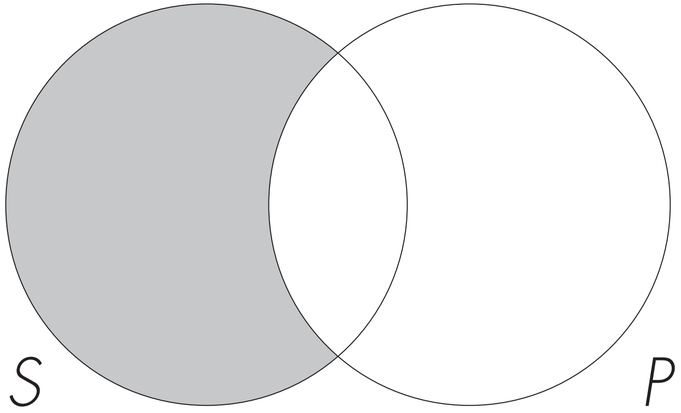
CRITICAL THINKING

Lecture #20

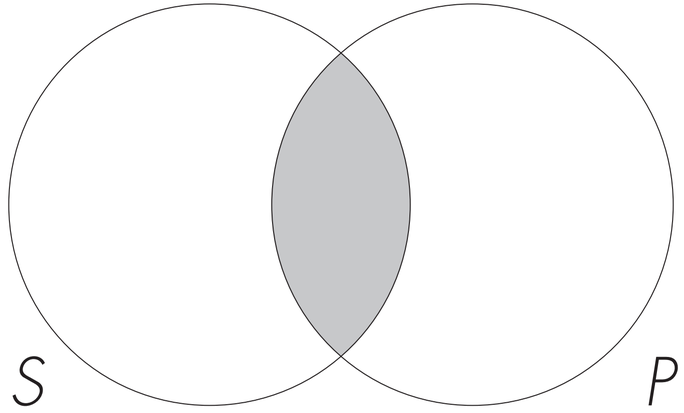
The Square of Opposition

Professor David Emmanuel Gray

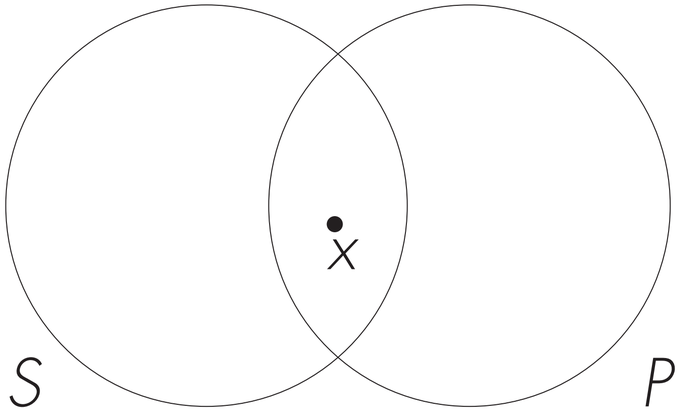
Four Standard Forms of Categorical Statements



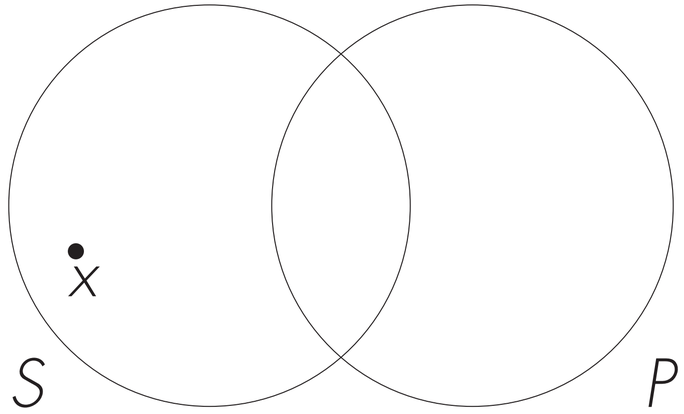
A: All *S* is *P*.
(Shade in all of *S* not shared with *P*.)



E: No *S* is *P*.
(Shade in all of *S* shared with *P*.)



I: Some *S* is *P*.
(Dot-*x* in *S* shared with *P*.)



O: Some *S* is not *P*.
(Dot-*x* in *S* not shared with *P*.)

Four Standard Forms of Categorical Statements (Generalized)

Universal Positive

A: All *X* is *Y*.

Shade in all of *X* not shared with *Y*.

Universal Negative

E: No *X* is *Y*.

Shade in all of *X* shared with *Y*.

Particular Positive

I: Some *X* is *Y*.

Dot-*x* in *X* shared with *Y*.

Particular Negative

O: Some *X* is not *Y*.

Dot-*x* in *X* not shared with *Y*.

Note: A complement like non-*S* or non-*P* can substitute *X* or *Y*.

Inferences with Categorical Statements: *Instructions*

Given that a categorical statement is true or false, draw a Venn diagram representing that statement, being sure to label its subject term (*S*) and predicate term (*P*). (Be sure to put the subject term (*S*) on the left and the predicate term (*P*) on the right.)

Now given that Venn diagram, what can you infer about other categorical statements? That is, are these other statements true, false, or unknown? Use a Venn diagram to justify each of your answers (being sure to keep each statement's subject term on the left and predicate term on the right). You may assume that neither *S* nor *P* is empty.

Inferences with Categorical Statements: *Illustration* #1

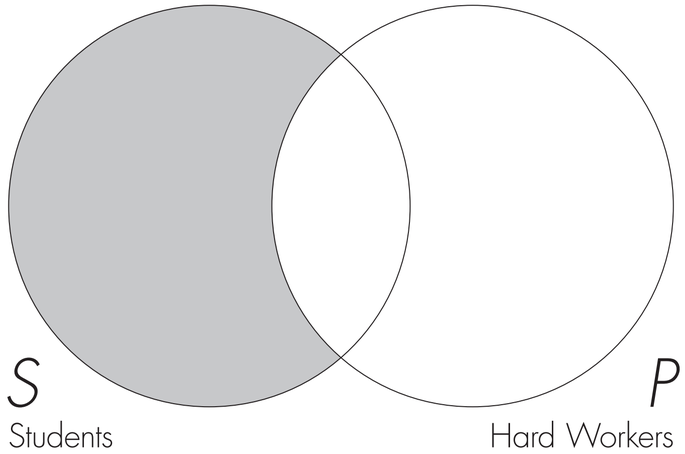
Assume that the following categorical statement is *true*:

All students are hard workers.

Draw the Venn diagram representing this statement, being sure to label its subject term (*S*) and predicate term (*P*).

Inferences with Categorical Statements: *Illustration* #1

All students are hard workers.



A Statement: All *S* is *P*.

True

Drawing this Venn diagram will help us make interferences concerning the truth of other categorical statements involving students (*S*) and hard workers (*P*).

Inferences with Categorical Statements: *Illustration* #1

All students are hard workers.

Now given the truth of this statement (and its associated Venn diagram), what can you infer about the following categorical statement?

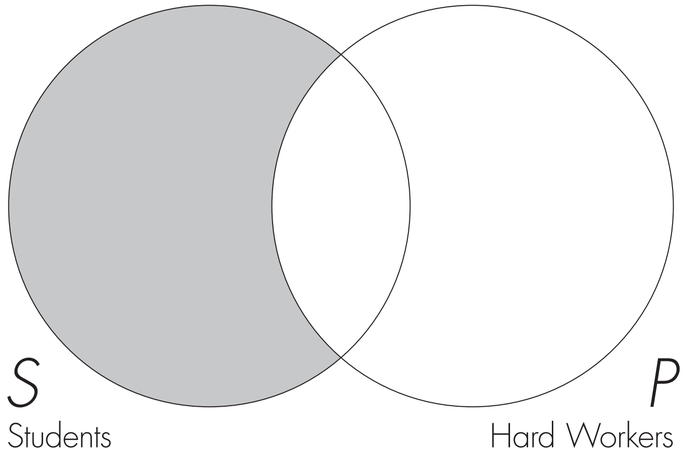
Some students are not hard workers.

That is, is this second statement true, false, or is its truth/falsity unknown? Use a Venn diagram to justify your answer.

(Again, this is an example of making an inference from one categorical statement to a second one that involves the *same* subject (*S* = students) and predicate (*P* = hard workers) terms.)

Inferences with Categorical Statements: *Illustration* #1

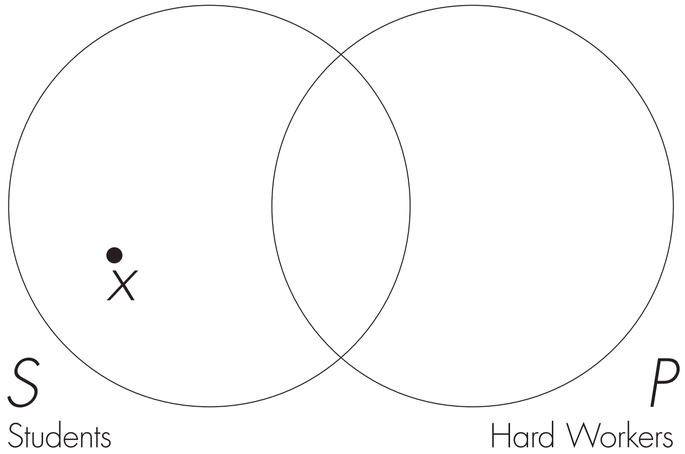
All students are hard workers.



A Statement: All *S* is *P*.

True

Some students are not hard workers.



O Statement: Some *S* is not *P*.

??

When making a categorical inference, it is easiest to just compare the associated Venn diagrams for the categorical statements.

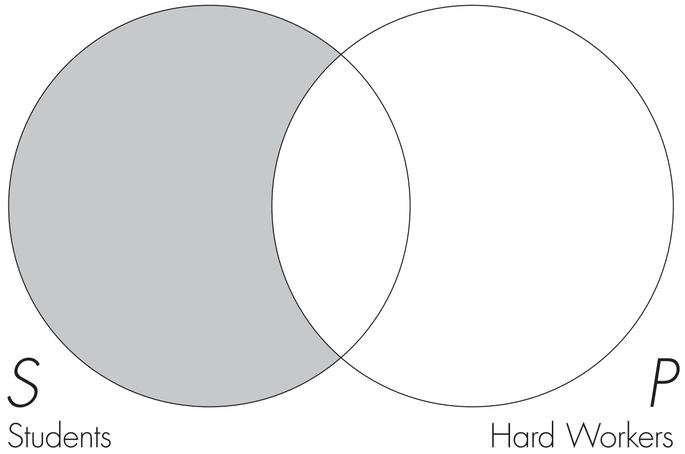
Inferences with Categorical Statements: *Illustration* #1

The **A** statement (All *S* is *P*) claims there is *nothing* in the area of S outside of P (the **A** diagram shades that zone). Meanwhile, the **O** statement (Some *S* is not *P*) says there is *something* in that very same area (the **O** diagram has a dot-*x* there). This means that both statements cannot be true at the same time!

So given that "All students are hard workers" (**A**) is *true*, the claim that "Some students are not hard workers" (**O**) *cannot* also be true. So that second claim (the **O** statement) must be *false*.

Inferences with Categorical Statements: *Illustration* #1

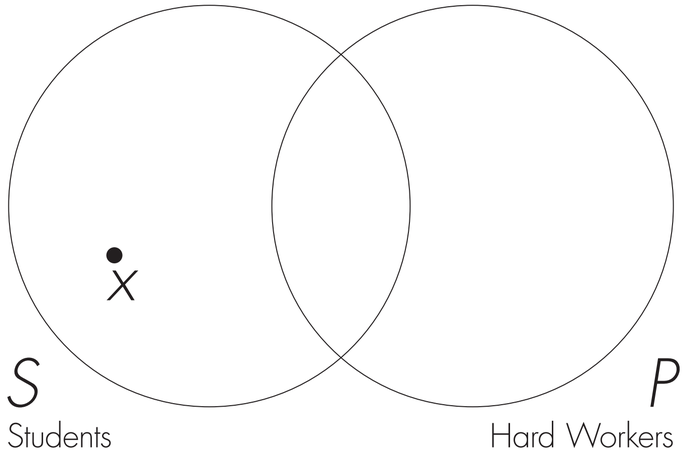
All students are hard workers.



