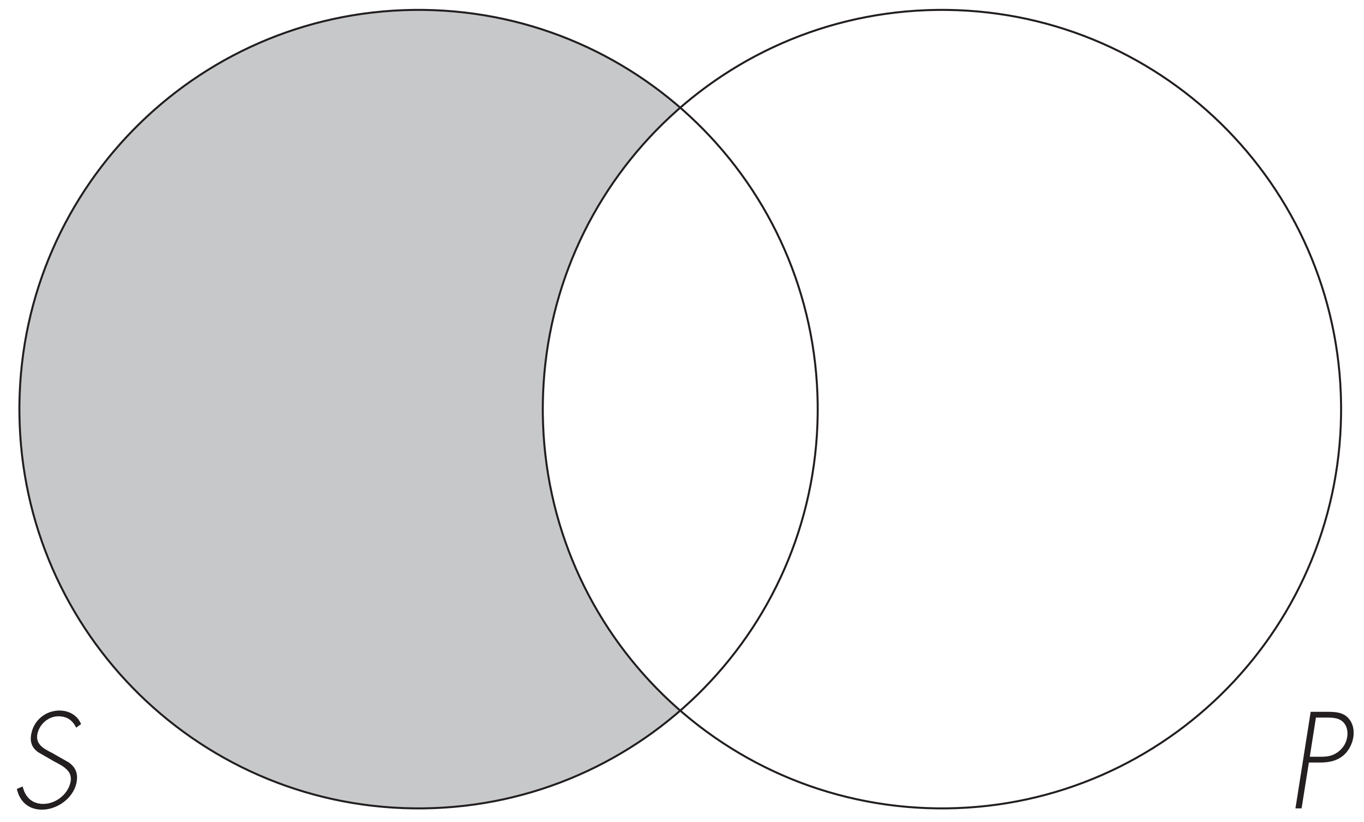
It turns out that the obversion of each of the standard four categorical statements is logically equivalent to the original statement. So, for instance, "All students are lazy people" (**A**: All *S* is *P*) is logically equivalent to its obversion: "No students are non-lazy people" (No *S* is non-*P*).

Obversion: **A**

In general, any **A** statement and its obversion are logically the same.



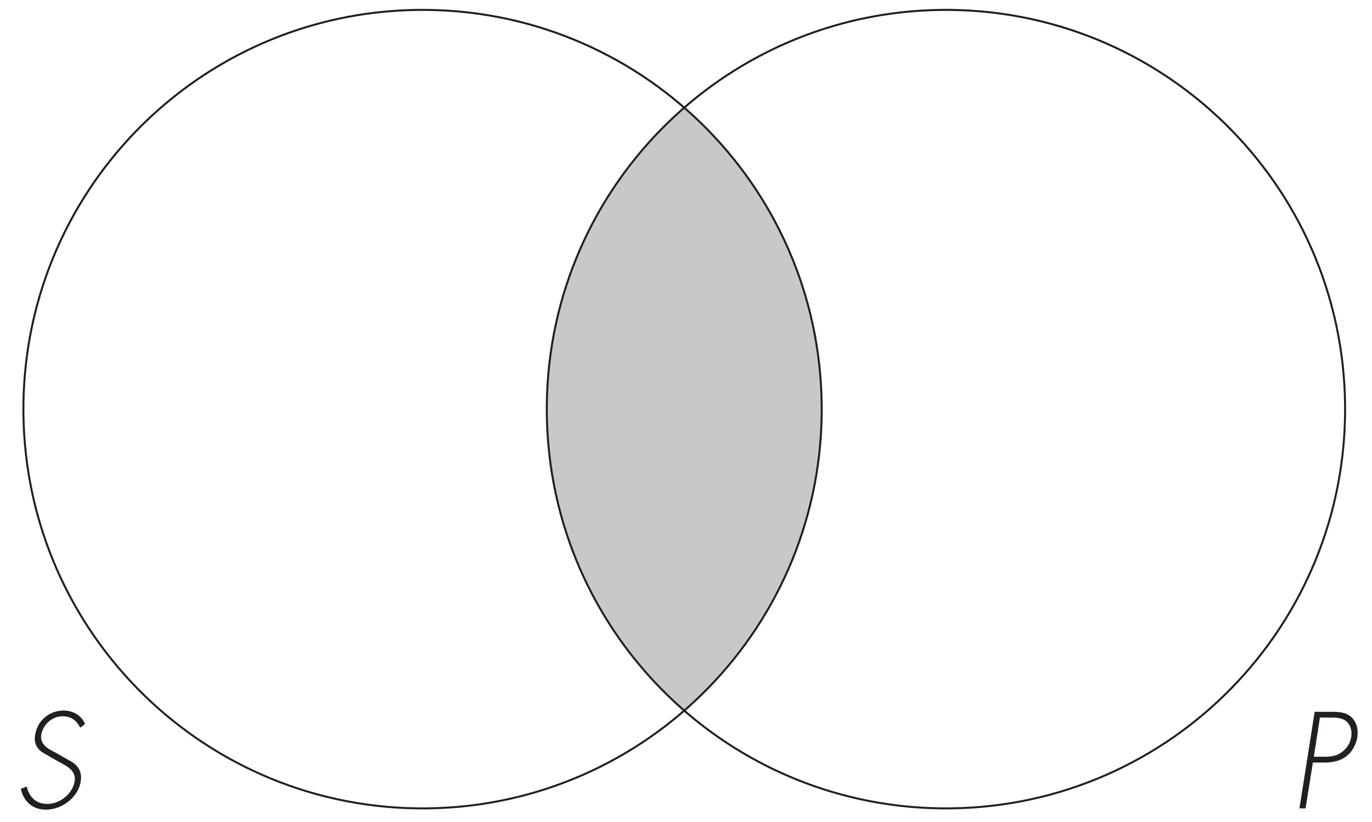
A Statement: All *S* is *P*.



