CRITICAL THINKING

Lecture #18

Categorical Statements

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Categories

Category: A collection or set of things. A category is denoted by italicized capital letters, e.g., *A*, *B*, *C*, . . .

E.g., *P* could be the category of professors teaching at CMU.

Element: A thing that is in a category. An element is denoted by italicized lowercase letters, e.g., *x*, *y*, *z*, . . .

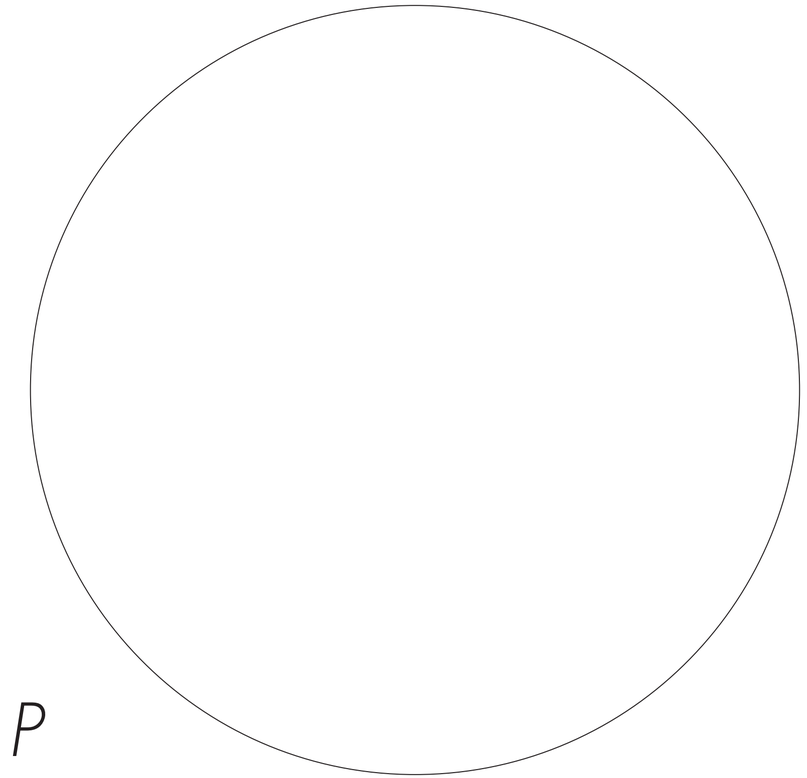
E.g., *x* could be Professor Gray, an element of category *P*.

Empty category: A category that contains no elements.

E.g., the category, *M*, of CMU professors living on Mars is (currently) an empty category.

Venn Diagrams

Categories may be diagrammed using circles called Venn diagrams. For instance, category *P* (of CMU professors) may be diagrammed:



Venn Diagrams

This diagram can also illustrate that category *P* (CMU professors) has at least one element *x* (Professor Gray):

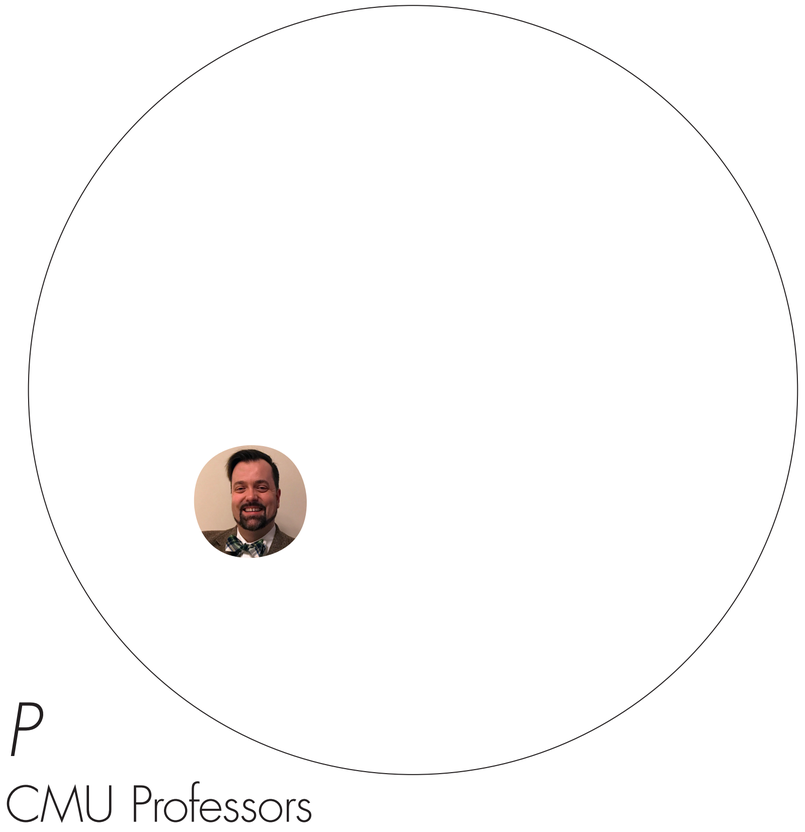
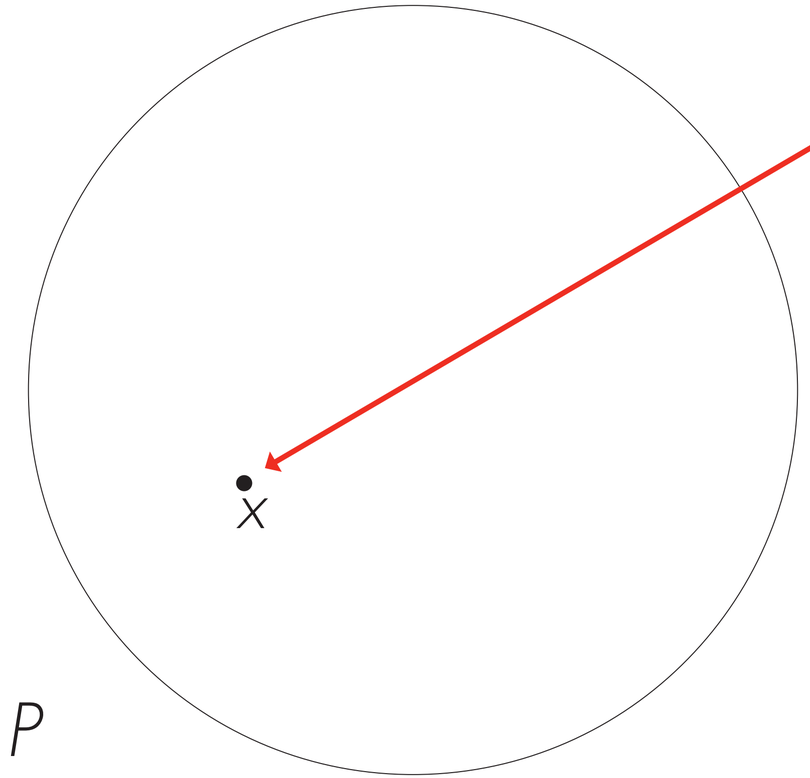


Image: J. A. Gray, 2017.