A Statement: All *S* is *P*.

True

Some students are not hard workers.



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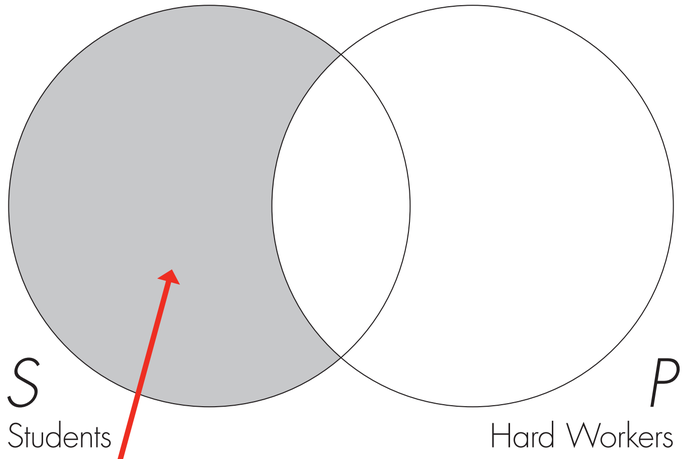
Inferences with Categorical Statements: *Illustration* #1

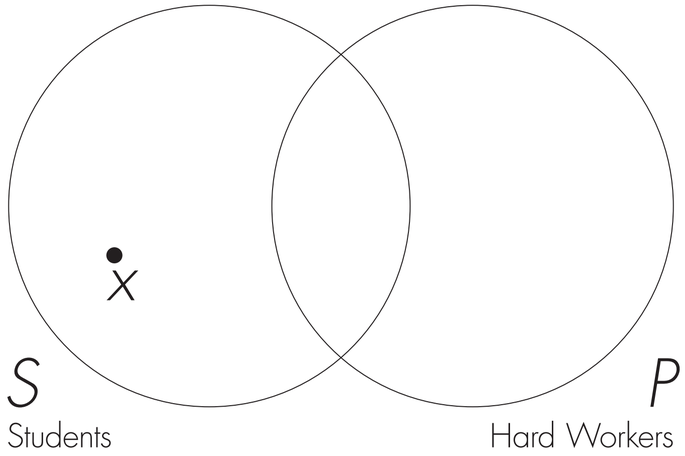
All students are hard workers.



A Statement: All *S* is *P*.

True

Some students are not hard workers.

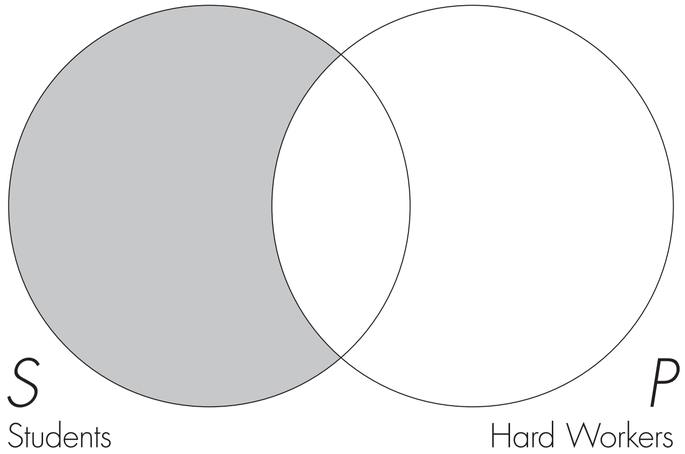


O Statement: Some *S* is not *P*.

The truth of the statement on the left implies that there is nothing in the area of students outside of hard workers...

Inferences with Categorical Statements: *Illustration* #1

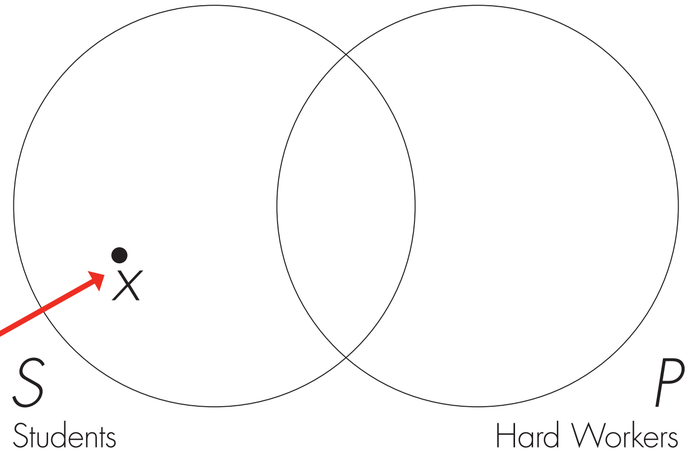
All students are hard workers.



A Statement: All *S* is *P*.

True

Some students are not hard workers.

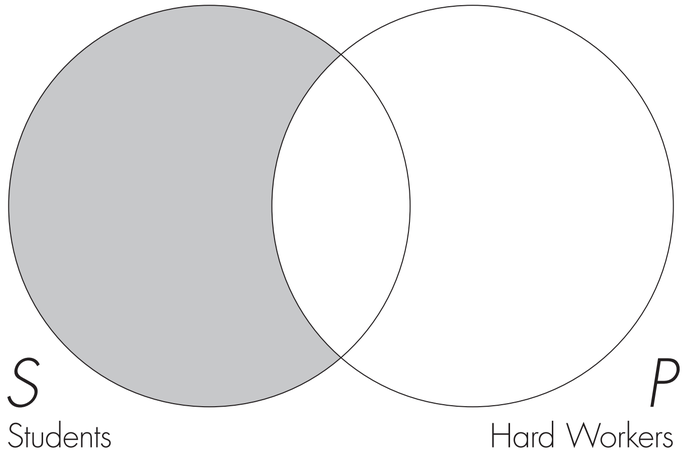


O Statement: Some *S* is not *P*.

The truth of the statement on the left implies that there is nothing in the area of students outside of hard workers, which the Venn diagram on the right clearly denies.

Inferences with Categorical Statements: *Illustration* #1

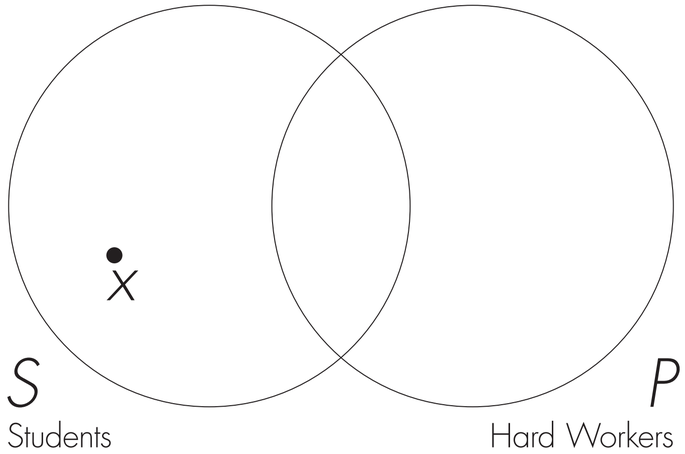
All students are hard workers.



A Statement: All *S* is *P*.

True

Some students are not hard workers.



O Statement: Some *S* is not *P*.

False!

So the truth of the statement on the left shows that the statement on the right is *false*.

Inferences with Categorical Statements: *Illustration #2*

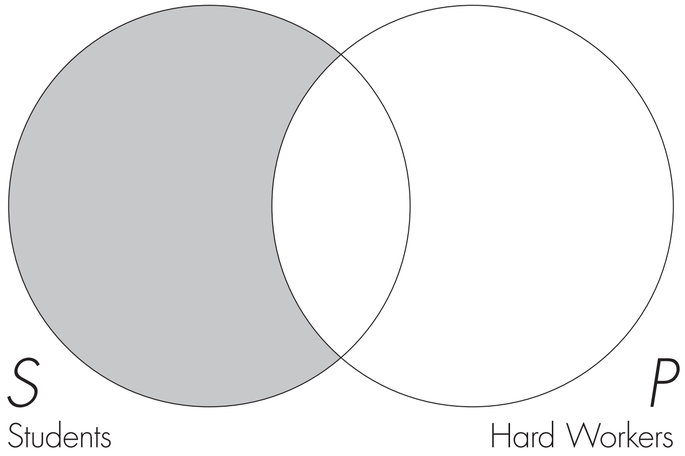
Assume that the following categorical statement is *false*:

All students are hard workers.

Since this is false, draw the Venn diagram representing what is actually true, being sure to label the subject term (*S*) and predicate term (*P*).

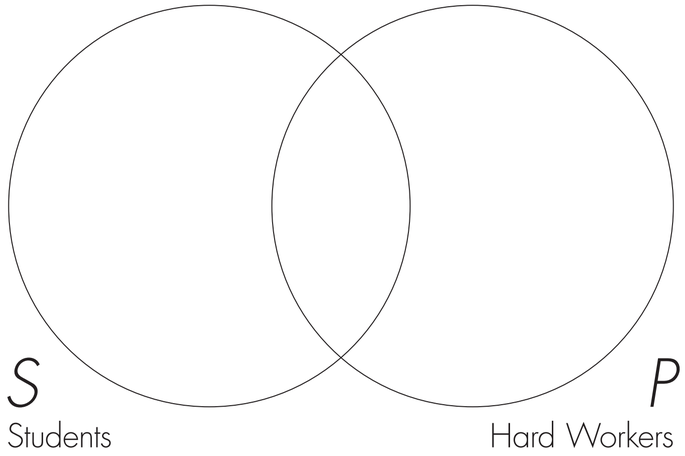
Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



A Statement: All *S* is *P*.

False

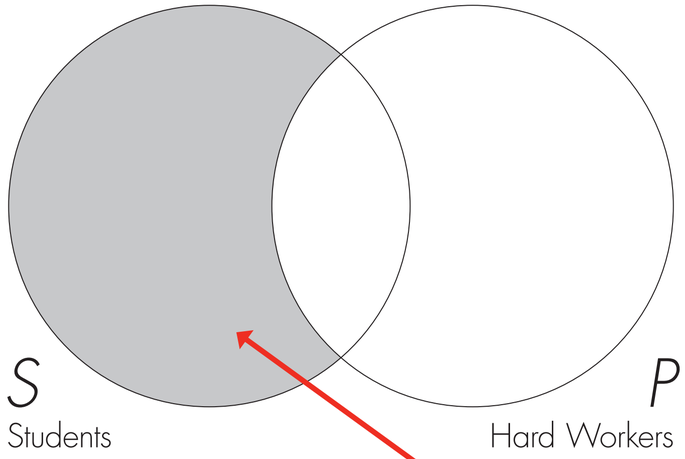


True?

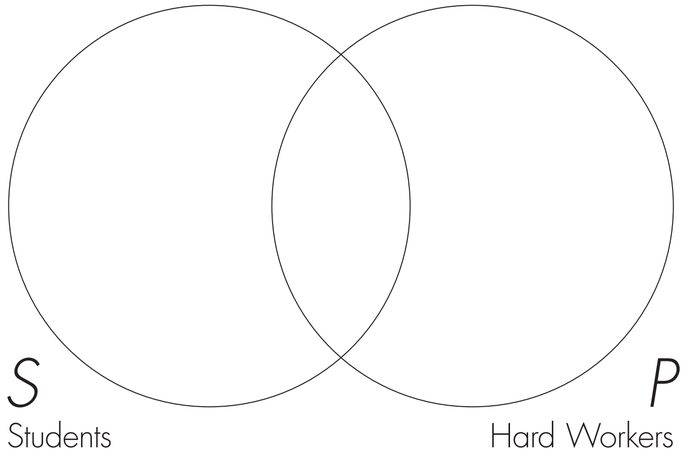
Given that the statement on the left is false, what is true relationship between students and hard workers? We can diagram the truth on the right.