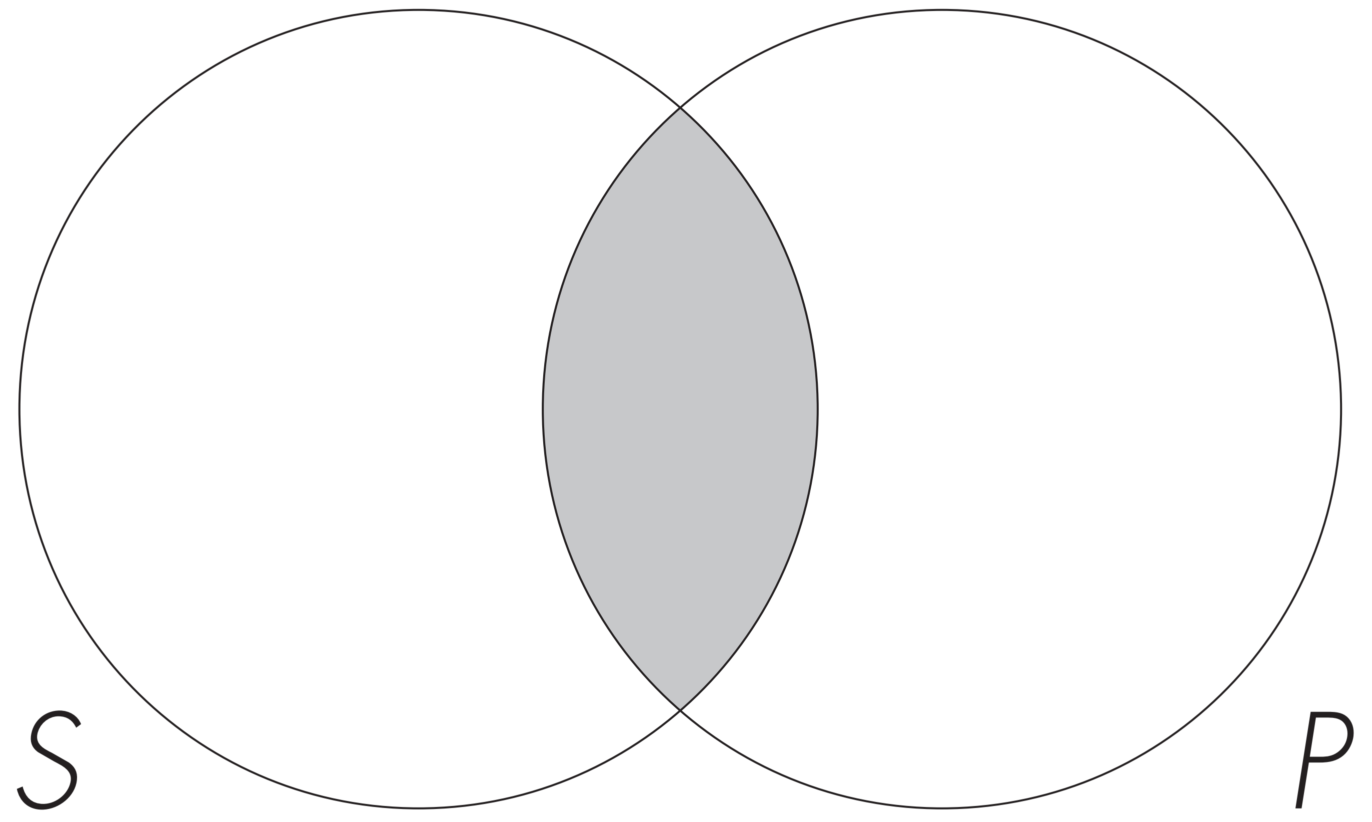
A's Obversion: No *S* is non-*P*.

Obversion: **E**

Similarly, any **E** statement and its obversion are logically the same.



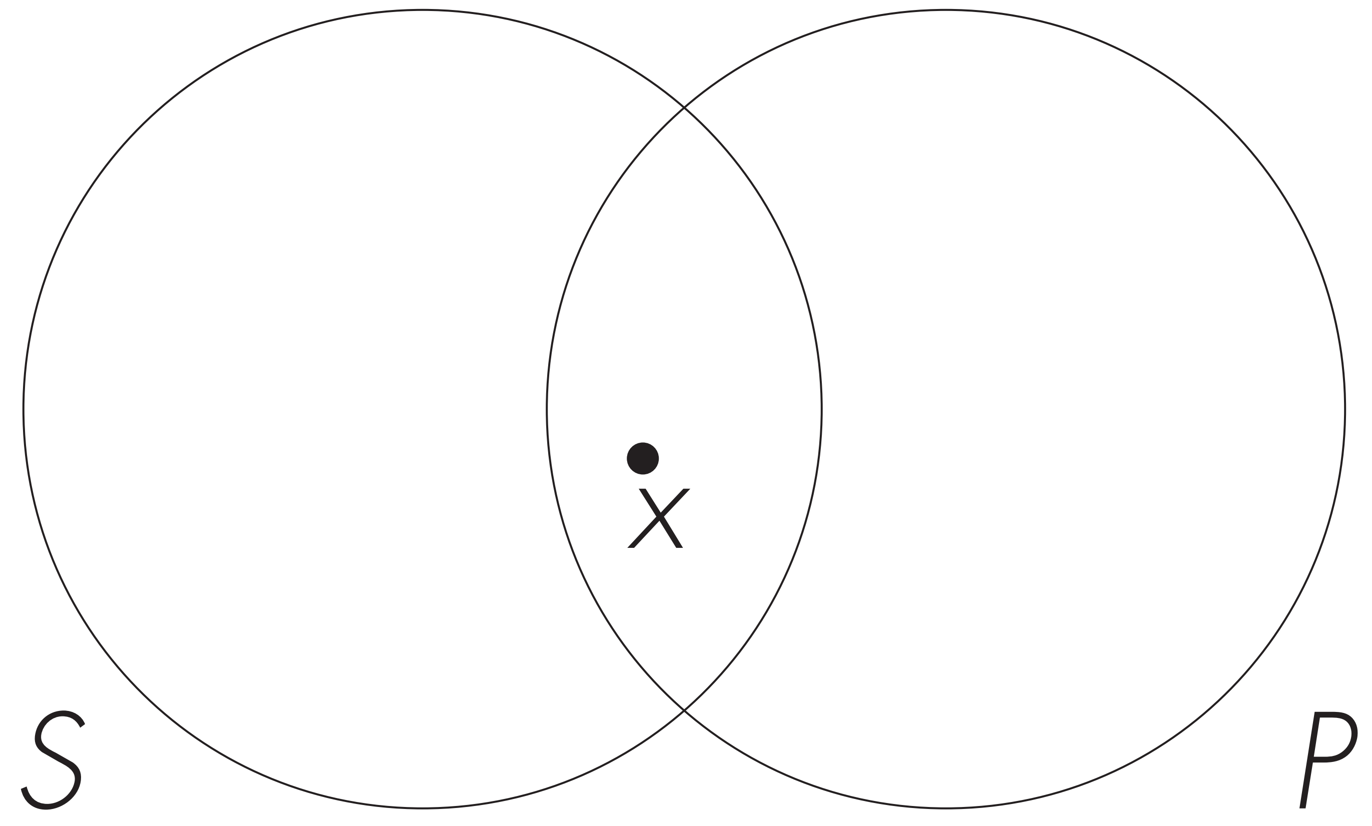
E Statement: No *S* is *P*.