Venn Diagrams

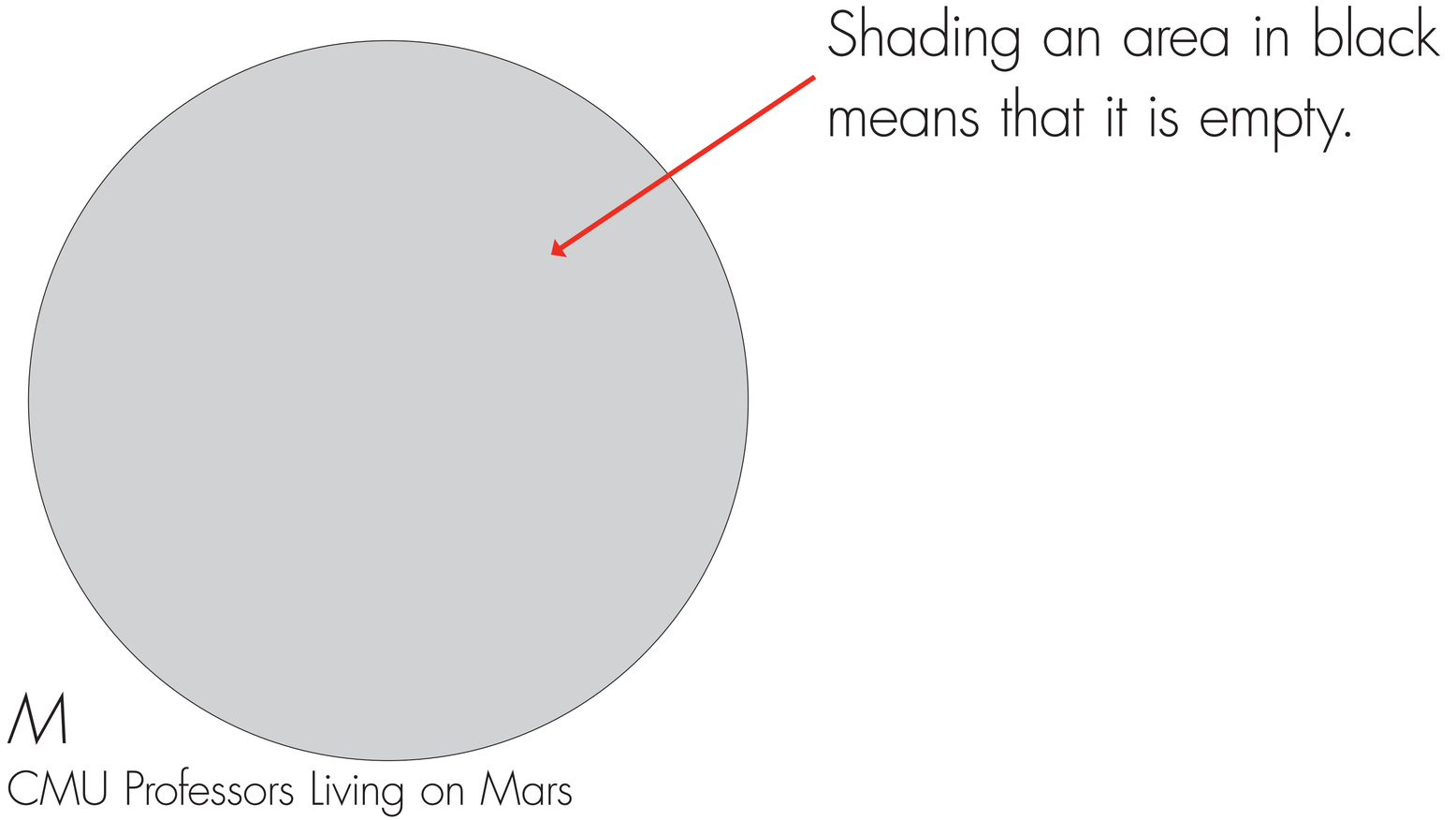
This diagram can also illustrate that category *P* (CMU professors) has at least one element *x* (Professor Gray):



Putting a dot-x (or any other lowercase italic letter) means that there is *at least one* thing in this area.