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



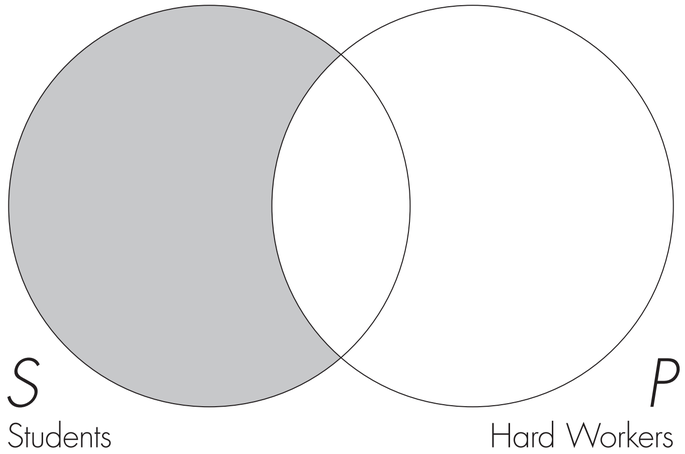
A Statement: All *S* is *P*.



The statement on the left says there is *nothing* in the area of students outside of hard workers...

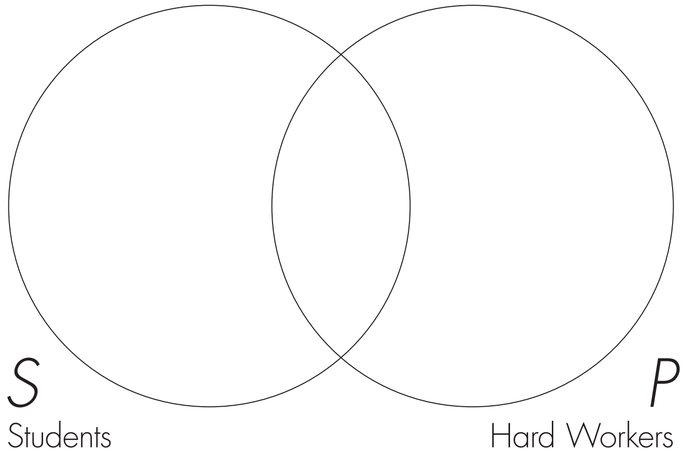
Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



A Statement: All *S* is *P*.

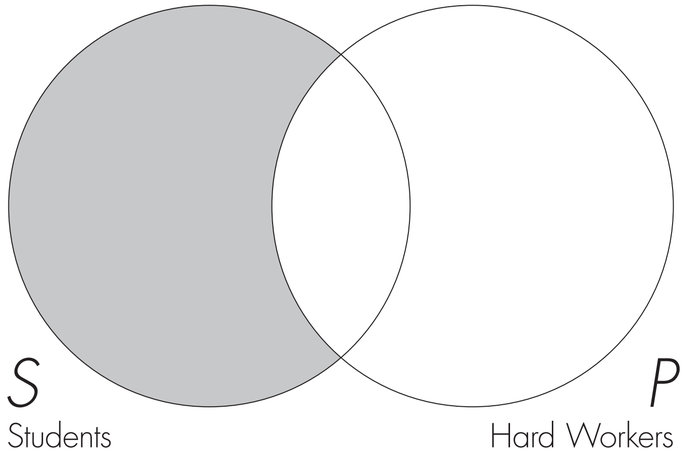
False



The statement on the left says there is *nothing* in the area of students outside of hard workers. So if that statement is actually false…

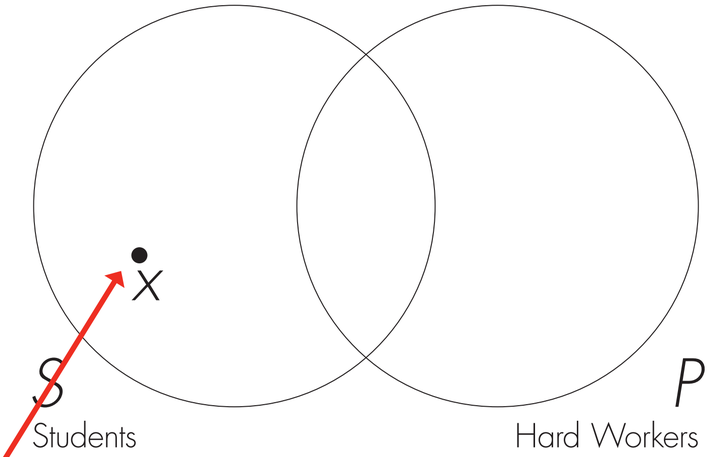
Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



A Statement: All *S* is *P*.

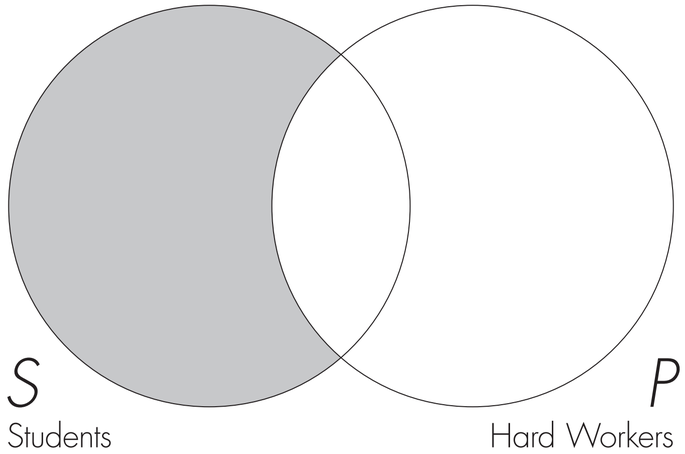
False



The statement on the left says there is *nothing* in the area of students outside of hard workers. So if that statement is actually false, there must be *something* in that same area.

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



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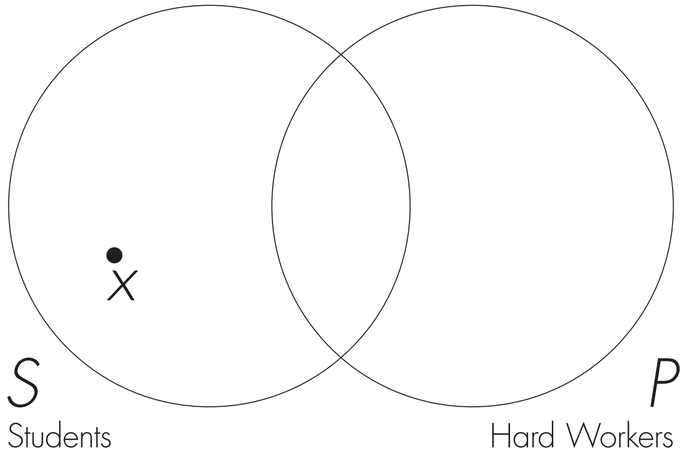


True!

Now we know the truth, and we can make categorical inferences based on this.

Aside: *Diagramming False Categorical Statements*

In general, if you are told that a categorical statement is false, you should do the following:

1. Create the Venn diagram of the statement as it is, being sure to clearly label the subject (*S*) and predicate (*P*) terms, and then

2. Create a second Venn diagram, where any shaded areas now have a dot-*x* instead and any areas with a dot-*x* are now shaded.

This second Venn diagram will now show the true relationship between the subject (*S*) and predicate (*P*) terms. It will be easier to make inferences using this second diagram.

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.

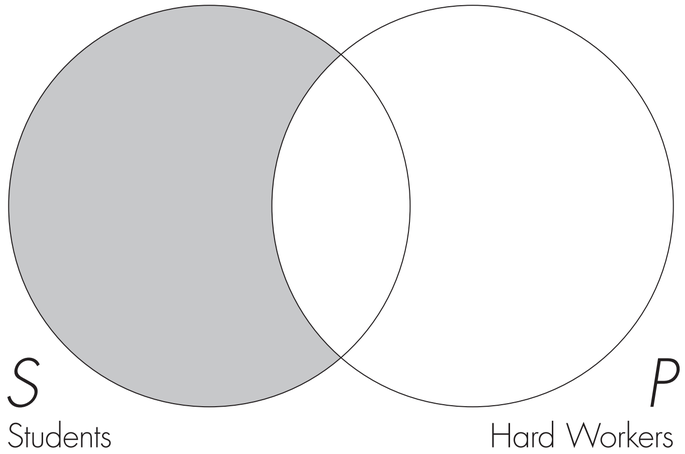
Now given the truth of this second Venn diagram (that shows what is actually true when the above statement is false), what can you infer about the following categorical statement?

Some students are not hard workers.

That is, is this second statement true, false, or is its truth/falsity unknown? Use a Venn diagram to justify your answer.

Inferences with Categorical Statements: *Illustration #2*

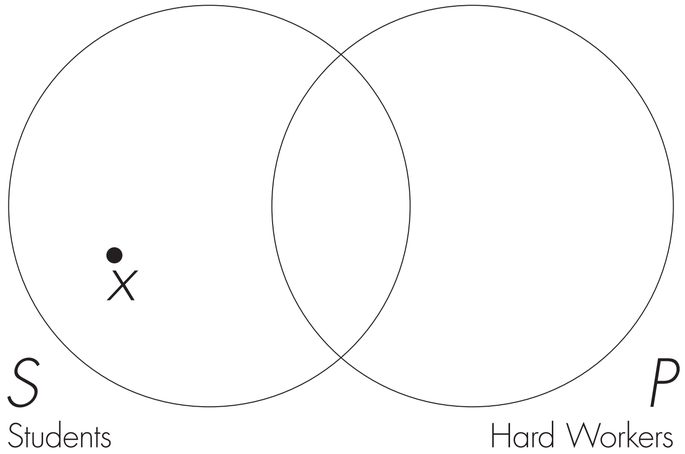
All students are hard workers.



A Statement: All *S* is *P*.

False

Some students are not hard workers.



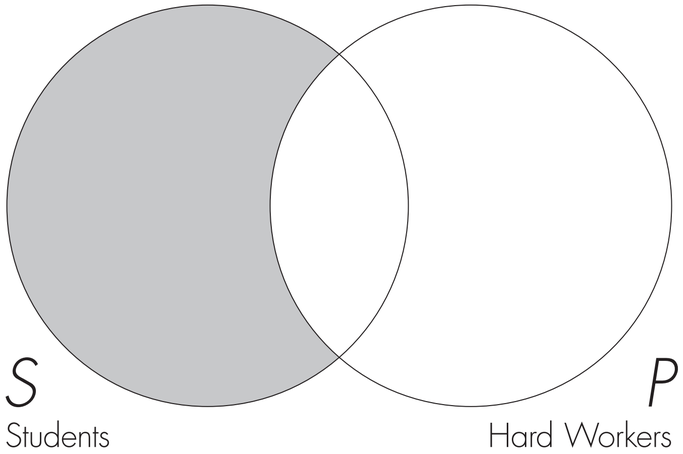
O Statement: Some *S* is not *P*.

??

This is what we are being asked to determine...