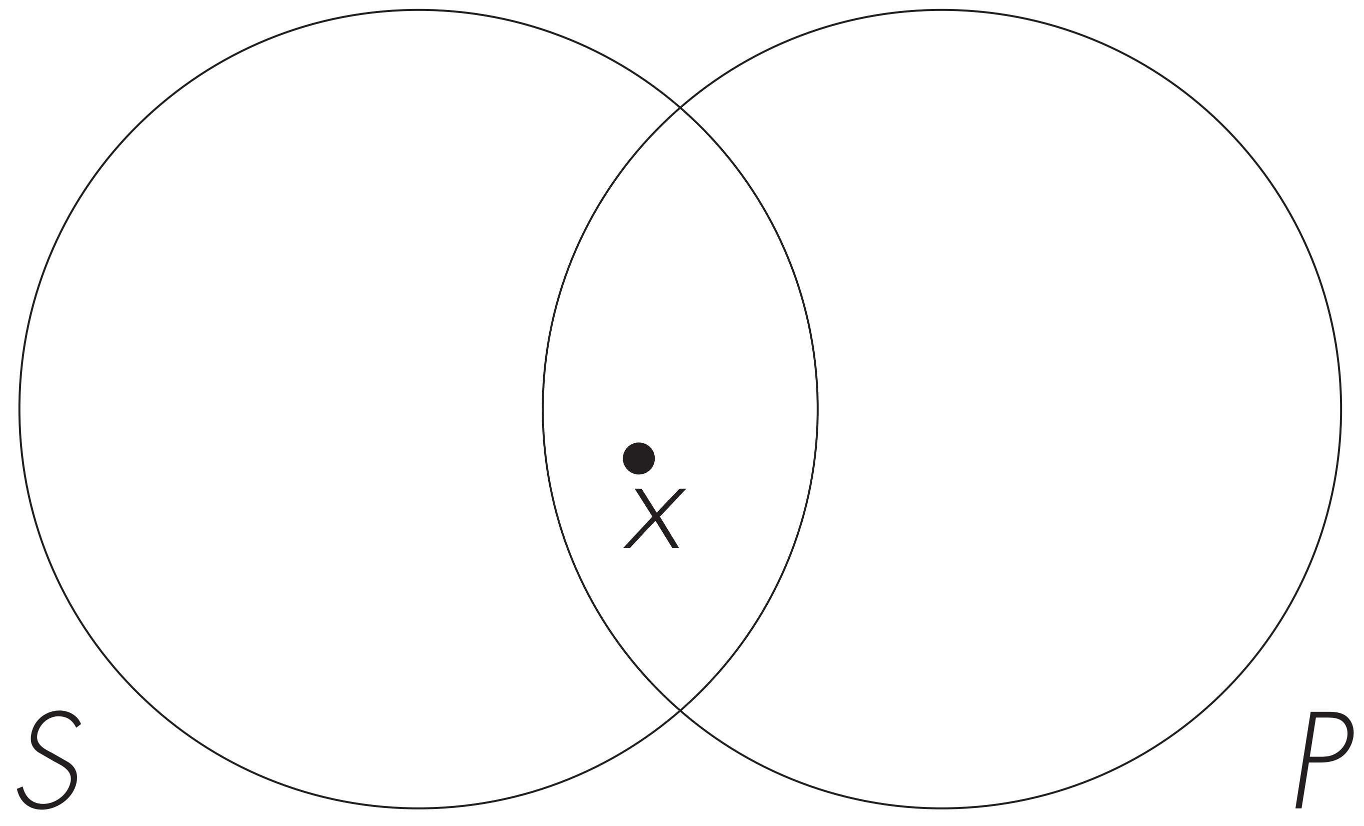
E's Obversion: All *S* is non-*P*.

Obversion: **I**

And so for any **I** statement and its obversion.



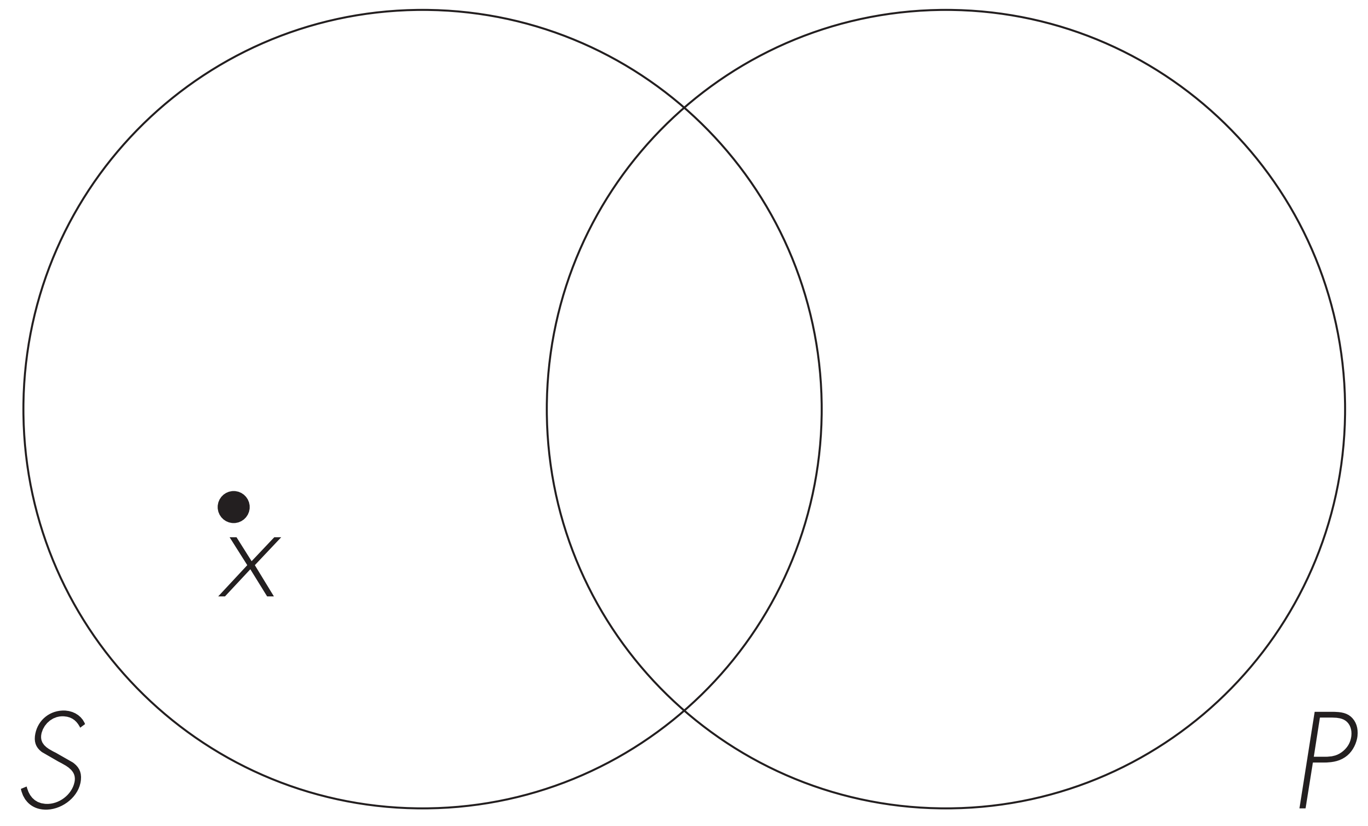
I Statement: Some *S* is *P*.