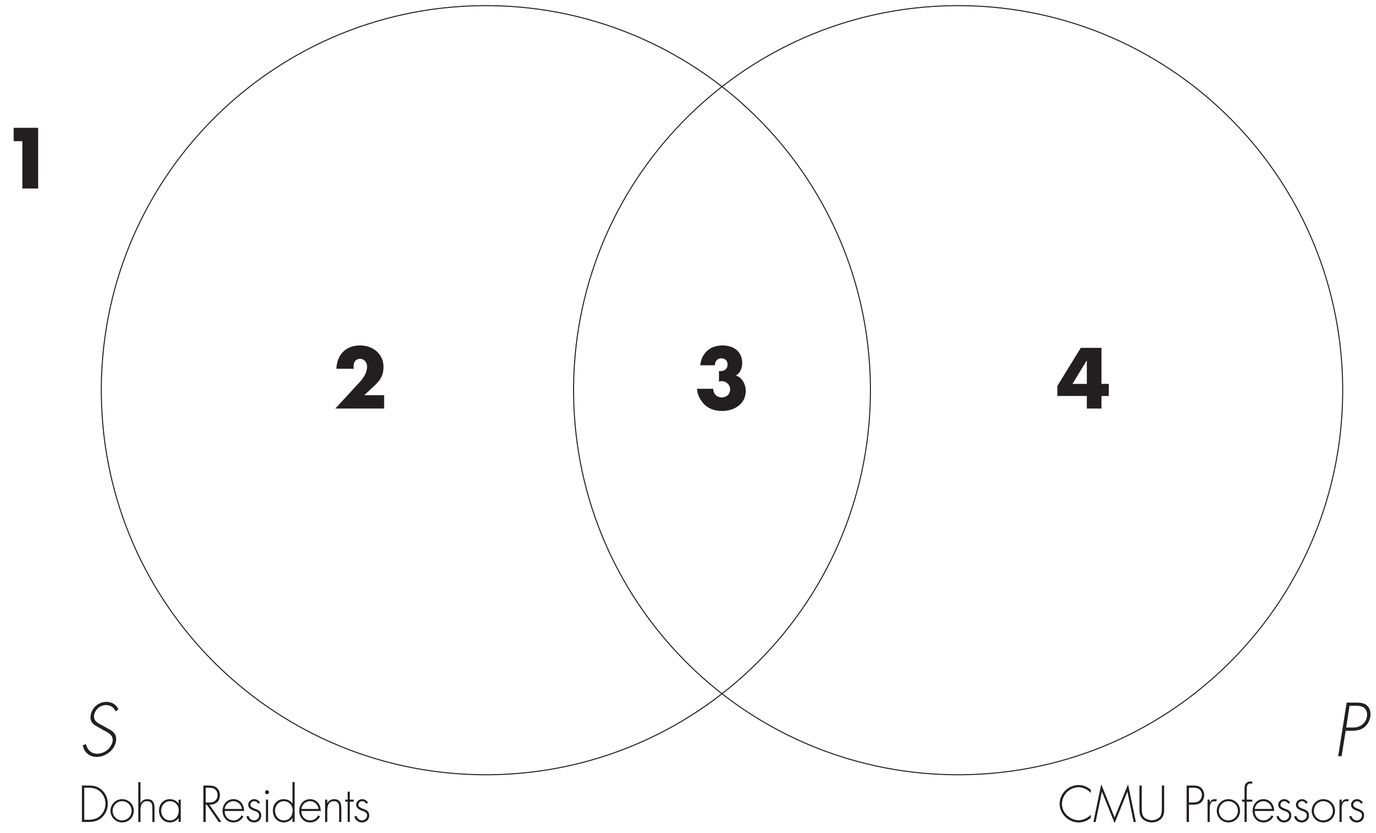
Venn Diagrams

Venn diagrams can also illustrate that category *M* (CMU professors living on Mars) is empty:



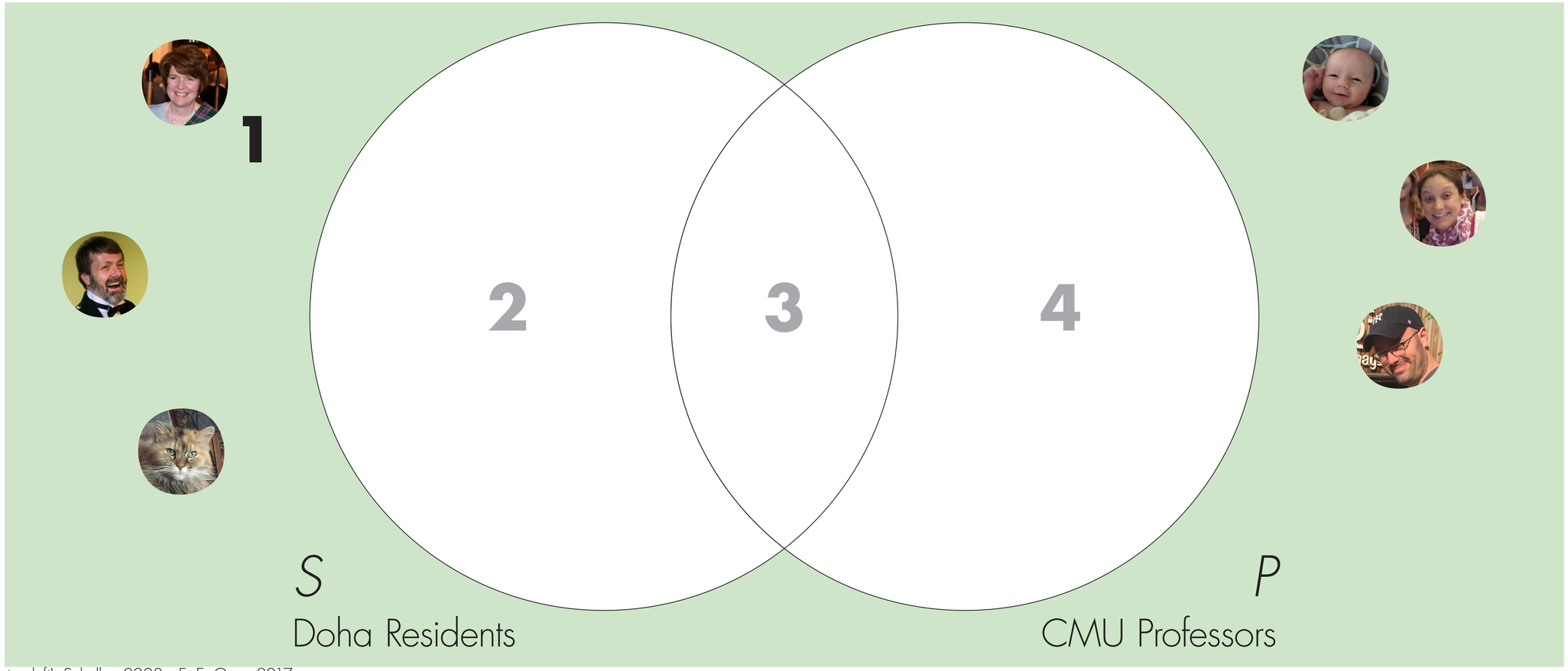
Venn Diagrams

A Venn diagram can also illustrate the possible relationships between two different categories. This is the diagram for categories *S* (Doha residents) and *P* (CMU professors):