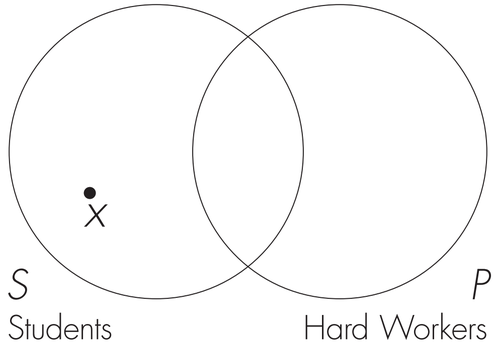
Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



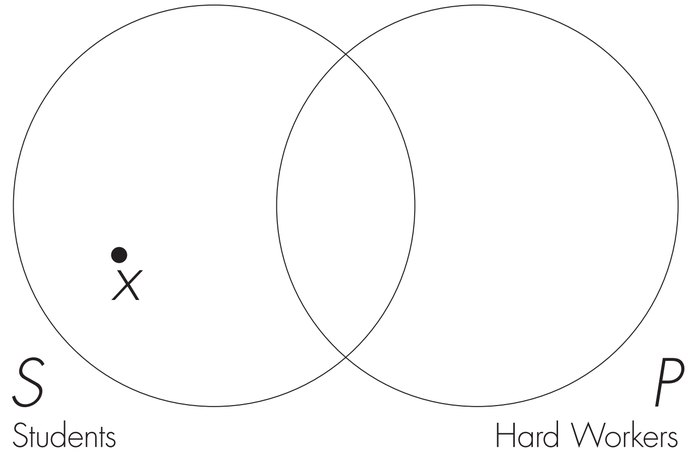
A Statement: All *S* is *P*.

False



True

Some students are not hard workers.

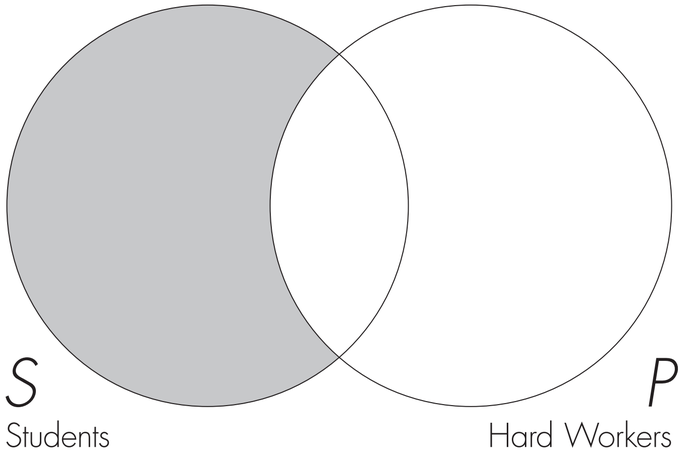


O Statement: Some *S* is not *P*.

But fortunately, we already made that Venn diagram showing us what is actually true when the statement on the left is false…

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



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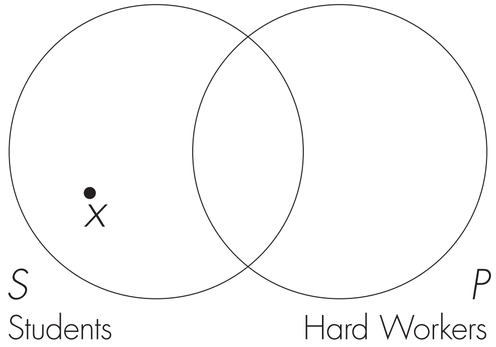

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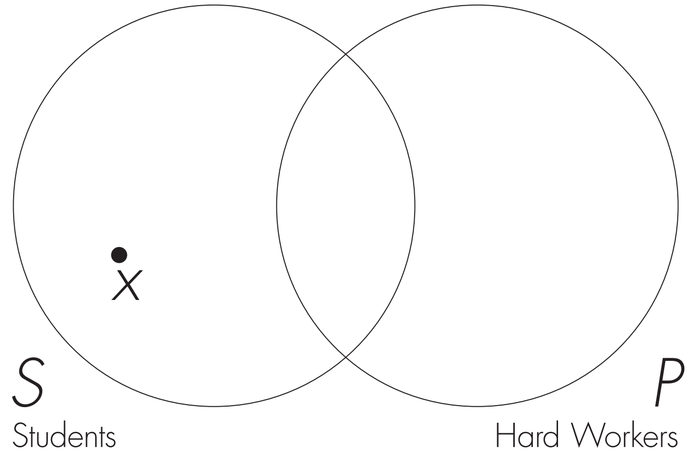


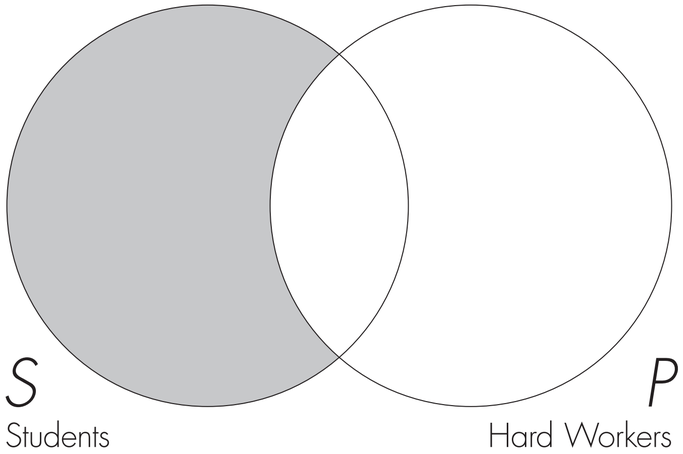
O Statement: Some *S* is not *P*.

??

So we can use that new Venn diagram (in the middle above) to determine the truth value of the statement on the right...

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



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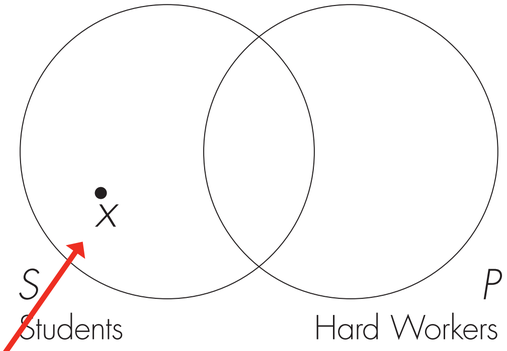

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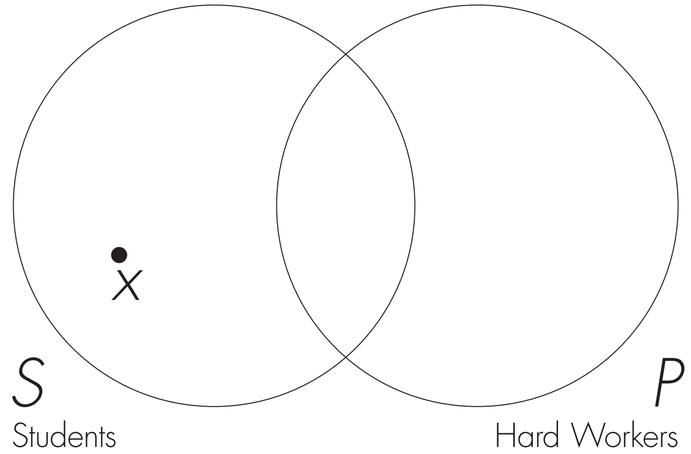
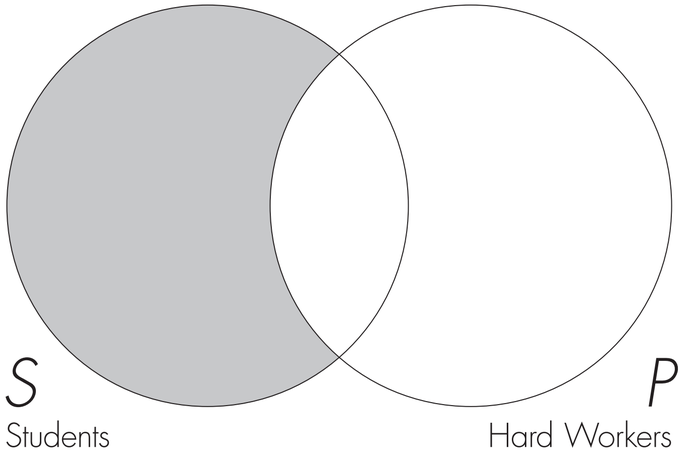


O Statement: Some *S* is not *P*.

The middle Venn diagram says there is something in the area of students outside of hard workers...

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



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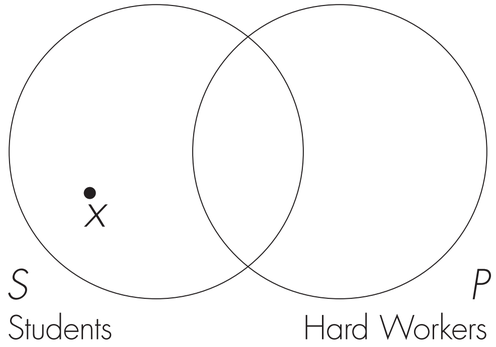
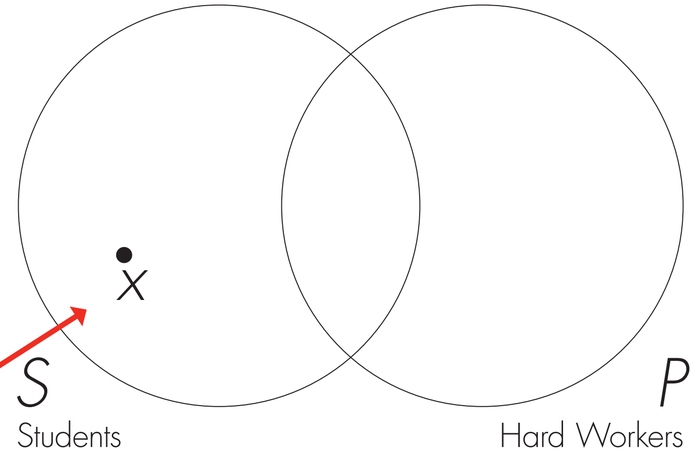


True

Some students are not hard workers.

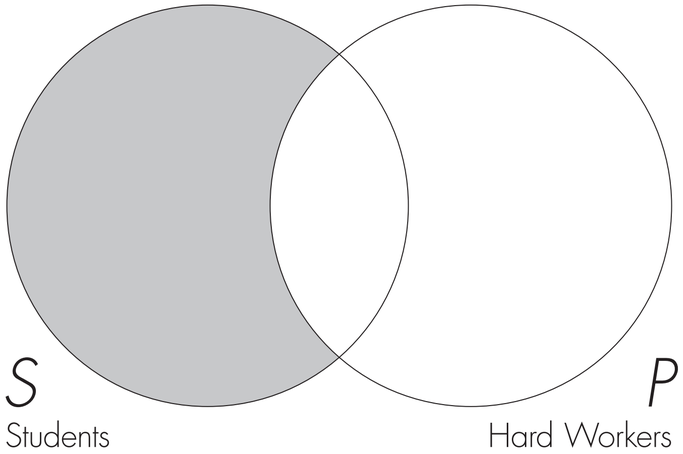


O Statement: Some *S* is not *P*.

The middle Venn diagram says there is something in the area of students outside of hard workers, with which the statement on the left agrees.

Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



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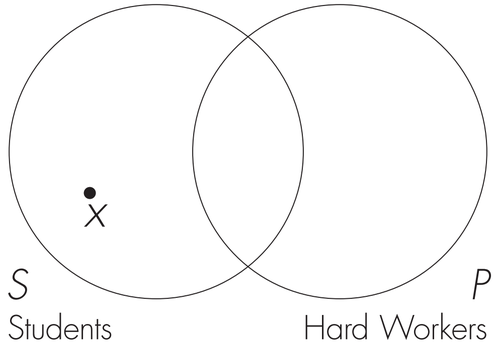

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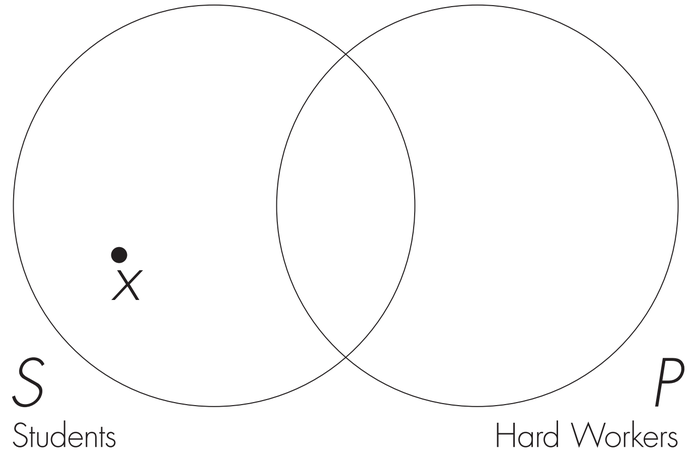


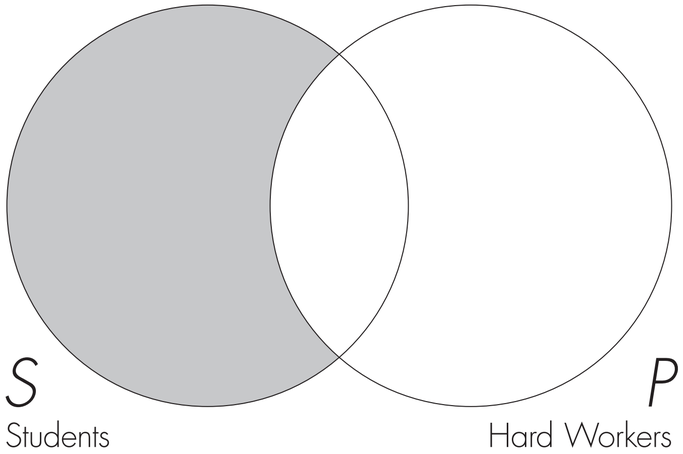
O Statement: Some *S* is not *P*.

True!

The middle Venn diagram says there is something in the area of students outside of hard workers, with which the statement on the left agrees. So that statement must also be true!

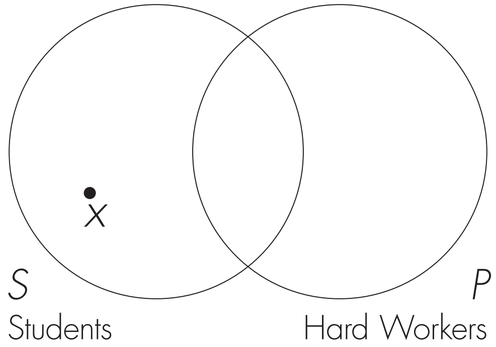
Inferences with Categorical Statements: *Illustration #2*

All students are hard workers.



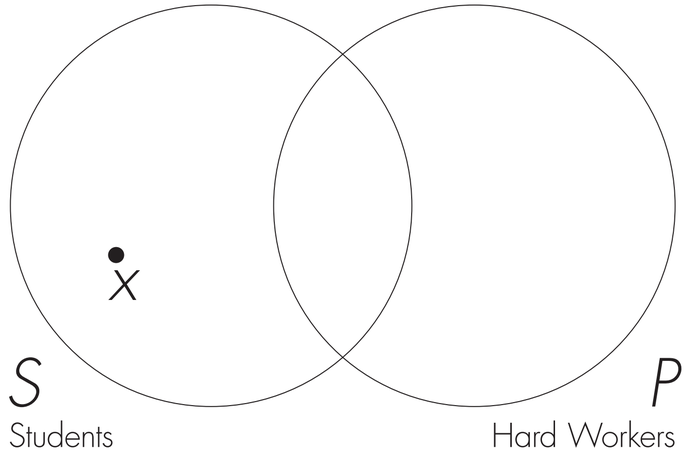
A Statement: All *S* is *P*.

False



True

Some students are not hard workers.



O Statement: Some *S* is not *P*.

True

Indeed, the middle Venn diagram is exactly the same as the one on the right! So these two must obviously have the same truth value.

Exercise #1

Assume that the following categorical statement is *true*:

No students are hard workers.

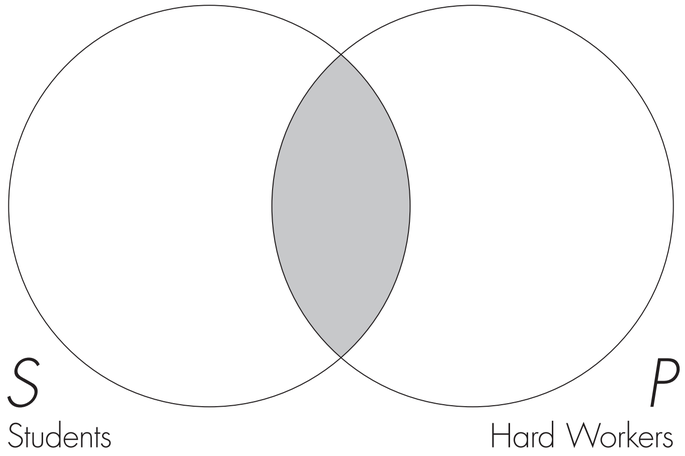
Given the truth of this statement, what can you infer about the following categorical statement?

Some students are hard workers.

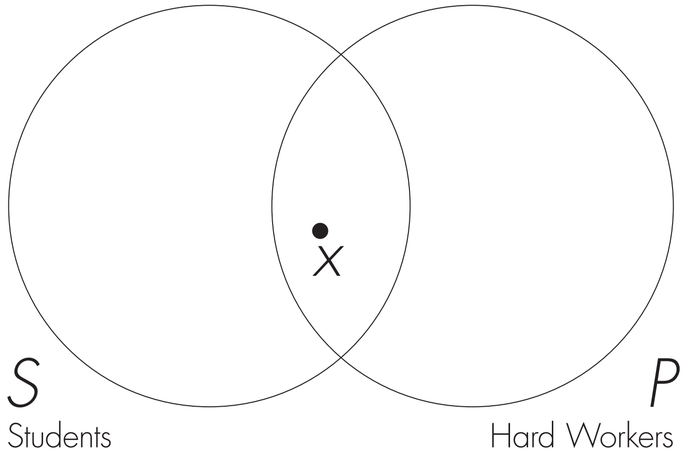
That is, is this second statement true, false, or unknown?

Exercise #1: *Venn Diagrams*

No students are hard workers.

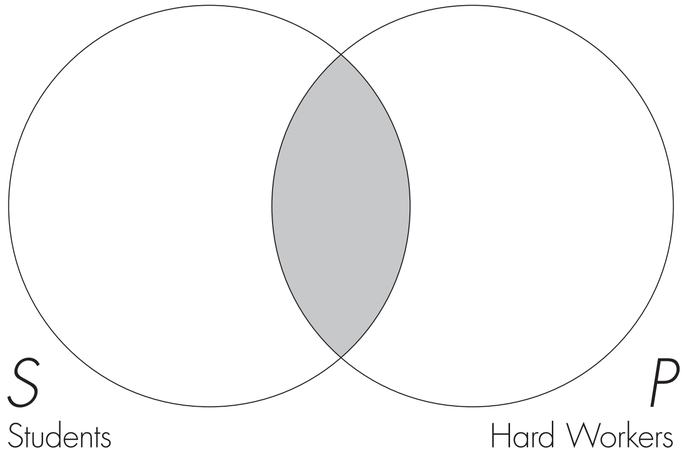


Some students are hard workers.

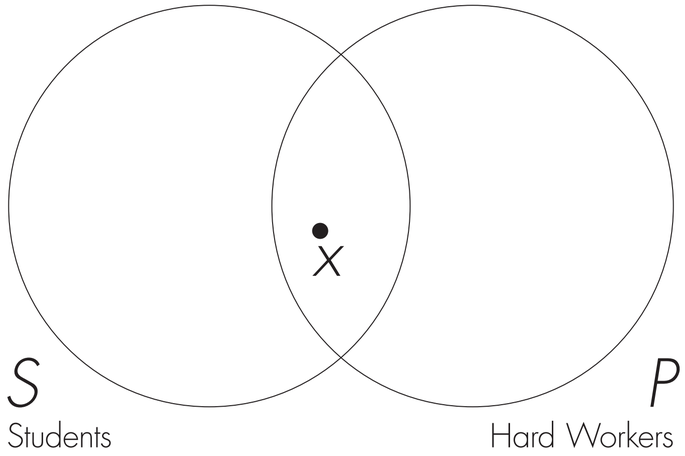


Exercise #1: *Inference Determined*

No students are hard workers.



Some students are hard workers.



The statement on the right is *false*.

The truth of the original statement (on the left) implies that there is nothing in the area of overlap between students and hard workers, which the other statement (on the right) clearly denies. So the statement on the right cannot be true.

Exercise #2

Assume that the following categorical statement is *false*:

No students are hard workers.

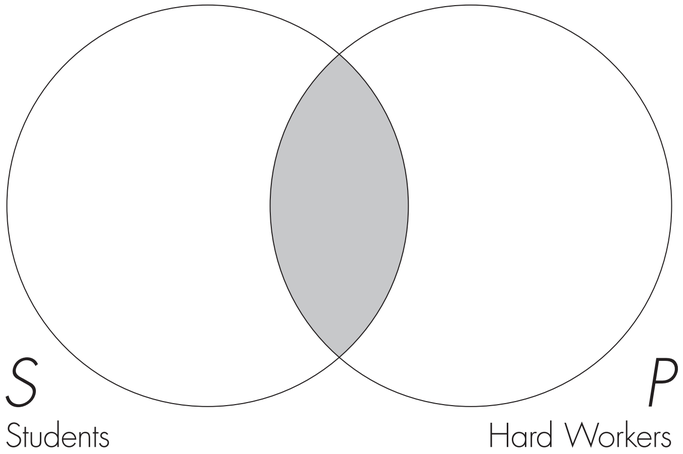
Given the truth of this statement, what can you infer about the following categorical statement?

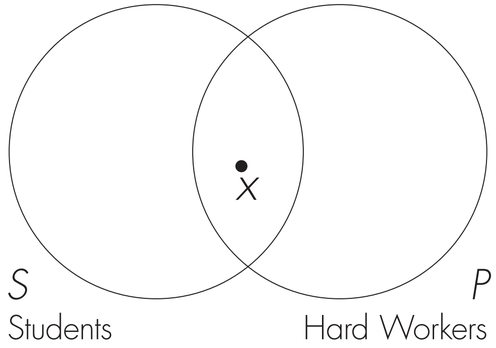
Some students are hard workers.

That is, is this second statement true, false, or unknown?

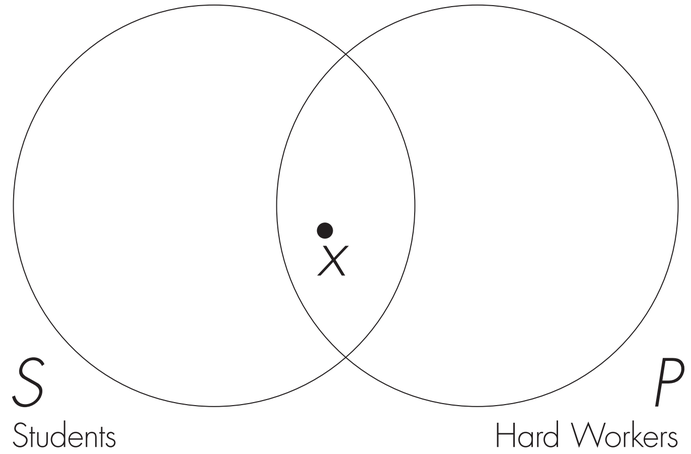
Exercise #2: *Venn Diagrams*

No students are hard workers.