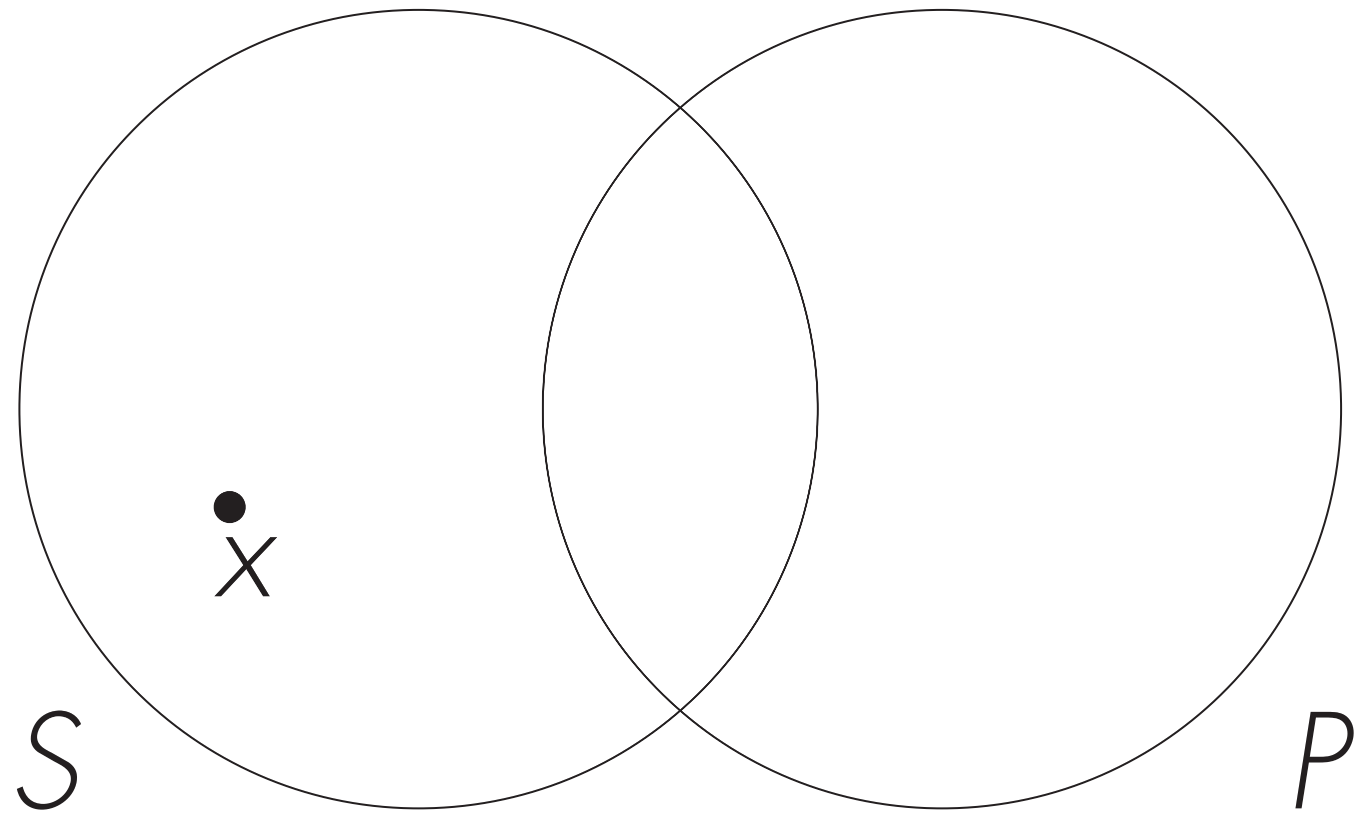
I's Obversion: Some *S* is not non-*P*.

Obversion: O

And finally for any **O** statement and its obversion.



O Statement: Some *S* is not *P*.



O's Obversion: Some *S* is non-*P*.

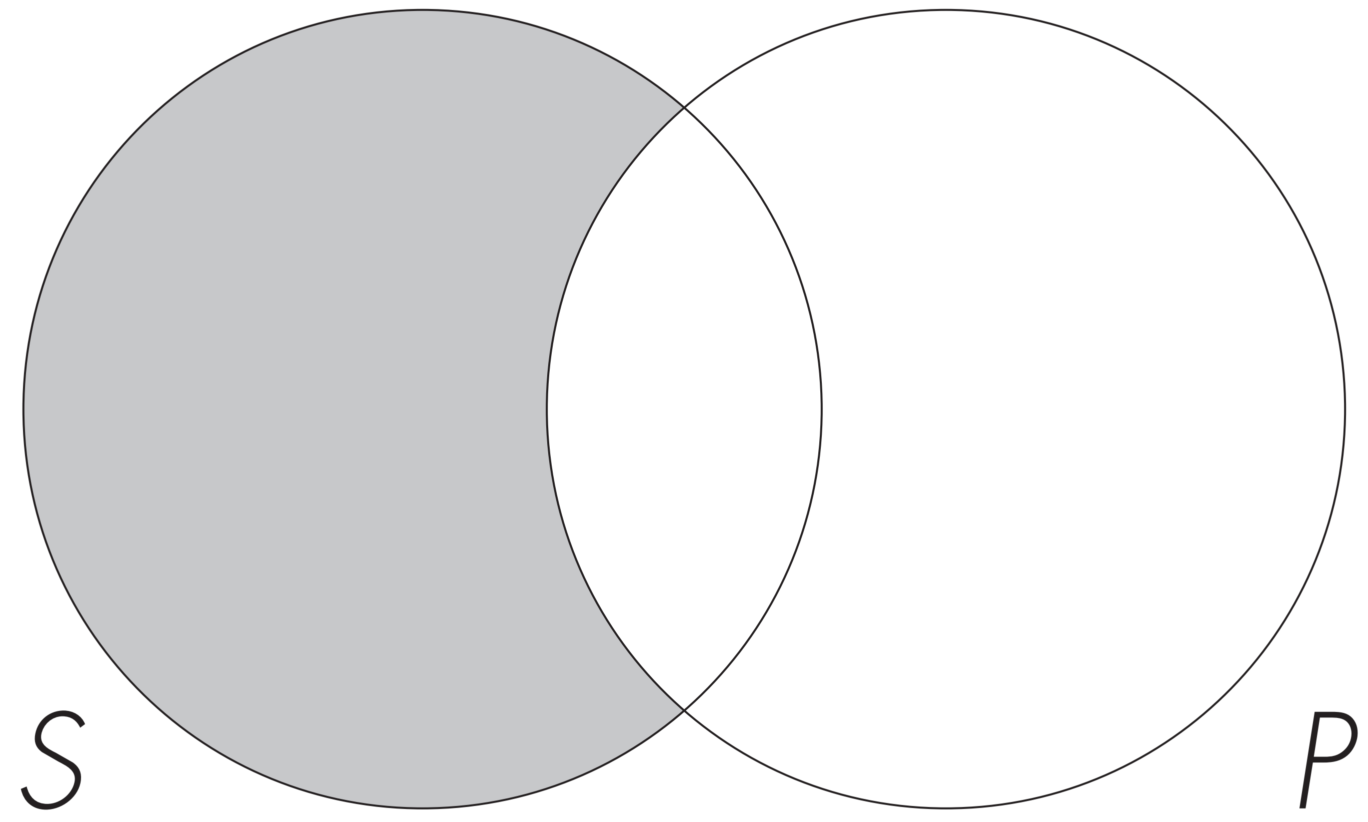
Categorical Statements: *Contraposition*