Venn Diagrams

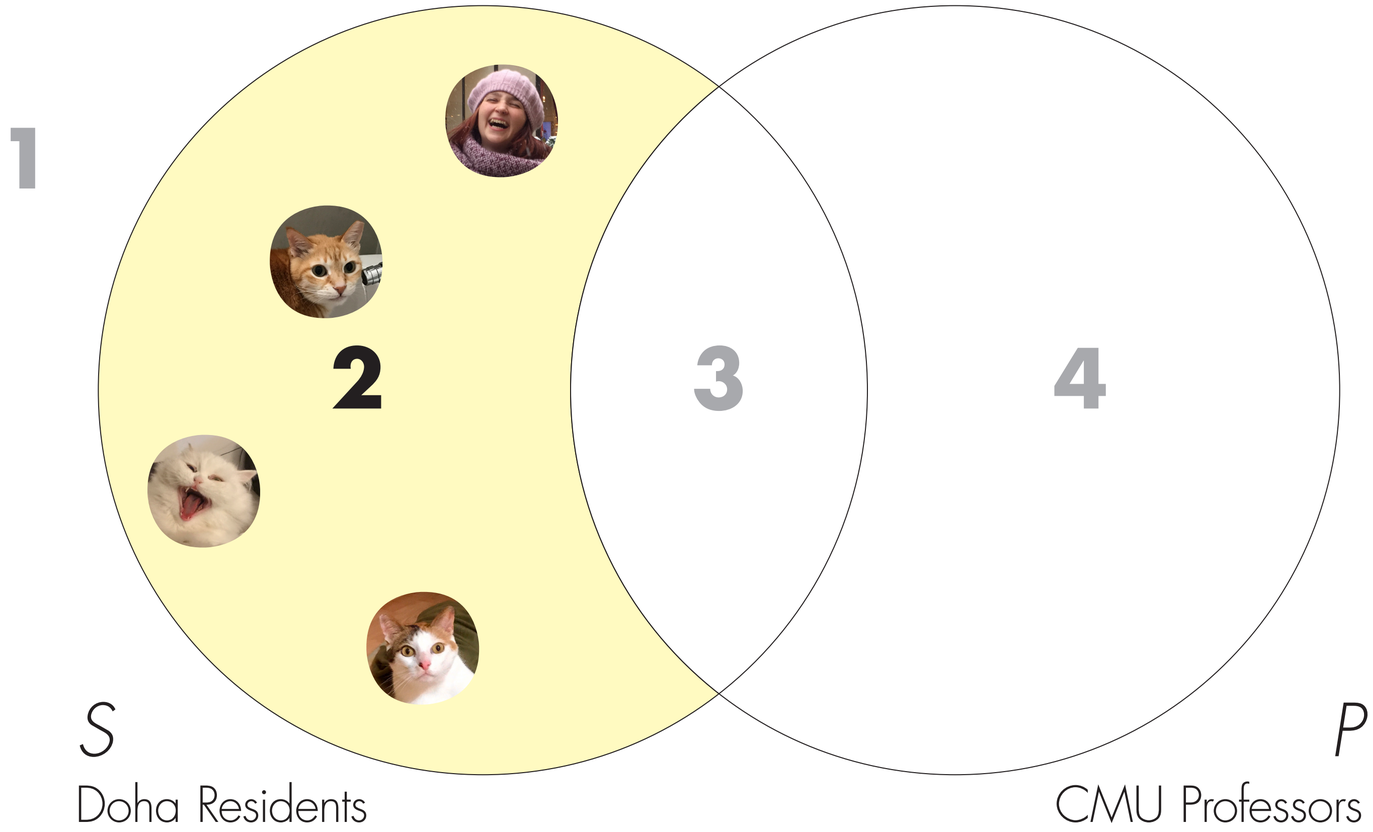
Zone 1 ("donut") has things that are outside of categories *S* (Doha residents) and *P* (CMU professors). It is made up of whatever is *neither* an *S nor* a *P*.



Images (clockwise from top-left): Scheller, 2008B; E. E. Gray, 2017; [Crista], 2017; D. E. Gray, 2016D, E; Scheller, 2008A.

Venn Diagrams

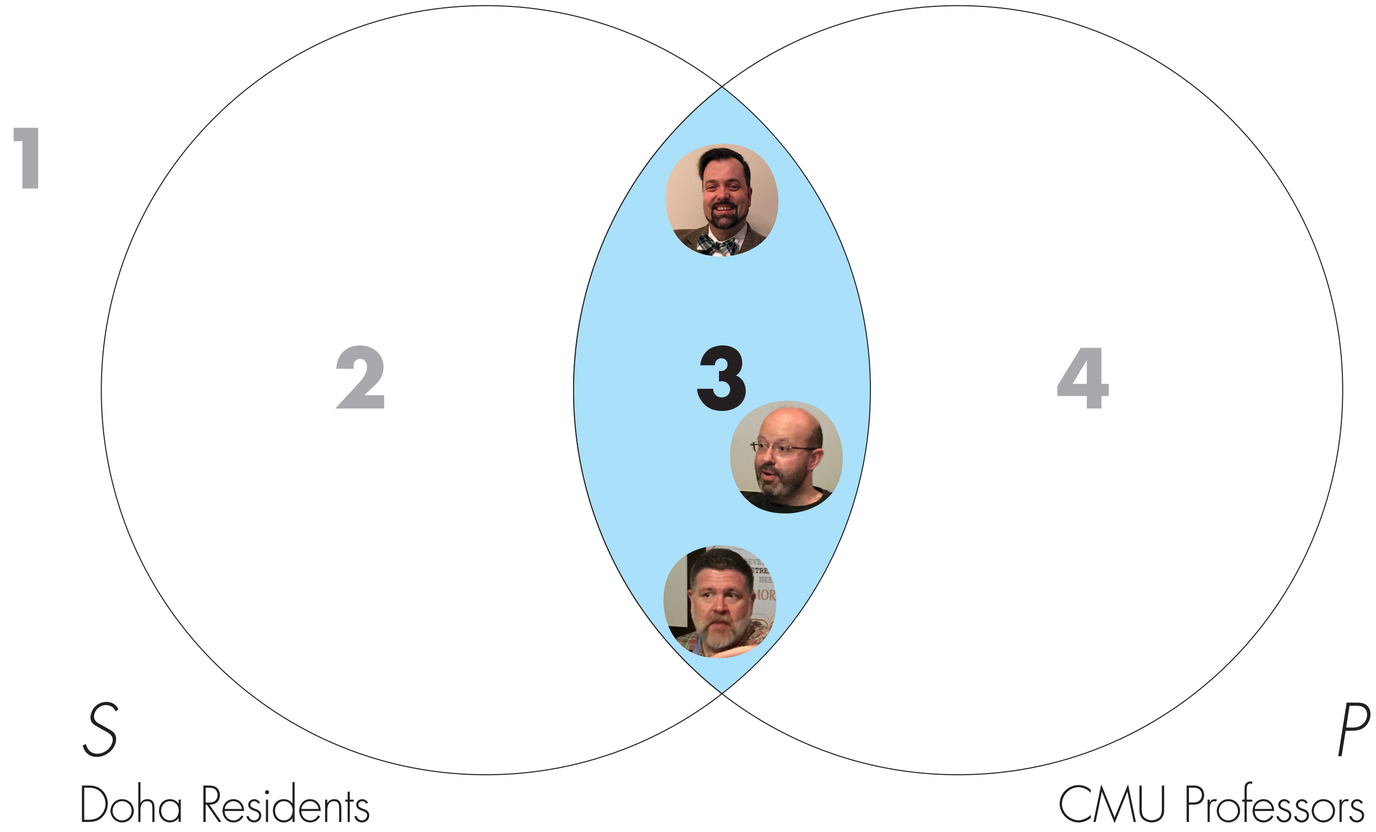
Zone 2 ("Pacman") has things that are inside of category *S* (Doha residents) but outside of category *P* (CMU professors). It is made up of whatever *is* an *S* but *not* a *P*.