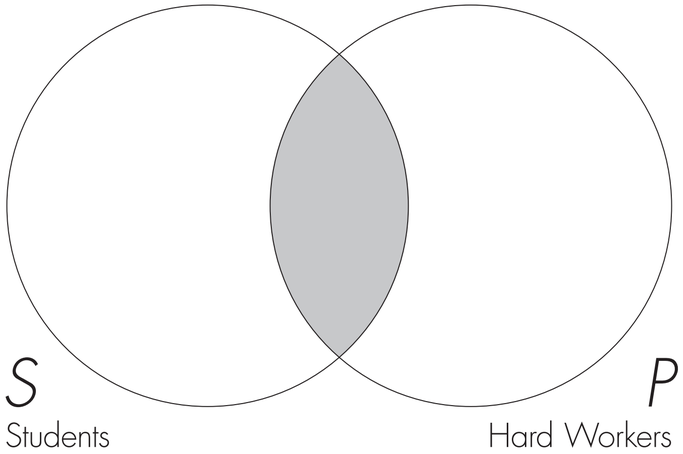


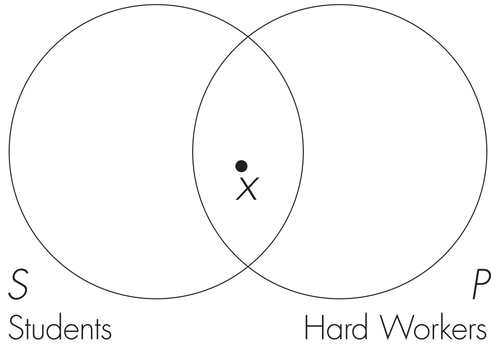
Some students are hard workers.



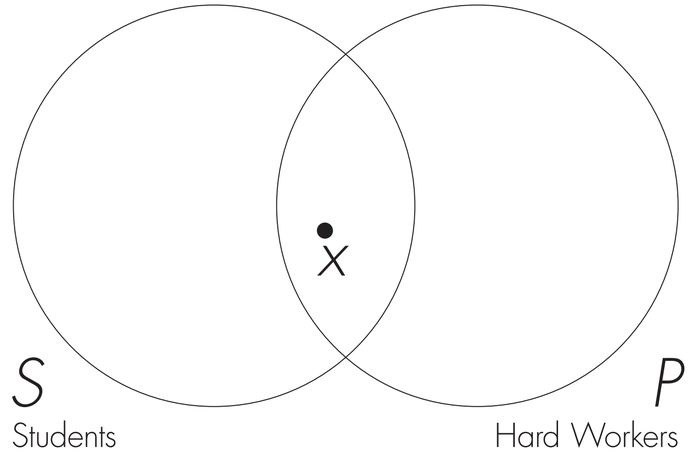
Exercise #2: *Inference Determined*

No students are hard workers.





Some students are hard workers.



The statement on the right is *true*.

The falsehood of the statement on the left implies that there is something in the area of overlap between students and hard workers, as seen in the middle Venn diagram. Indeed this middle diagram is the same as that for the truth of the statement on the right. So the statement on the right must be true.

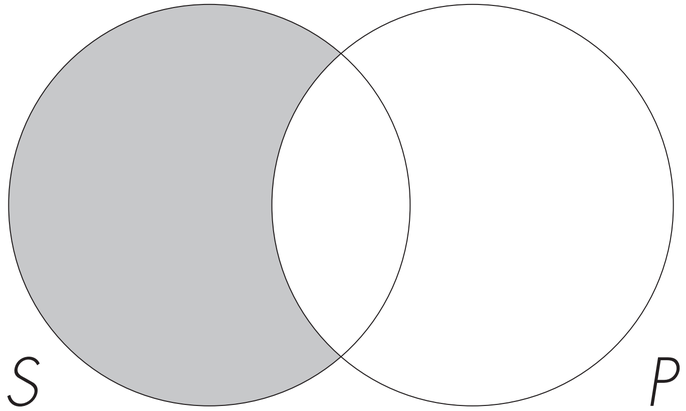
Categorical Statements: *Contradictories*

Two statements are **contradictories** if they both cannot be true at the same time *and* they both cannot be false at the same time.

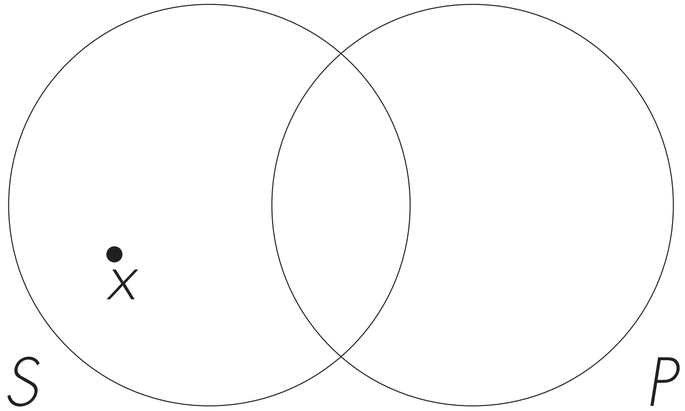
For instance, the statements "All students are hard workers" (**A**: All *S* is *P*) and "Some students are not hard workers" (**O**: Some *S* is not *P*) are contradictories. Both cannot be true and both cannot be false. So if you know one is true, the other must be false, and vice versa.

Contradictories: **A** & **O**

In general, **A** and **O** statements involving the same subject (*S*) and predicate (*P*) terms are always contradictories.



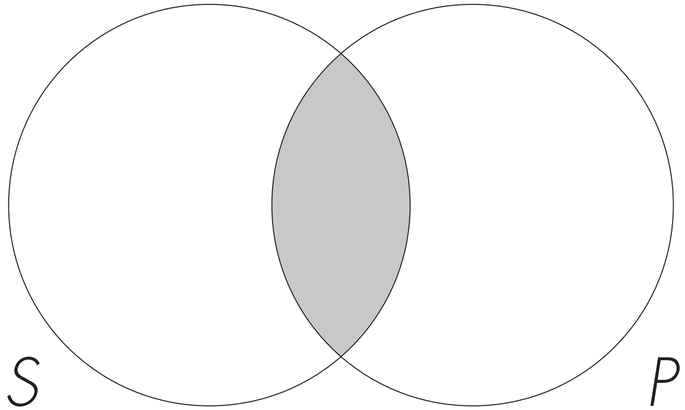
A Statement: All *S* is *P*.



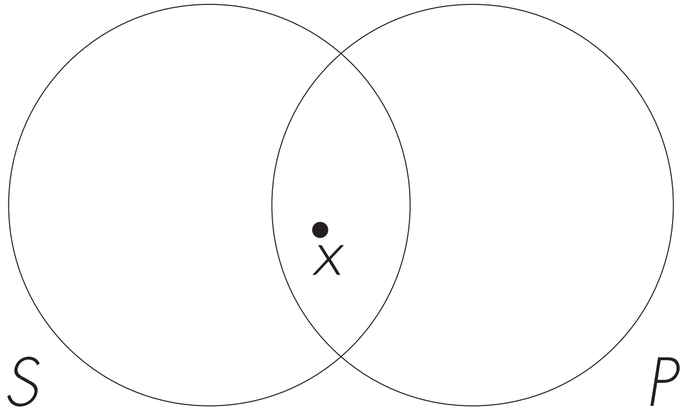
O Statement: Some *S* is not *P*.

Contradictories: **E** & **I**

Similarly, **E** and **I** statements involving the same subject (*S*) and predicate (*P*) terms are also always contradictories.



E Statement: No *S* is *P*.



I Statement: Some *S* is *P*.

Exercise #3

Assume that the following categorical statement is *true*:

All students are hard workers.

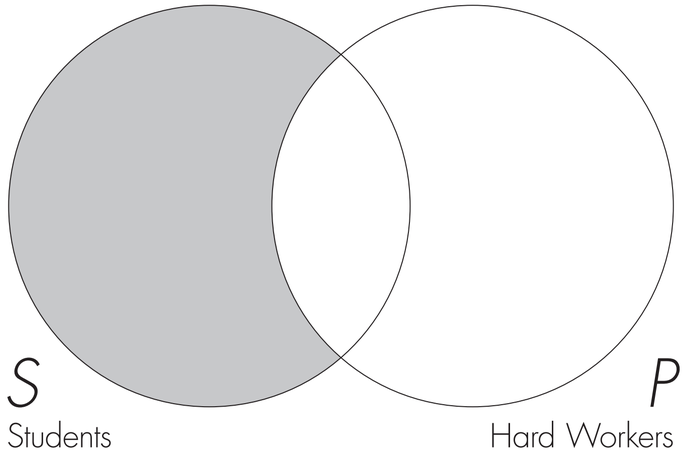
Given the truth of this statement, what can you infer about the following categorical statement?

No students are hard workers.

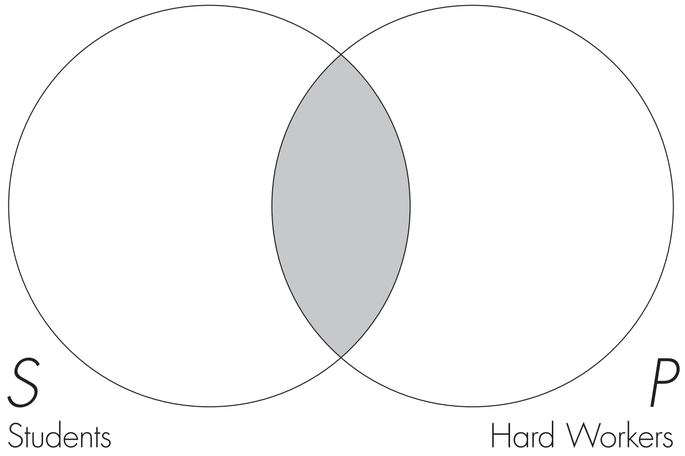
That is, is this second statement true, false, or unknown?

Exercise #3: *Venn Diagrams*

All students are hard workers.

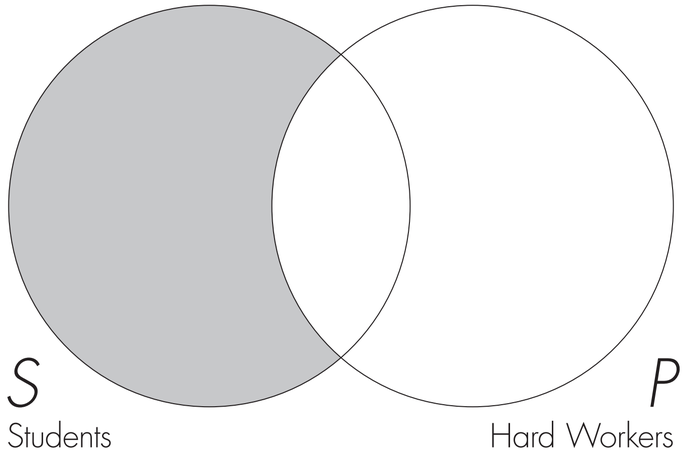


No students are hard workers.

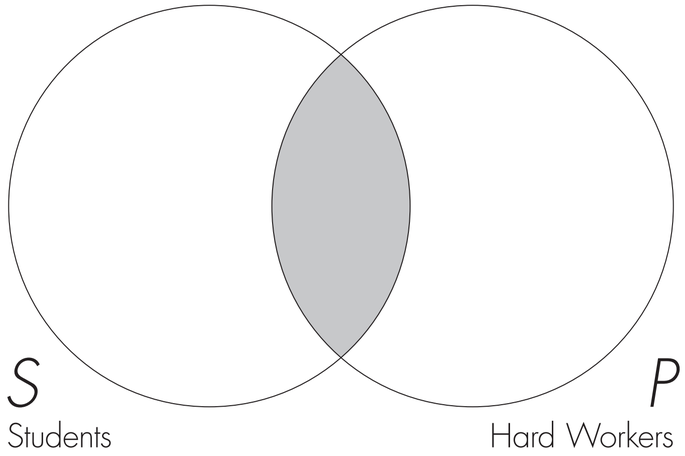


Exercise #3: *Inference Determined*

All students are hard workers.