According to **contraposition**, a categorical statement is changed by (1) replacing its subject (*S*) term with that subject's complement (non-*S*), (2) replacing its predicate (*P*) term with that predicate's complement (non-*P*), and (3) swapping the new subject and new predicate.

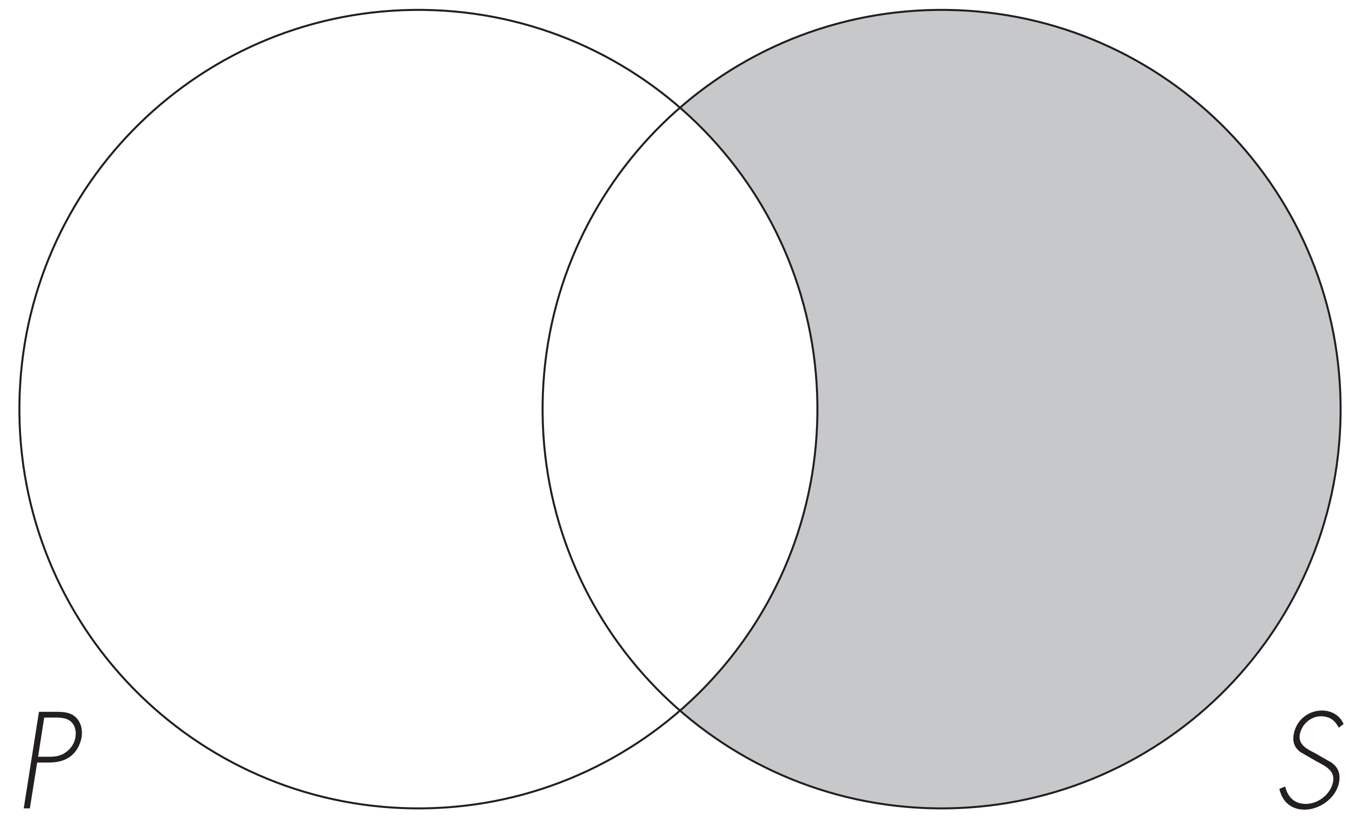
In *some* instances, the new statement will be logically equivalent to the original one. For example, the proposition "All students are lazy people" (**A**: All *S* is *P*) is logically the same as "All non-lazy people are non-students" (All non-*P* is non-*S*).

Contraposition: **A**

In general, any **A** statement and its contrapositive are logically the same.



A Statement: All *S* is *P*.