Images (from top to bottom): D. E. Gray, 2016A; 2017B, C, A.

Venn Diagrams

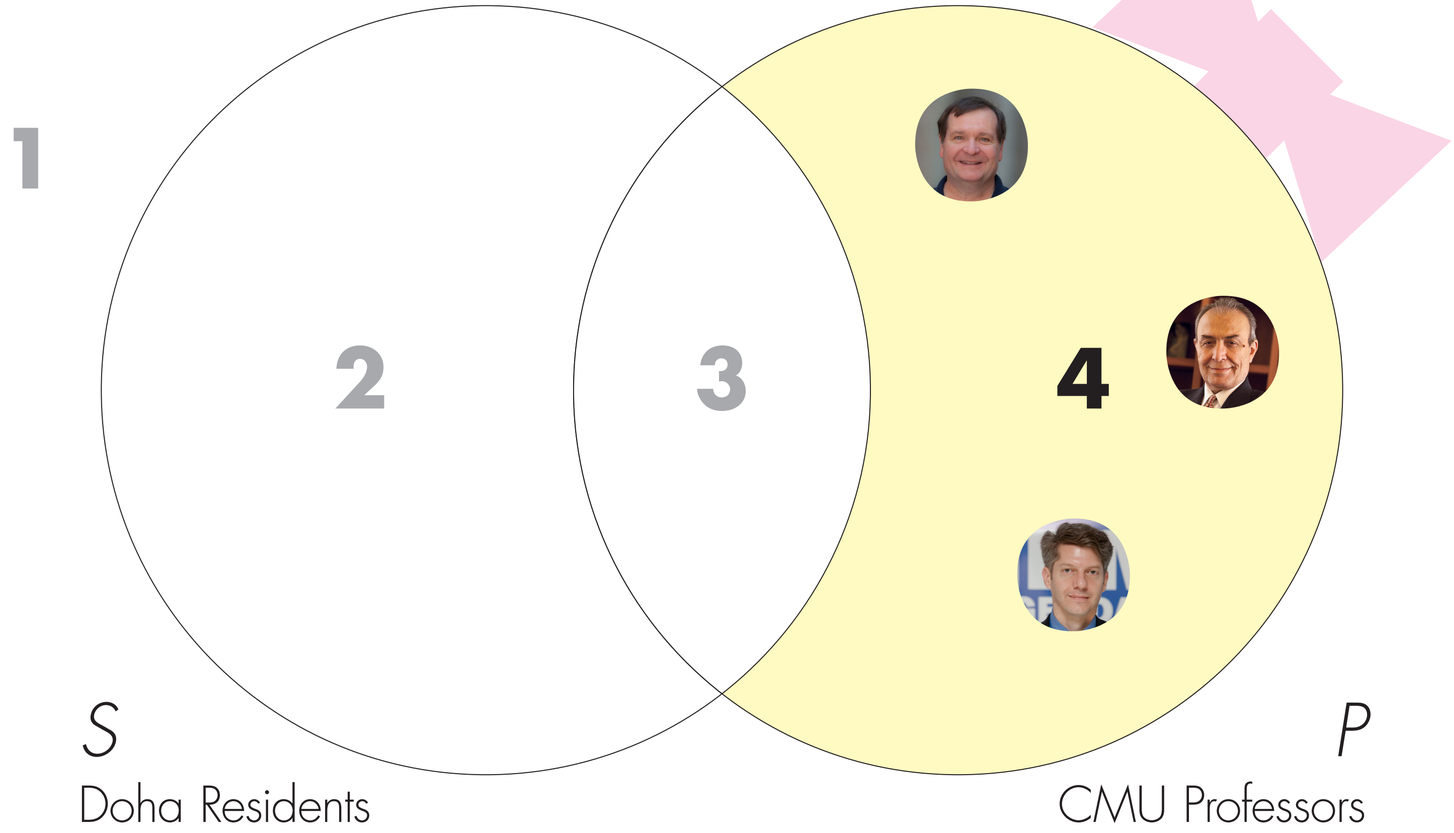
Zone 3 ("teardrop") has things that are inside both category *S* (Doha residents) and category *P* (CMU professors). It is made up of whatever is *both* an *S* *and* a *P*.



Images (from top to bottom): J. A. Gray, 2017; D. E. Gray 2016B, C.

Venn Diagrams

Zone 4 ("Ms. Pacman") has things that are outside of category *S* (Doha residents) but inside of category *P* (CMU professors). It is made up of whatever is *not* an *S* but *is* a *P*.



Images (from top to bottom): [Mark], n.d.; [Ilker], 2014; Wiesenhaan, 2013.

Categorical Statements

A **categorical statement** makes a claim concerning the relationship between two categories of things: the subject term (*S*) and the predicate term (*P*).

The **subject term** (*S*) names the main category the statement is about; the **predicate term** (*P*) names the category the statement is using to say something about that subject.

Categorical Statements: *Standard Forms*

There are four standard forms of categorical statements:

1. Universal Affirmative (**A**): All *S* is *P*.
2. Universal Negative (**E**): No *S* is *P*.
3. Particular Affirmative (**I**): Some *S* is *P*.
4. Particular Negative (**O**): Some *S* is not *P*.