No students are hard workers.



The statement on the right is *false*.

The truth of the statement on the left implies that there is nothing in the area of students outside of hard workers, and since there must be at least one student somewhere, that student must be in the area of overlap between students and hard workers. However, the Venn diagram for the statement on the right clearly denies this last point. So the statement on the right cannot be true.

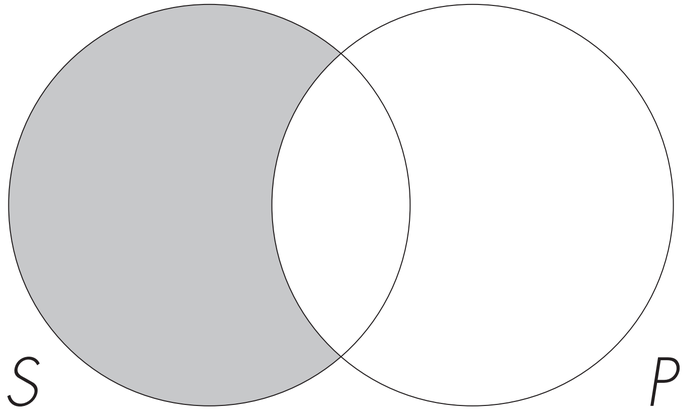
Categorical Statements: *Contraries*

Two statements are *contraries* if they both cannot be true, though both may be false.

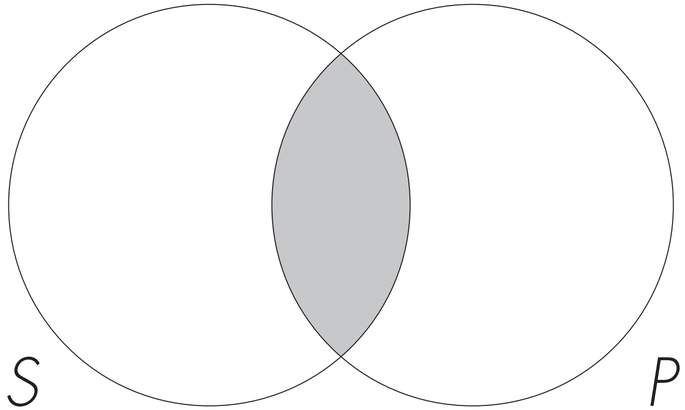
For instance, the statements "All students are hard workers" (**A**: All *S* is *P*) and "No students are hard workers" (**E**: No *S* is *P*) are contraries. Both cannot be true: if one is true, the other must be false. However, both statements could, in fact, be false: there might be some students who are hard workers and some other students who are not. (See the next slides for more explanation.)

Contraries: **A** & **E**

In general, **A** and **E** statements involving the same subject (*S*) and predicate (*P*) terms cannot both be true.*



A Statement: All *S* is *P*.

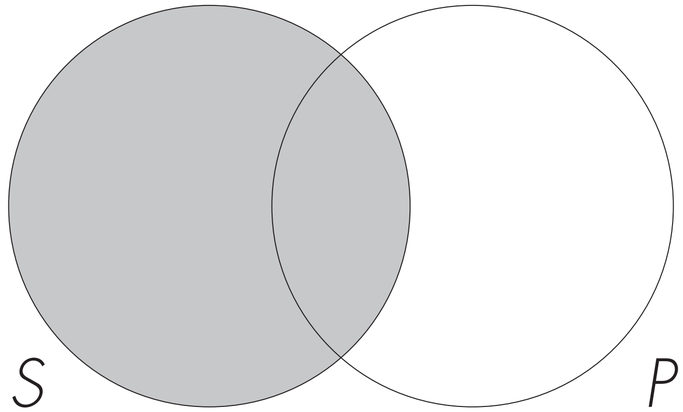


E Statement: No *S* is *P*.

*__Note:__ This only works when the subject term is not empty! Unless told otherwise, *always* assume the subject (*S*) term is not empty.

Contraries: **A** & **E**

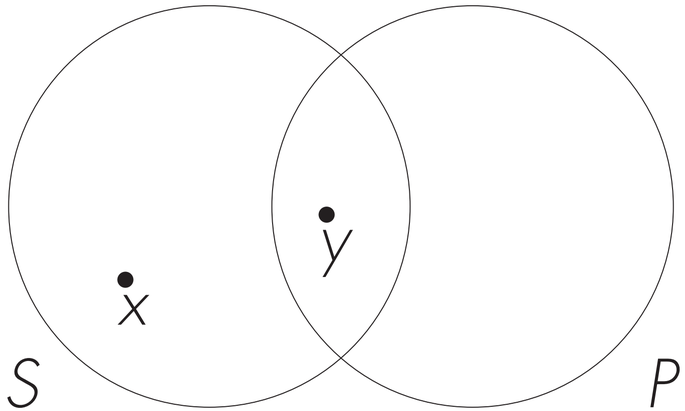
Both **A** and **E** statements cannot be true because that would mean the subject (*S*) term is empty.*



***Once more:** Unless told otherwise, always assume the subject (*S*) term is not empty.

Contraries: **A** & **E**

However, both **A** and **E** statements could be false. There is nothing problematic about that.



Therefore, **A** and **E** statements involving the same subject (*S*) and predicate (*P*) terms are contraries.

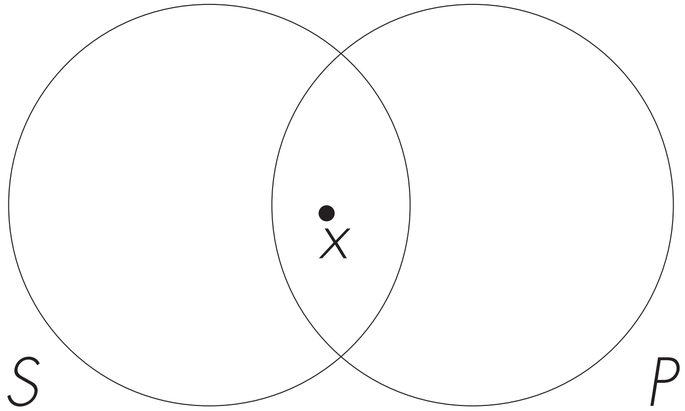
Categorical Statements: *Subcontraries*

Two statements are **subcontraries** if they both cannot be false, though they both may be true.

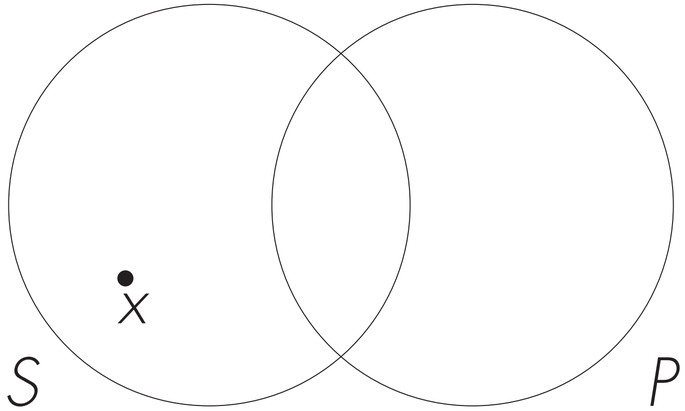
For instance, the statements "Some students are hard workers" (**I**: Some *S* is *P*) and "Some students are not hard workers" (**O**: Some *S* is not *P*) are subcontraries. Both cannot be false: if one is false, the other must be true. However, both statements could be true: as already noted, there might be some students who are hard workers and some other students who are not. (See the next slides for more explanation.)

Subcontraries: **I** & **O**

In general, **I** and **O** statements involving the same subject (*S*) and predicate (*P*) terms cannot both be false.*



I Statement: Some *S* is *P*.

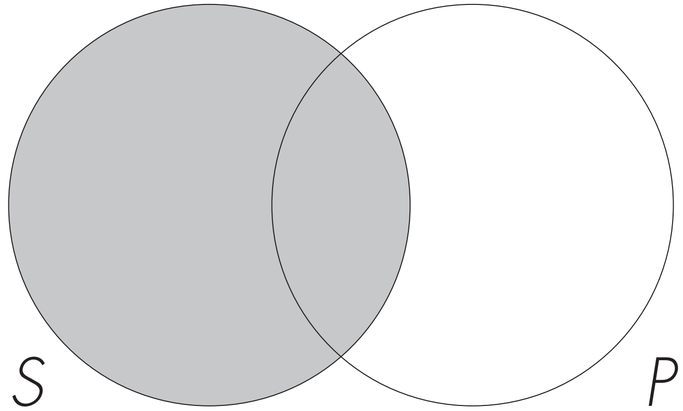


O Statement: Some *S* is not *P*.

*__Note:__ This also only works when the subject (*S*) term is not empty!

Subcontraries: **I** & **O**

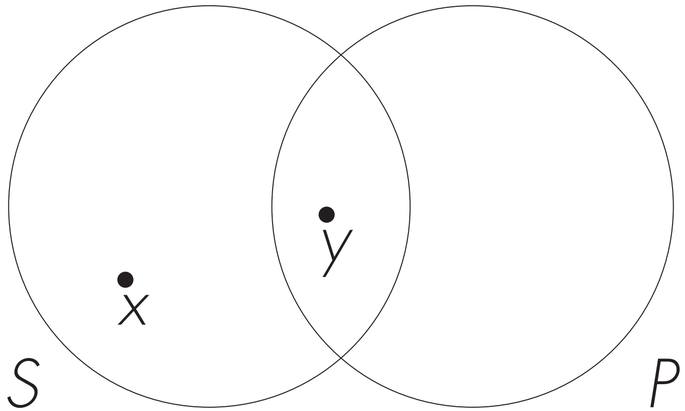
Both **I** and **O** statements cannot be false because that would mean the subject (*S*) term is empty.*



*__Once yet again:__ Unless told otherwise, *always* assume the subject (*S*) term is not empty.

Subcontraries: **I** & **O**

However, both **I** and **O** statements could be true. There is nothing problematic about that.



Therefore, **I** and **O** statements involving the same subject (*S*) and predicate (*P*) terms are subcontraries.

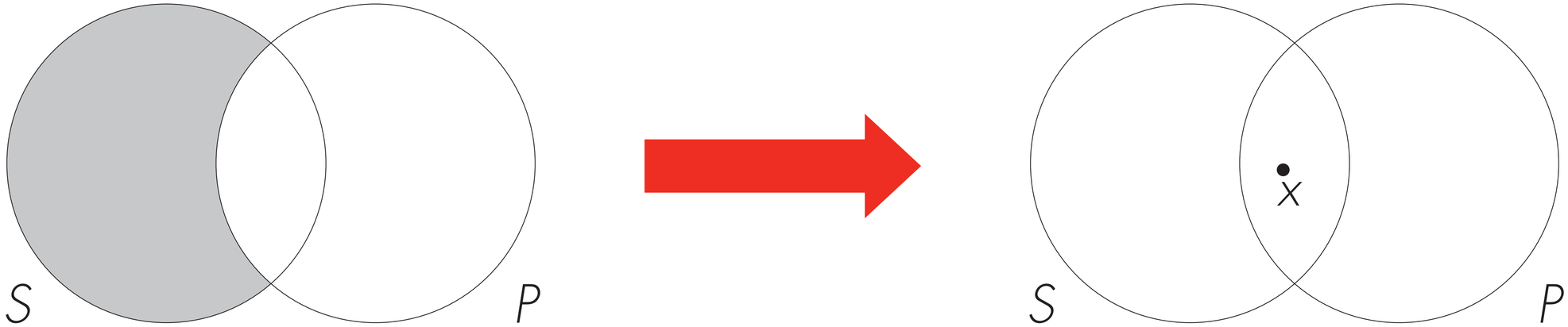
Categorical Statements: Subalternation

According to **subalternation**, any true *universal* categorical statement may be transformed into a true *particular* one. Going the other direction, subalternation says that any false particular categorical statement may be transformed into a false universal one.