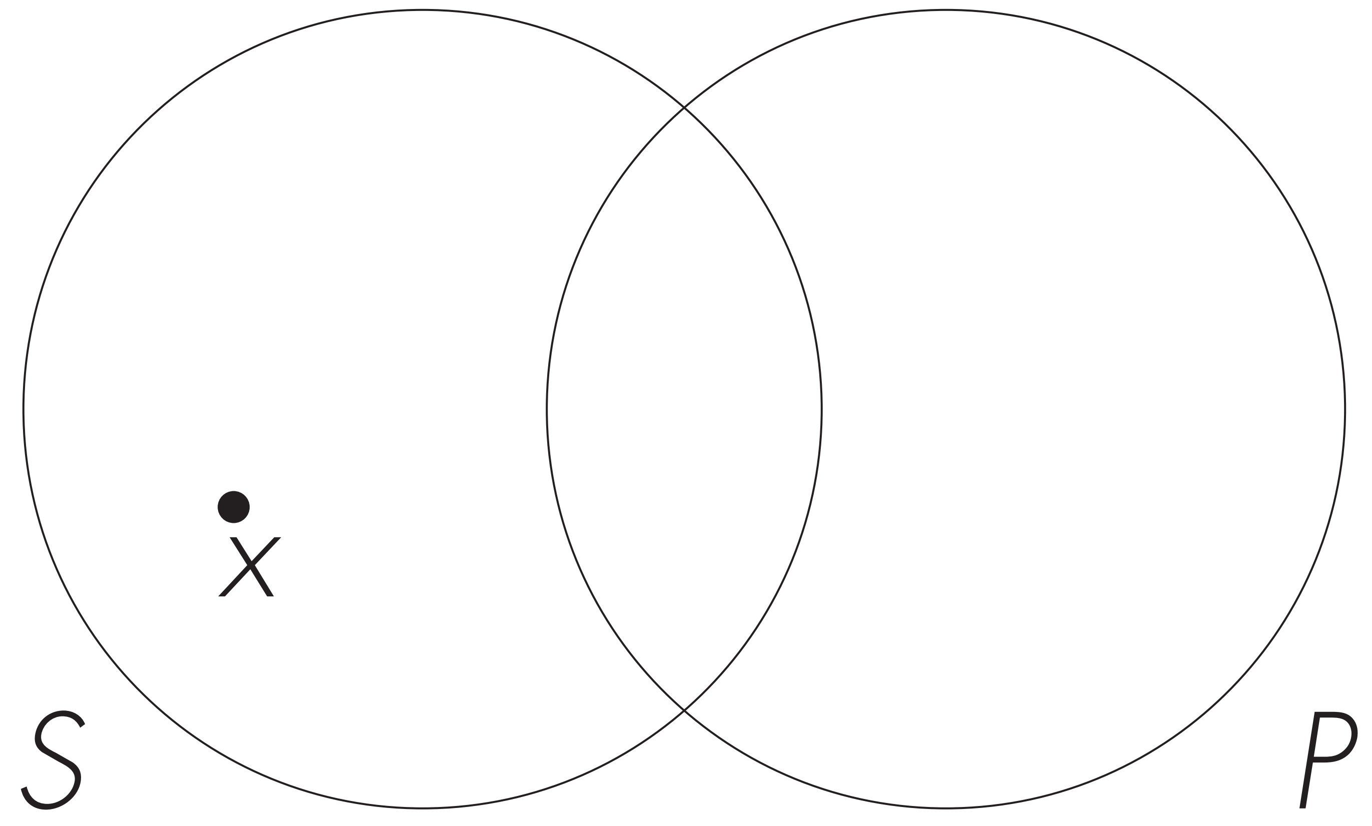


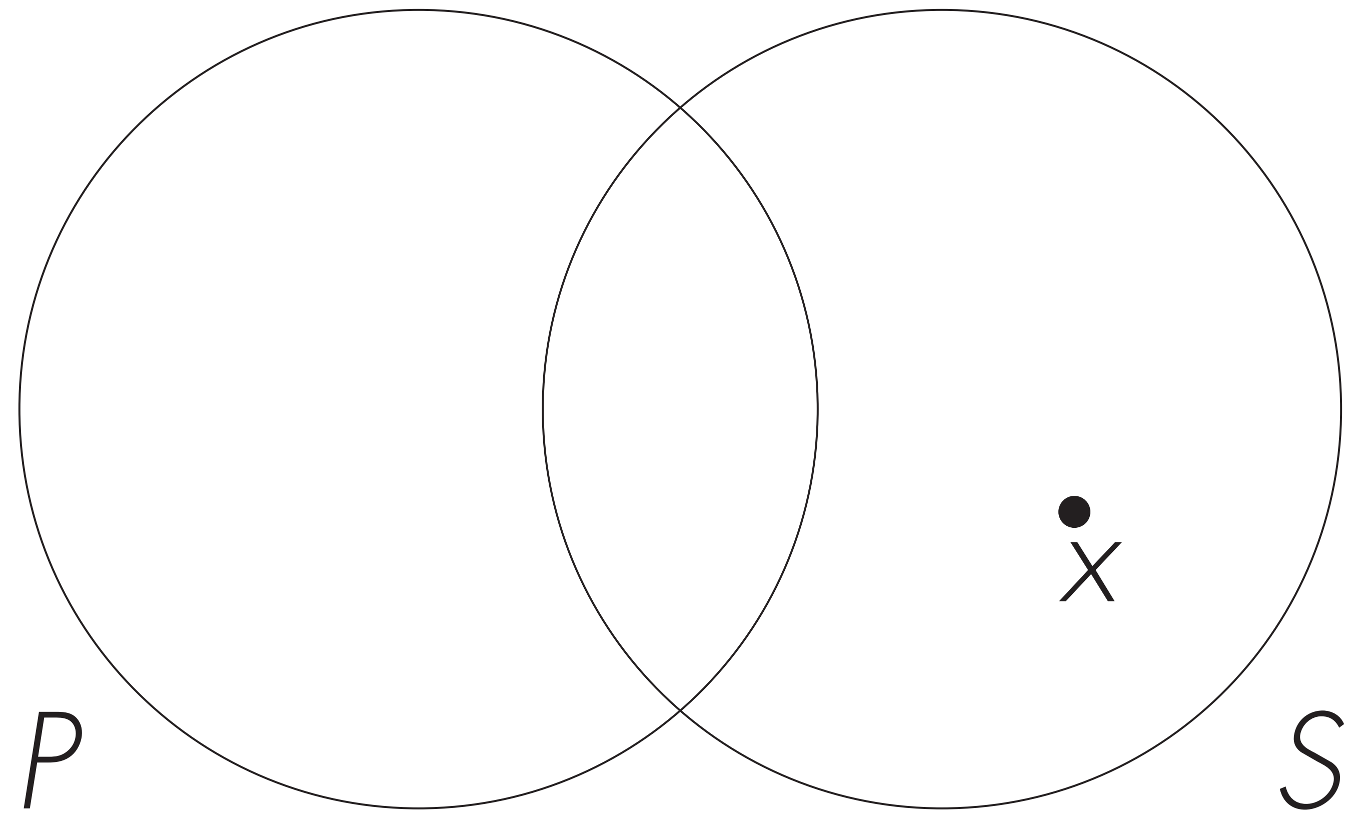
A's Contrapositive: All non-*P* is non-*S*.

Look closely and you will see that the shaded area is actually the same in both diagrams.

Contraposition: O

Similarly, any **O** statement and its contrapositive are logically the same.