Categorical Statements: Quality, Quantity, & Distribution

When analyzing a categorical statement, there are three questions to ask about it:

1. **Quantity:** Does it refer to *all* members of *S*, or only to *some* members of *S*?
   (E.g., if I pick an *S* at random, do I immediately know if it is also a *P* or not?)

2. **Quality:** Does the proposition *affirm* or *deny* (complete or partial) inclusion of *S* in *P*?

3. **Distribution:** Does it refer to *all* members of *P* or only to *some* members of *P*?
   (E.g., if I pick a *P* at random, do I immediately know if it is also an *S* or not?)

Analyzing Categorical Statements: *Instructions*

Analyzing the logical structure of categorical statements works as follows:

1. Identify the subject term (*S*) and predicate term (*P*),
2. Identifying its logical form (**A**, **E**, **I**, or **O**),
3. Draw the Venn diagram representing it (with the subject term (*S*) on the left and the predicate term (*P*) on the right), being sure to label the parts, and
4. Explain its quality, quantity, and distribution.

Analyzing Categorical Statements: *Universal Affirmative* (**A**)

All men are mortal.

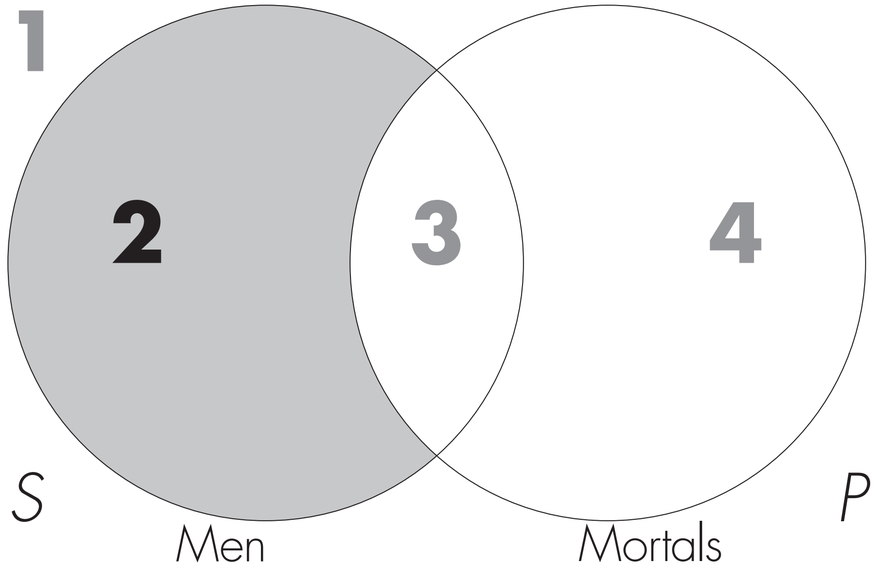
Subject (*S*): Men.

Predicate (*P*): Mortals.

Logical form: All *S* is *P*.

The idea is that the area of *S* that is *not* shared with *P* must be empty. All the *S*'s are inside *P*.

The Venn diagram:



For any statement of the form "All *S* is *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means all the *S*'s must be in that zone and all other *S* zones must therefore be empty. In this case, zone 2 is empty and so it is shaded in. Simply put: with an **A** statement, *shade in all zones of S not shared with* P.

Analyzing Categorical Statements: *Universal Affirmative* (**A**)

Quantity: Universal because it is referring to *all* the *S*'s. They are all *P*'s, as is seen in the Venn diagram. (In other words, if you were to pick a random *S*, you would immediately know that it is also a *P*.)

Quality: Affirmative because it *affirms* that *S*'s are also *P*'s. (In other words, this statement affirms that *S* is completely included inside of *P*.)

Distribution: *P* is *not* distributed because the statement only refers to *some* of the *P*'s, but not necessarily to all of them. As the Venn diagram shows, some of those *P*'s may be *S*'s, but some other *P*'s may not actually be inside *S*. (In other words, if you were to pick a random *P*, you would not immediately know if it was also an *S* or not.)

Analyzing Categorical Statements: *Universal Negative* (**E**)

No lawyers are honest.

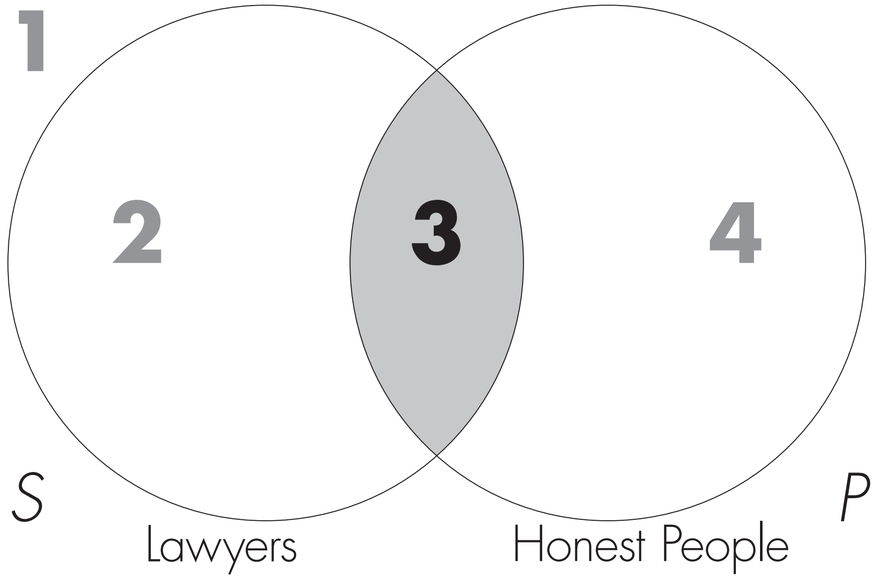
Subject (*S*): Lawyers.

Predicate (*P*): Honest people.

Logical form: No *S* is *P*.

The idea is that the area of *S* that *is* shared with *P* must be empty. None of the *S*'s are inside *P*.

The Venn diagram:



For any statement of the form "No *S* is *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means no *S*'s must be in that zone. In this case, zone 3 is empty and so it is shaded in. Simply put: with an **E** statement, *shade in all zones of* S *shared with* P.

Analyzing Categorical Statements: *Universal Negative* (**E**)

Quantity: Universal because it is referring to *all* the *S*'s. They are all not *P*'s, as is seen in the Venn diagram. (In other words, if you were to pick a random *S*, you would immediately know that it is not a *P*.)