So, for instance, if the statement "All students are hard workers" (**A**: All *S* is *P*) is true, then "Some students are hard workers" (**I**: Some *S* is *P*) is trivially true as well.

Subalternation: *From Truth of* **A** *to Truth of* **I**

In general, any true **A** statement may be transformed into a true **I** statement about the same subject (*S*) and predicate (*P*) terms.*



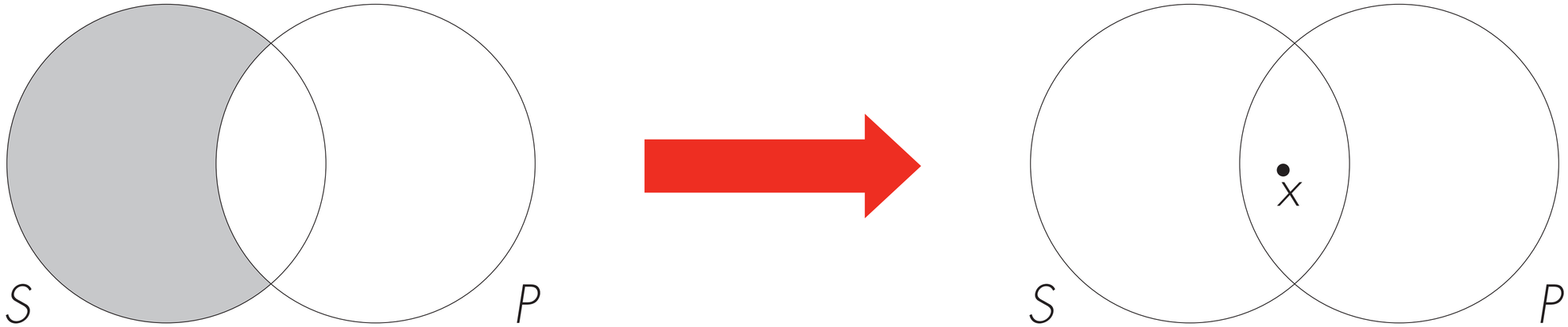
A Statement: All *S* is *P*.

I Statement: Some *S* is *P*.

*Note: This also only works when the subject (*S*) term is not empty!

Subalternation: *From Truth of* **A** *to Truth of* **I** *(Explanation)*

The idea is that there are no *S*'s outside of *P*, but those *S*'s have to be somewhere!* So we know that there is at least one *S* (call it *x*) inside of *P*.



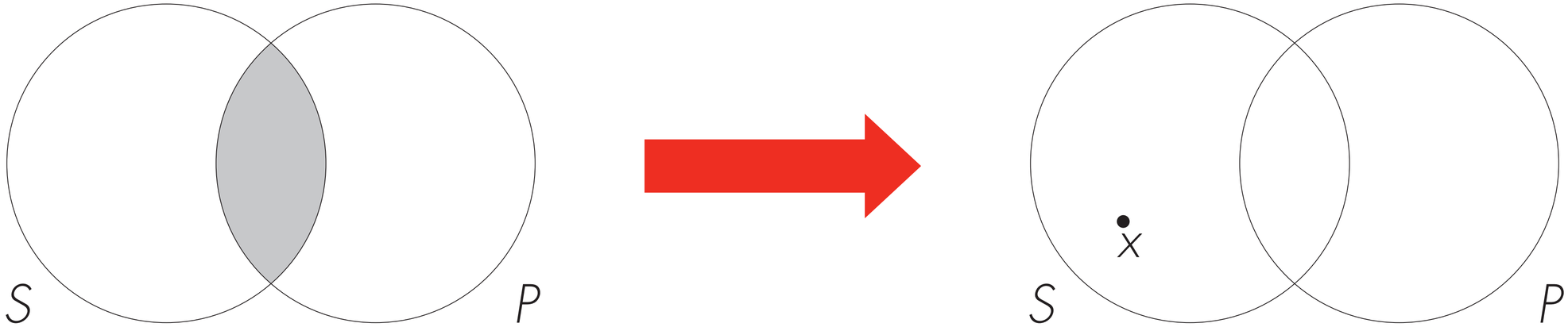
A Statement: All *S* is *P*.

I Statement: Some *S* is *P*.

*Note: This also only works when the subject (*S*) term is not empty!

Subalternation: *From Truth of* **E** *to Truth of* **O**

Similarly, any true **E** statement may be transformed into a true **O** statement about the same subject (*S*) and predicate (*P*) terms.*



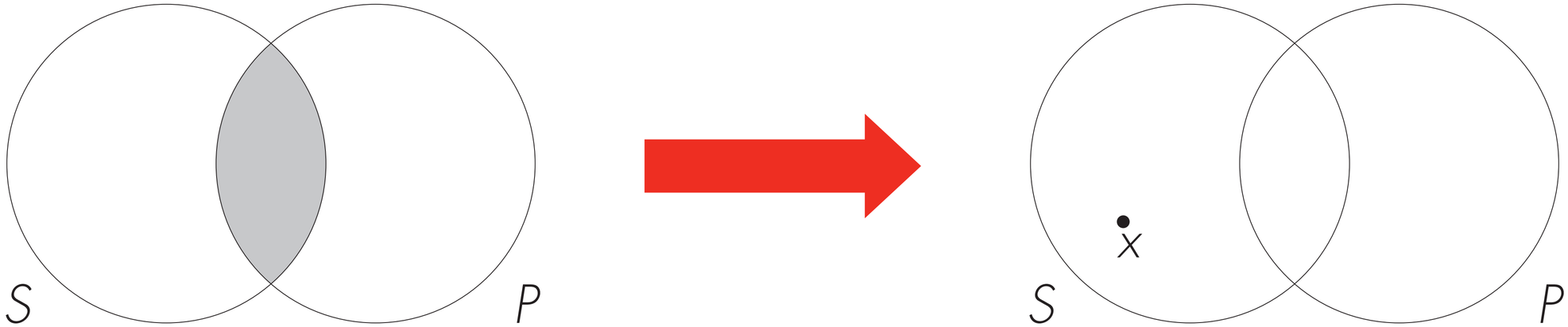
E Statement: No *S* is *P*.

O Statement: Some *S* is not *P*.

*__Note:__ This also only works when the subject (*S*) term is not empty!

Subalternation: *From Truth of* **E** *to Truth of* **O** *(Explanation)*

The idea is that there are no *S*'s inside of *P*, but those *S*'s have to be somewhere!* So we know that there is at least one *S* (call it *x*) outside of *P*.

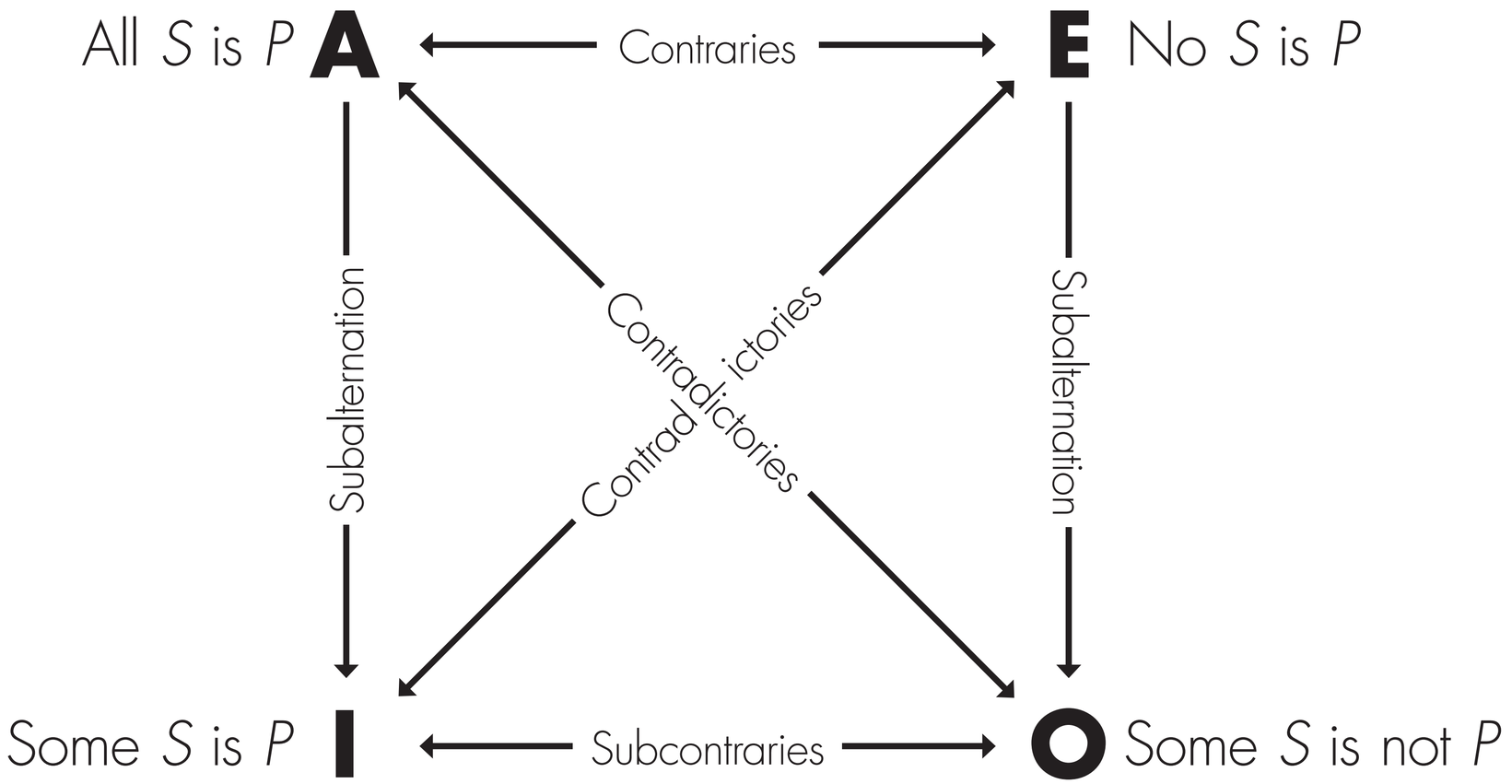


E Statement: No *S* is *P*.

O Statement: Some *S* is not *P*.

*__Note:__ This also only works when the subject (*S*) term is not empty!

The Square of Opposition



The Square of Opposition & Making Inferences

Fix the subject (*S*) and predicate (*P*) terms. The square of oppositions reveals these inferences:

– If **A** is true: **E** is false; **I** is true; **O** is false.
– If **A** is false: **O** is true; **E** and **I** are unknown.

– If **E** is true: **A** is false; **I** is false; **O** is true.
– If **E** is false: **I** is true; **A** and **O** are unknown.

– If **I** is true: **E** is false; **A** and **O** are unknown.
– If **I** is false: **A** is false; **E** is true; **O** is true.

– If **O** is true: **A** is false; **E** and **I** are unknown.
– If **O** is false: **A** is true; **E** is false; **I** is true.

The Square of Opposition & Making Inferences

The square of opposition contains a lot of useful information concerning what you can infer from a single categorical statement, but Venn diagrams provide more intuitive ways to figure out these inferences.

(So feel free to memorize the square, but only to check the work you have done with diagrams.)

Next Class…

We will look at some further inferences that can be made from a single categorical statement.