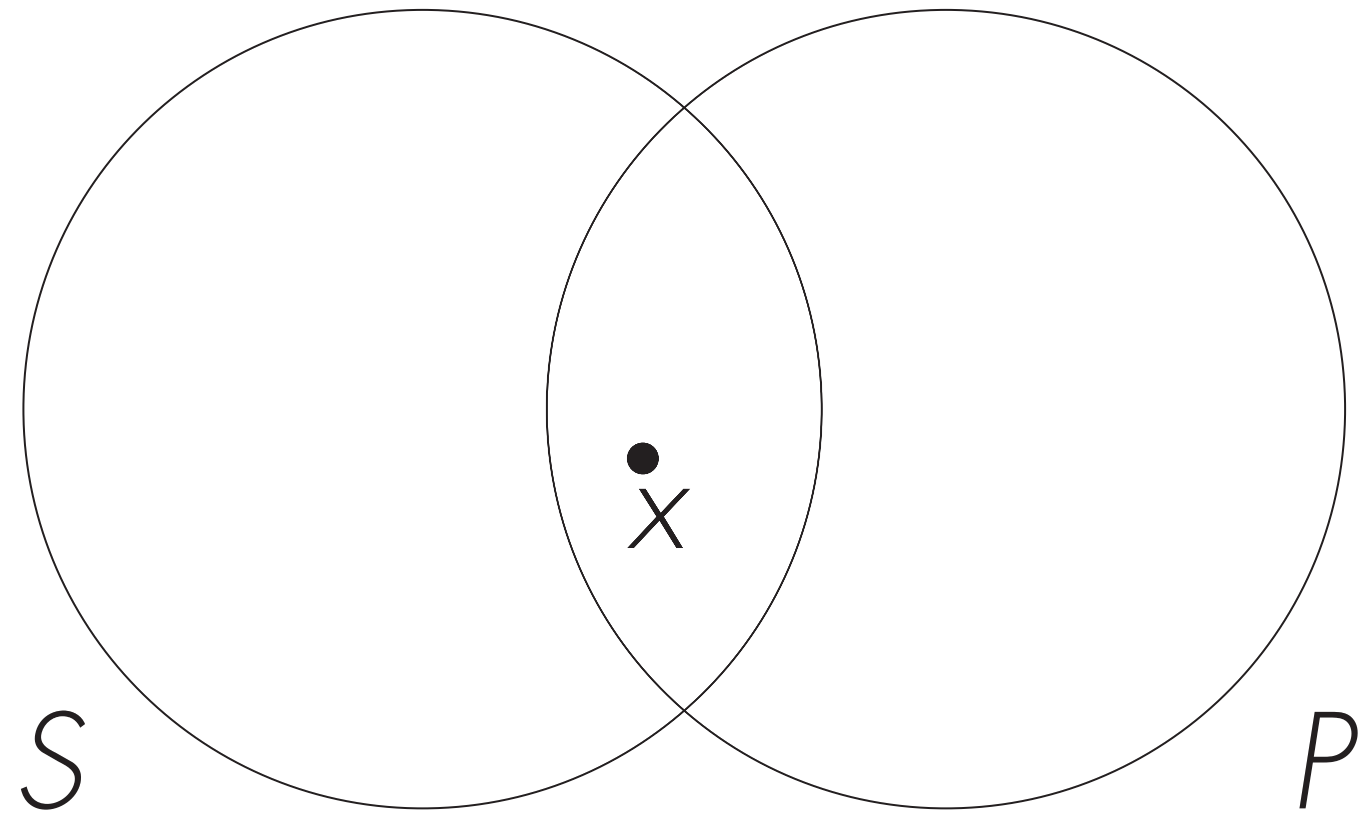
O Statement: Some *S* is not *P*.

O's Contrapositive: Some non-*P* is not non-*S*.

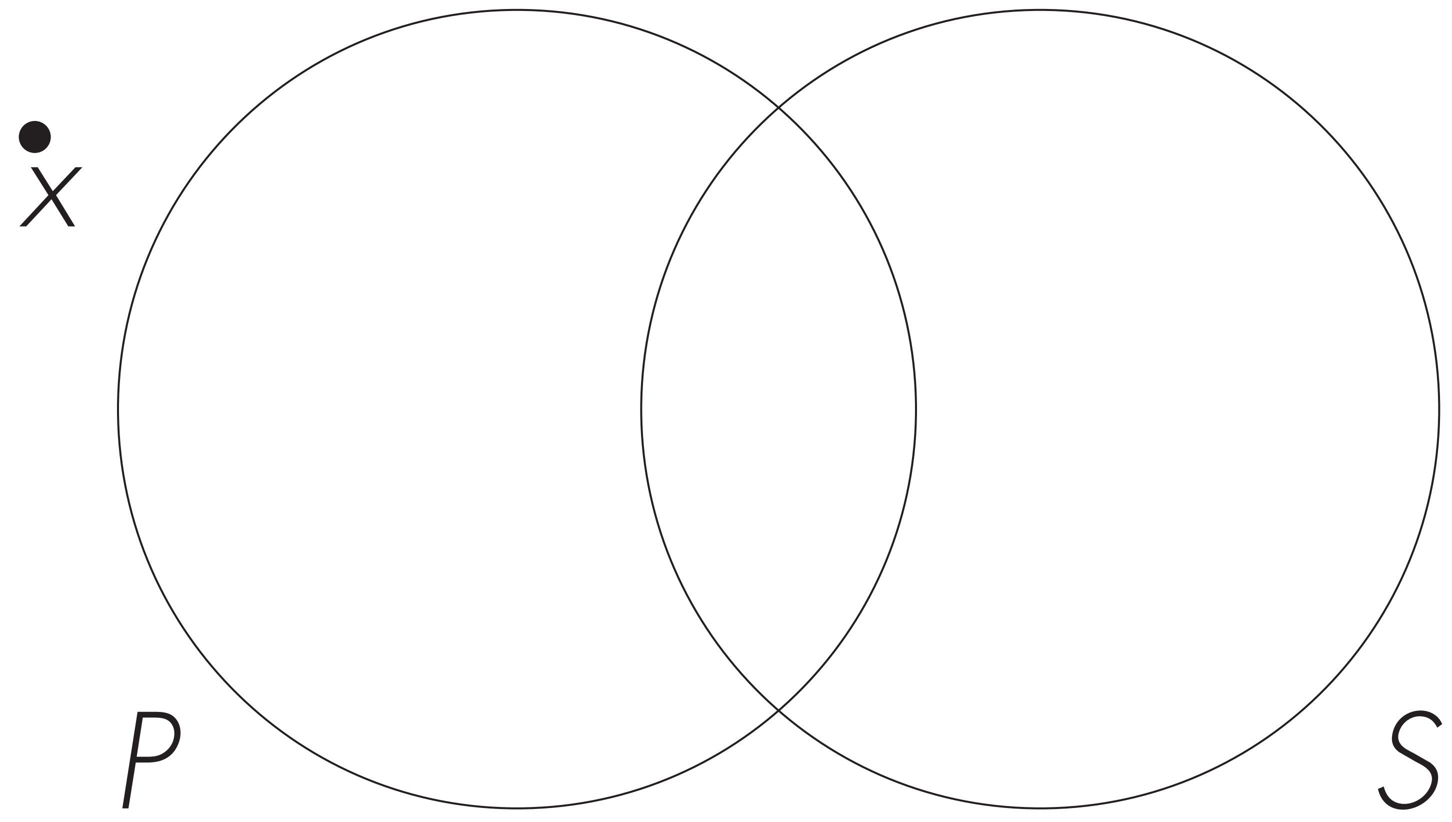
Look closely and you will see that the dot-*x* is actually in the same place in both diagrams.

Contraposition: **I**

However, any **I** statement and its contrapositive are *not* logically the same.