Quality: Negative because it *denies* that *S*'s are also *P*'s. (In other words, this statement denies that *S* is included inside of *P*.)

Distribution: *P is* distributed because the statement only refers to *all* of the *P*'s. As the Venn diagram shows, they are all not *S*'s. (In other words, if you were to pick a random *P*, you would immediately know that it is not an *S*.)

Analyzing Categorical Statements: *Particular Affirmative* (I)

Some students are hard workers.

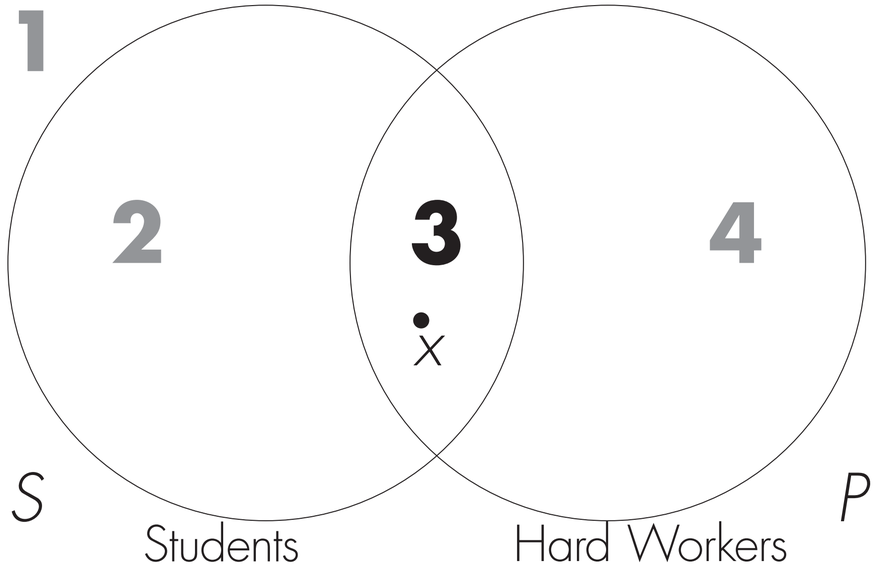
Subject (*S*): Students.

Predicate (*P*): Hard workers.

Logical form: Some *S* is *P*.

The idea is that the area of *S* that *is* shared with *P* must have something in it. At least one *S* is inside *P*.

The Venn diagram:



For any statement of the form "Some *S* is *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means that there is at least one *S* in that zone. In this case, zone 3 has something and so it has a dot-x inside it. Simply put: with an I statement, *a dot-x goes in the zone(s) of* S *shared with* P.

Analyzing Categorical Statements: *Particular Affirmative* (I)

Quantity: Particular because it is referring to *some* of the *S*'s, but not necessarily to all of them. Some *S*'s are *P*'s, but some other *S*'s may not be *P*'s, as is seen in the Venn diagram. (In other words, if you were to pick a random *S*, you would not immediately know if it was also a *P* or not.)

Quality: Affirmative because it *affirms* that at least one *S* is also a *P*. (In other words, this statement affirms that *S* is partially included inside of *P*.)

Distribution: *P* is *not* distributed because the statement only refers to *some* of the *P*'s, but not necessarily to all of them. As the Venn diagram shows, some *P*'s are *S*'s, but some other *P*'s may not be *S*'s. (In other words, if you were to pick a random *P*, you would not immediately know if it was also an *S* or not.)

Analyzing Categorical Statements: *Particular Negative* (O)

Some professors are not lazy.

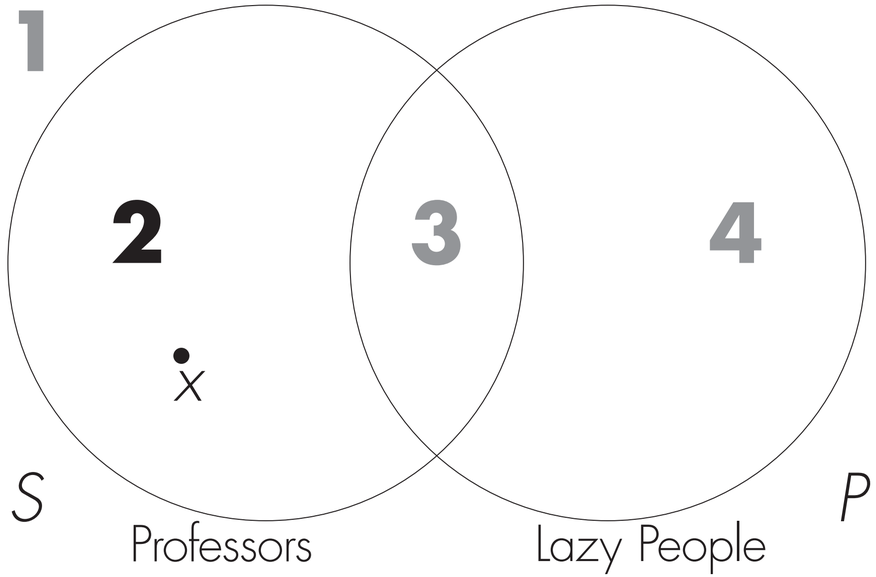
Subject (*S*): Professors.

Predicate (*P*): Lazy people.

Logical form: Some *S* is not *P*.

The idea is that the area of *S* that is *not* shared with *P* must have something in it. At least one *S* is not inside *P*.

The Venn diagram:



For any statement of the form "Some *S* is not *P*", look at the zone(s) *S* and *P* share. In this case, it is only zone 3. This means that there is at least one *S* outside that zone. In this case, zone 2 has something and so it has a dot-x inside it. Simply put: with an O statement, *a dot-x goes in the zone(s) of S not shared with* P.

Analyzing Categorical Statements: *Particular Negative* (O)

Quantity: Particular because it is referring to *some* of the *S*'s, but not necessarily to all of them. Some *S*'s are not *P*'s, but some other *S*'s may actually be *P*'s, as is seen in the Venn diagram. (In other words, if you were to pick a random *S*, you would not immediately know if it was also a *P* or not.)

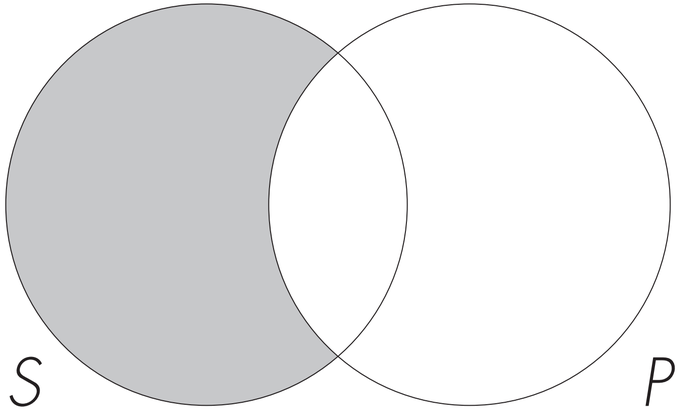
Quality: Negative because it *denies* that at least one *S* (marked with an *x*) is also a *P*. (In other words, this statement denies that part of *S* (i.e., *x*) is included inside of *P*.)

Distribution: *P is* distributed because the statement only refers to *all* of the *P*'s. As the Venn diagram shows, all *P*'s are not that one *S* marked with an *x*. (In other words, if you were to pick a random *P*, you would immediately know that it is not *x*.)

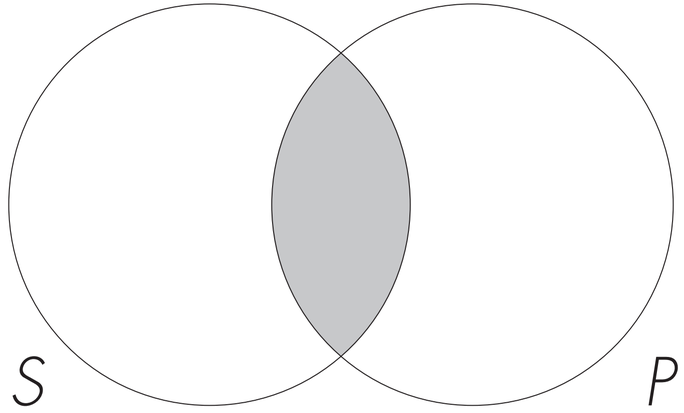
Categorical Statements

Just keep in mind, if you can draw a picture of the categorical statement in a Venn diagram, then you can much more easily analyze its logical structure.

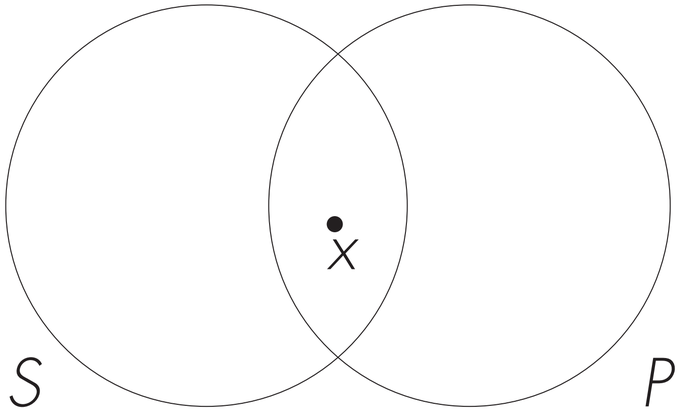
Four Standard Forms of Categorical Statements



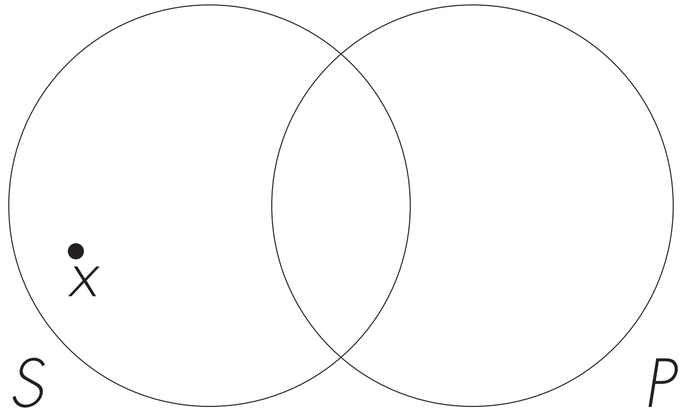
A: All *S* is *P*.
(Shade in all of *S* not shared with *P*.)



E: No *S* is *P*.
(Shade in all of *S* shared with *P*.)



I: Some *S* is *P*.
(Dot-*x* in *S* shared with *P*.)



O: Some *S* is not *P*.
(Dot-*x* in *S* not shared with *P*.)

Next Class…

We will examine more complicated forms of categorical propositions.

References

[Crista Crittenden]. (2017, July 27). [Digital image]. (Unpublished).

Gray, D. E. (2016A, March 4). [Jennifer Gray]. [Digital image]. (Unpublished).

Gray, D. E. (2016B, March 31). [John Gasper]. [Digital image]. (Unpublished).

Gray, D. E. (2016C, March 31). [Patrick McGinnis]. [Digital image]. (Unpublished).

Gray, D. E. (2016D, May 6). [Alex Cheek]. [Digital image]. (Unpublished).

Gray, D. E. (2016E, August 8). [Schilling cat]. [Digital image]. (Unpublished).

Gray, D. E. (2017A, July 6). [Sunny cat]. [Digital image]. (Unpublished).

Gray, D. E. (2017B, July 21). [Pumpkin cat]. [Digital image]. (Unpublished).

Gray, D. E. (2017C, July 23). [Snowflake cat]. [Digital image]. (Unpublished).

Gray, E. E. (2017, August 1) [Finn “Grapefruit Nostradamus” Demacek]. [Digital image]. (Unpublished).

Gray, J. A. (2017, March 28). [Professor Gray]. [Digital image]. (Unpublished).

[Ilker Baybars]. (2014). [Online image]. Retrieved August 2, 2017 from http://tepper.cmu.edu/our-faculty-and-research/about-our-faculty/faculty-profiles/baybars/baybars-ilker.

[Mark Stehlik]. (n.d.). [Online image]. Retrieved August 2, 2017 from https://www.cs.cmu.edu/~mjs/.

Scheller, D. (2008A, October 18). [Edward Gray]. [Digital image]. (Unpublished).

Scheller, D. (2008B, October 18). [Kay Gray]. [Digital image]. (Unpublished).

Wiesenhaan, H. (2013, October 31). [Dr. Alex John London attends the International Commission on Missing Persons (ICMP) Convention]. [Online image]. Getty Images. Retrieved August 2, 2017 from http://www.gettyimages.com/license/186464589.