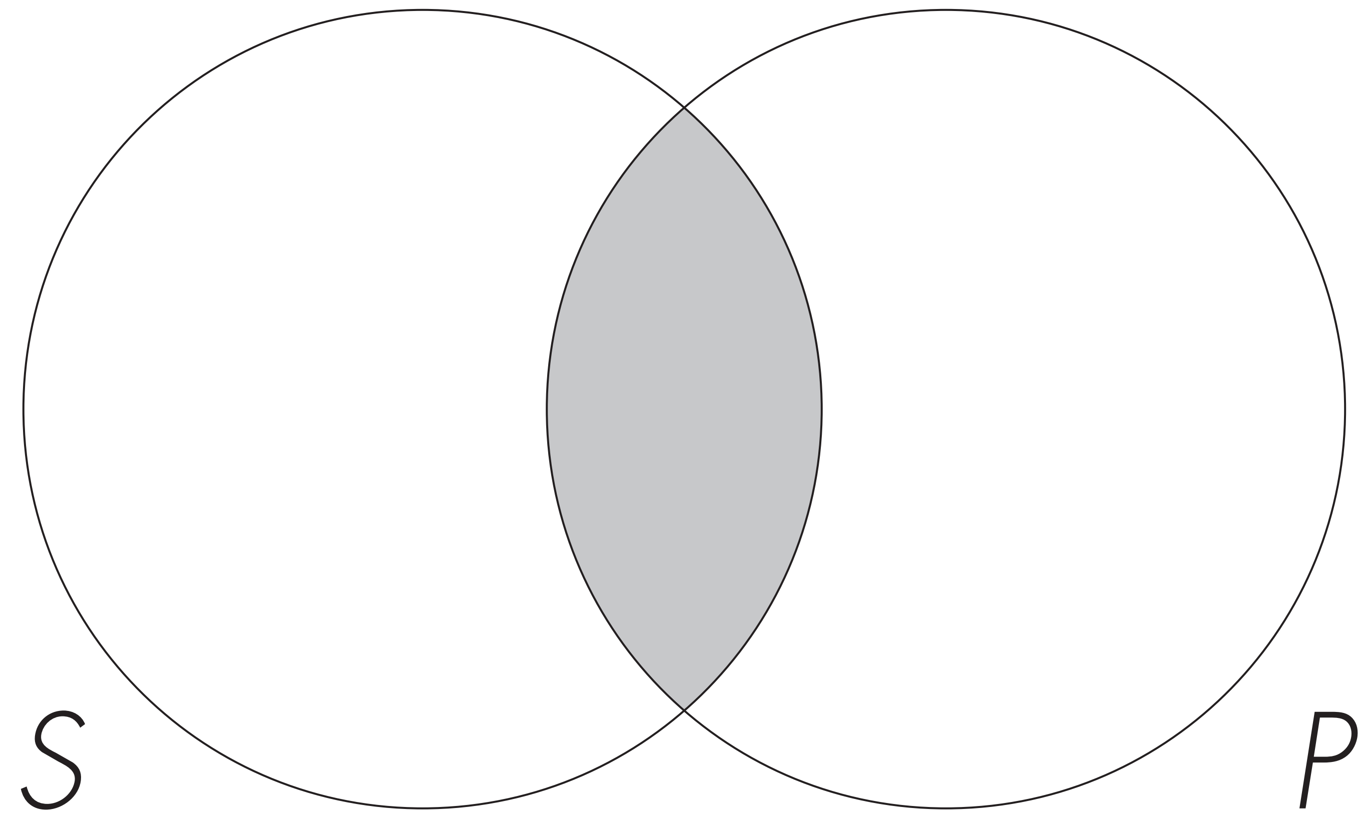
I Statement: Some *S* is *P*.



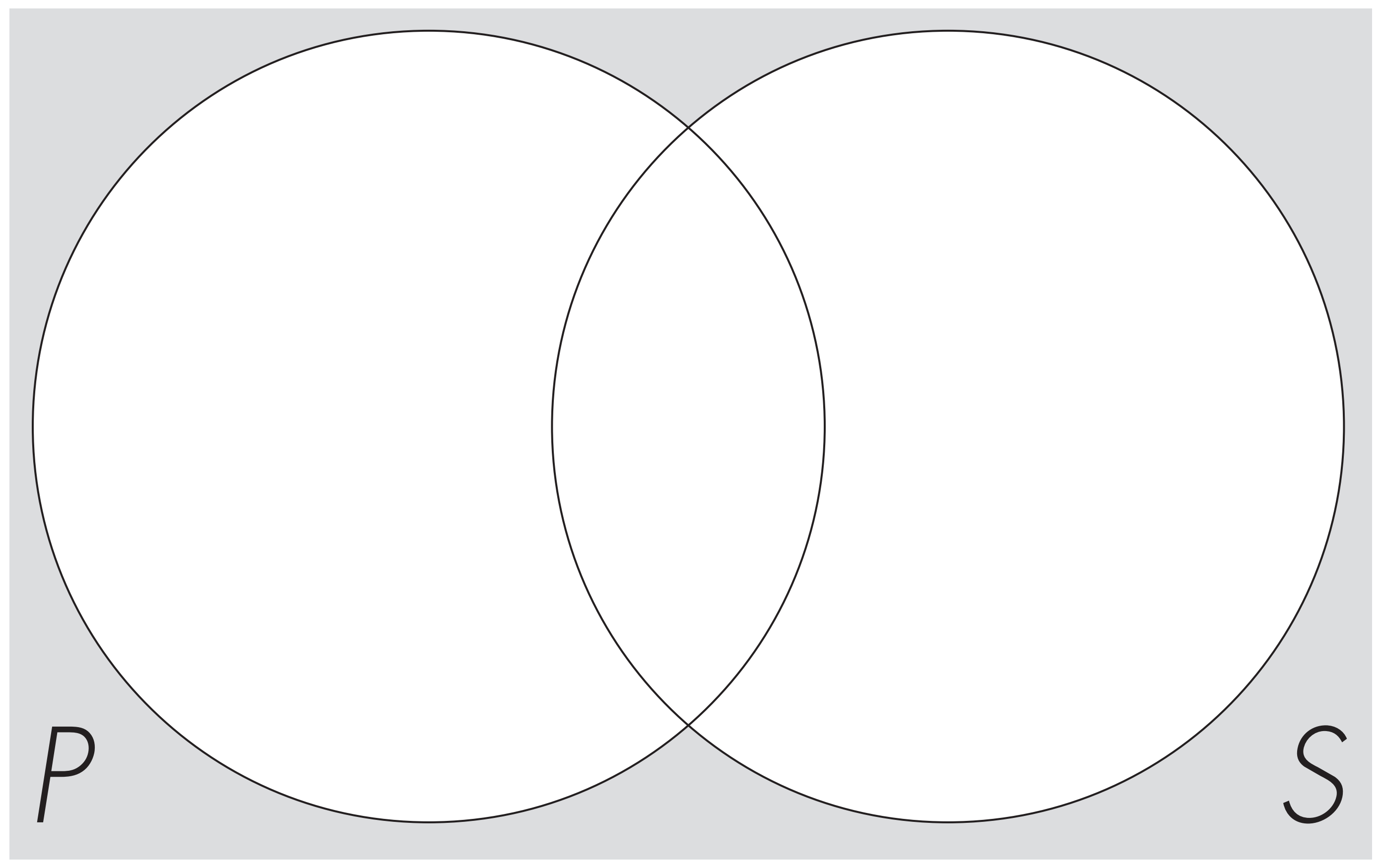
I's Contrapositive: Some non-*P* is non-*S*.

Contraposition: **E**

However, any **E** statement and its contrapositive are *not* logically the same.



E Statement: No *S* is *P*.



E's Contrapositive: No non-*P* is non-*S*.

Categorical Inferences

All this can certainly seem overwhelming!

The solution: if you ever get lost, just make a Venn diagram. From that simple diagram, you should be able assess any inferences between categorical statements.

Next Class…

We will have a workshop on using Venn diagrams for making inferences from one categorical statement to another.